MEAM 520 Robot Trajectories

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Lecture 14: November 1, 2012

Homework 4 due Friday 11/2

Homework 4: Velocity Kinematics and Jacobians

MEAM 520, University of Pennsylvania Katherine J. Kuchenbecker, Ph.D.

October 23, 2012

This assignment is due on **Friday, November 2 (updated)**, by 5:00 p.m. sharp. You should aim to turn the paper part in during class the day before. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. Late submissions will be accepted until 5:00 p.m. on Monday, November 5, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

Written Problems (60 points)

This entire assignment is written and consists of two significantly adapted problems from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions on both questions. Write in pencil, show your work clearly, box your answers, and staple your pages together.

- 1. Adapted SHV 4-20, page 160 Three-link Cylindrical Manipulator (30 points) The book works out the DH parameters and the transformation matrix T_3^0 for this robot on pages 85 and 86; you are welcome to use these results directly without rederiving them.
 - (a) Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for the three-link cylindrical manipulator of Figure 3.7 on page 85.
 - (b) Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.
 - (c) Find the 3 \times 3 angular velocity Jacobian J_{ω} for the same robot.
 - (d) Find this robot's 6×3 Jacobian J.
 - (e) Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $d_3 = -0.05$ m/s. What is v_3^0 , the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.
 - (f) For the same situation, what is ω_3^0 , the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.
 - (g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.
 - (h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

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- 2. Adapted SHV 4-18, page 160 Three-link Spherical Manipulator (30 points)
 - The book does not seem to work out the forward kinematics for this robot anywhere. Please use the diagram on the left side of Figure 1.12 on page 15 in SHV to define the positive joint directions and the zero configuration for the robot. If we additionally choose the x_0 axis to point in the direction the robot arm points in the zero configuration, you can calculate that the tip of the spherical manipulator is at $[x \ y \ z]^T = [c_1c_2d_3 \ s_1c_2d_3 \ d_1 s_2d_3]^T$. In this expression θ_1 , θ_2 , and d_3 are the joint variables; s_i is $\sin \theta_i$ and c_i is $\cos \theta_i$; and d_1 is a constant.
 - (a) Calculate the 3×3 linear velocity Jacobian J_v for the spherical manipulator with no offsets shown in the left side of Figure 1.12 on page 15 of SHV. You may use any method you choose.
 - (b) Find the 3 \times 3 angular velocity Jacobian J_{ω} for the same robot.
 - (c) Find this robot's 6×3 Jacobian J.
 - (d) Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0 , the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and \dot{d}_3 ? Make sure to provide units for any coefficients in these equations, if needed.
 - (e) For the same configuration described in the previous question, what is v_3^0 , the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and \dot{d}_3 ? Provide units for any coefficients in these equations, if needed.
 - (f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = [0 \text{ m/s} 0.5 \text{ m/s} 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.
 - (g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.
 - (h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.
 - (i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

3. Optional Extra Credit – Visualizing the Linear Velocity Jacobian (unknown points)

If you have time and interest, feel free to try this optional extra-credit problem. Modify your solution for the PUMA robot animation in Homework 3 (puma_robot_yourpennkey.m) in the following ways:

- Rename the file jacobian_yourpennkey.m
- Eliminate the spherical wrist, so that end-effector is at the origin of frame 3 (the wrist center).
- $\bullet\,$ Remove the offsets by setting b and d to zero. This should give you an articulated manipulator.
- Change the zero configuration as follows: when all three angles are zero, the arm should be horizontal and pointing in the direction of the positive x_0 axis. Although this is not what is shown in Figure 4.5 on page 145 in SHV, I think this is the zero configuration they used.
- Use the expression for J_{11} on page 144 in SHV to augment the visualization of the robot with three lines that go through the tip of the robot and show the direction in which the tip will move if you have only one non-zero joint velocity. Make the line for $\dot{\theta}_1$ red, the line for $\dot{\theta}_2$ green, and the line for $\dot{\theta}_3$ blue. Feel free to adjust other plotting parameters as needed.
- Check your solution with the provided motion modes, and feel free to create a new motion mode that showcases the Jacobian augmentation you added.

Submit your code as an attachment to an email to mean520@seas.upenn.edu with the subject Jacobian Extra Credit: Your Name, replacing Your Name with your name.



Project I : PUMA Light Painting



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MEAM.Design : MEAM520-12C-P01-Real

GENERAL	MEAM.Design - MEAM 520 - PUMA Light Painting: Reality
Hall of Fame	Once you have the PUMA painting in simulation, it's time to make the real robot paint with light. This assignment is due by 5:00 p.m. on Tuesday, November 6.
Laboratories	Your task is to record a light painting of the PUMA moving an LED around in space. Two steps are required in order to earn permission to work on the real robot:
Contact Into	1. Get trained to use the PUMA. Training takes 30 minutes and involves your whole team. See the plazza post on robot training.
COURSES MEAM 101	2. Get a TA's approval on your code. After you submit your simulation of the robot painting, a TA will respond with comments and suggestions to help you get your code ready to run on the real robot. You need to address their comments and resubmit. They will let you know when you have permission to run your code on the real robot.
MEAM 201	Submission
MEAM 410/510 MEAM 520 IPD 501 SAAST	 Make a folder named "final" inside your team's folder on the PUMA computer. Put all of your final code inside your "final" folder. Put the movie and time-lapse image inside the "final" folder as well.
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GUIDES	
Materials	
Laser Cutting	
3D Printing	
Machining	
ProtoTRAK	
PUMA 260	
PHANToM	
BeagleBoard	
MAEVARM	
Tap Chart	
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SOFTWARE	
SolidWorks	
Matlab	
NX	
Nastran	





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10pm							
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Events sh	nown in time zone: East	ern Time					🚹 Google Calenda
Towne Today	B2 Oct 28 - 	Nov 3, 2012 👻				Print Week	Month Agenda
	Sun 10/28	Mon 10/29	Tue 10/30	Wed 10/31	Thu 11/1	Fri 11/2	Sat 11/3
10am			10:30 - 12p	9:30 – 11 MEAM 347 Wind Tunnel	10 – Puma Training S		
11am		11 – 1p MEAM 245: Wind Tunnel	MEAM 247 MTS machine	11 - 12:30p MEAM 247 M	11 – Puma Training S		
12pm			12p - 3p MEAM 347 Wind Tunnel	Machi 12p – MEAM 5 12:30p – MEAM 520:	12p - 1:30p MEAM 347 Wind Tunnel		
1pm				1:30p - MEAM 520: P	1:30p - Pu1:30p - Pu		
2pm			· · · · ·	2p - MEAF 2p - MEAF	2p – Puma Training S		
3pm		3p – 4p MEAM 245 Wind Tunnel		3p – 6p MEAM 247 MTS machine	<mark>3p – 4:30p</mark> MEAM 347 Wind Tunnel		
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6pm		6p – MEAP6p – MEAP 6:30p – MI6:30p – MI	6p – MEAN 6p – MEAN 6:30p - 6:30p - 6:30p				
7pm		7p – MEAM 520: Pum 7:30p – MEAM 520: P					
Events sh	nown in time zone: East	ern Time					Google Calenda



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+ E https://docs.google.com/spreadsheet/viewform?formkey=dHhDVjNxUkp5bVdiTFVwWUpaR3ozdVE6MQ#gid=0



MEAM 520 Mid-Semester Course Evaluation

We appreciate your taking the time to complete this evaluation; your feedback will help us improve the class and our teaching for everyone's benefit. Please try to complete this survey before midnight on Friday, November 2. Your responses are anonymous, so you should feel comfortable being honest.

What is your overall rating of MEAM 520?

- O Don't Know
- 0: Poor
- 🔘 1: Fair
- 2: Good
- ③ 3: Very Good
- 4: Excellent

What is your overall rating of Professor Kuchenbecker?

- Don't Know
- O: Poor
- 1: Fair
- 2: Good
- 3: Very Good
- 4: Excellent

What is Professor Kuchenbecker doing well?

What specific things could Professor Kuchenbecker do to improve her teaching?

What is your overall rating of the teaching assistants in this class? Please answer "Don't Know" if you have not interacted with any of them yet this semester. The TAs are Philip, Denise, and Ryan

Confirmed Midterm Date Thursday, November 8, in class

Covers everything on Homework I through 4 plus Project I

Questions ?

Trajectory Planning

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Slides created by Jonathan Fiene





Chapter 5 in SHV

Configuration *q* Complete specification of the location of every point on the robot

Configuration Space *Q*

The set of all possible configurations

Collision

When any part of the robot contacts an obstacle in the workspace A(q)

Configuration Space Obstacle The set of configurations for which the robot collides with an obstacle

 $QO = \{q \in Q \mid A(q) \cap O \neq \emptyset\}$

Computationally

intensive to calculate! **Free Configuration Space** The set of all collision-free configurations $Q_{\text{free}} = Q \setminus QO$

What does the free configuration space look like for this round mobile robot (planar PP) with one small obstacle in the workspace?

y

What does the free configuration space look like for this square nonrotating mobile robot (planar PP) with one small obstacle in the workspace?

y



MATLAB simulation!

What does the free configuration space look like for this planar RR manipulator with one small obstacle in the workspace?

 θ_2



How do we prevent our robot from colliding with things?

Artificial Potential Fields

Treat robot as a point particle in the configuration space.

Robot feels forces from an artificial potential field U defined across its configuration space.

We design U to attract the robot to the desired final configuration and repel it from the boundaries of obstacles.

Want one global minimum at goal with no local minima. This is often really difficult to construct!

Potential Fields

Goal Attractive Force (Parabolic)

$$\mathbf{F}_{att,i}(q) = -\zeta(\mathbf{o}_i(q) - \mathbf{o}_i(q_f))$$

Obstacle Repulsive Force

$$\mathbf{F}_{rep,i}(q) = \eta_i \left(\frac{1}{\rho_i} - \frac{1}{\rho_{0,i}}\right) \frac{1}{\rho_i^2} \frac{\mathbf{o}_i(q) - \mathbf{b}}{||\mathbf{o}_i(q) - \mathbf{b}|}$$



MATLAB simulation!

Limit forces when very far away from goal.

Finite region of influence.

Potential Fields

Goal Attractive Force (Parabolic)

$$\mathbf{F}_{att,i}(q) = -\zeta(\mathbf{o}_i(q) - \mathbf{o}_i(q_f))$$

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Limit forces when very far away from goal.

Finite region of influence.

How do we apply these forces to the robot?

Jacobian Transpose

The transpose of the linear velocity Jacobian relates joint torques and forces to Cartesian end-effector forces



Where does this come from?

Principle of Virtual Work

$$\vec{\tau} \cdot d\vec{q} = \vec{F} \cdot d\vec{x}$$

$$\vec{\tau}^T d\vec{q} = \vec{F}^T d\vec{x}$$

$$\vec{\tau}^T d\vec{q} = \vec{F}^T J_v d\vec{q}$$

$$\vec{\tau}^T = \vec{F}^T J_v$$

$$\vec{\tau} = J_v^T \vec{F}$$



Beginning with the standard Jacobian

$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

We can solve for the joint torques necessary to exert a desired force at the end effector using the Jacobian transpose

$$\tau = \mathbf{J}^{\top}(q) \mathbf{F}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -a_1s_1 - a_2s_{12} & a_1c_1 + a_2c_{12} \\ -a_2s_{12} & a_2c_{12} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

This is really useful!

Chapter 5 goes into more detail on potential fields and then explains **probabilistic road maps** and **trajectory planning**.

Who is interested in these topics?