

MEAM 520

More Velocity Kinematics

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General Robotics, Automation, Sensing, and Perception Lab (GRASP)
MEAM Department, SEAS, University of Pennsylvania





CARBON DANCE THEATRE PRESENTS

SCIENCE PER FORMS

OCTOBER 25 (7:30PM)
OCTOBER 27 (7:30PM)
OCTOBER 28 (2:30PM)

CHRIST CHURCH NEIGHBORHOOD HOUSE
20 NORTH AMERICAN STREET, PHILADELPHIA
[OFF OF MARKET STREET AND 2ND AVENUE]

\$20 GENERAL, \$20 SENIOR, \$15 STUDENT & DANCEPASS HOLDERS



DANCE WITH CYBORGS



Carbon Dance Theatre presents a new dance piece by co-choreographers Meredith Rainey & Marcel Williams Foster exploring digital technology and robotics through dance.

Featuring Digital Technology and Robotic Collaborators:

Simon Kim
School of Design (Architecture), UPenn

Mark Yin
Mechanical Engineering & Applied Mechanics, UPenn

Georgia Guthrie
Jewish, The Hacktory



PARTY WITH US: THE CARBON CLUB

Join Carbon Dance Theatre for our annual fundraiser, The Carbon Club, a spin-off of the whimsical speak-easy 'The Carbon Club'. Get ready to dance and party the night away with live entertainment, auction, raffle, food, and drinks in the best juke joint in town.

Time Bar and Taproom
Monday, October 29th, starting at 8:30pm
\$55: General Admission
\$85: General Admission and Two Discounted Tickets

DANCE WITH US: SCIENCE PER FORMS

Thursday October 25 at 7:30PM
Talkback with Megan Bridge, <rdg> space

Saturday October 27 at 7:30PM
Talkback with Sharon Norris, <rdg> productions

Sunday October 28 at 2:30PM

Performances at Christ Church Neighborhood House
25 North American Street, off of Market and 2nd Ave
\$25 general, \$20 Senior, \$15 Student & DancePass Holders
Buy tickets here: tinyurl.com/scifdance

THINK WITH US: SYMPOSIUM BRYN MAWR COLLEGE

October 27th at 2:30PM at Bryn Mawr College
Haffner Hall, Dorothy Vernon Room. More info: tinyurl.com/bkwhall

Linda Caruso-Haviland, Director of Dance at Bryn Mawr College, will moderate a discussion on digital technologies and robotics in contemporary time and movement based art forms.

John Baker, Choreographer & Media Artist
Anne Barnes, Associate Professor of Architecture, Harvard University
Georgia Guthrie, Director of The Hacktory, Philly's First Hacker Space

This free event will feature hors d'oeuvres and beverages, no RSVP necessary.

CarbonDanceTheatre

CARBON DANCE THEATRE PRESENTS

SCIENCE PER FORMS

OCTOBER 25 (7:30PM)

OCTOBER 27 (7:30PM)

OCTOBER 28 (2:30PM)

CHRIST CHURCH NEIGHBORHOOD HOUSE
10 NORTH AMERICAN STREET, PHILADELPHIA
[OFF OF MARKET STREET AND 2ND AVENUE]

\$25 GENERAL, \$20 SENIOR, \$15 STUDENT & DANCEPASS HOLDERS

SYMPOSIUM AT BRYN MAWR COLLEGE: OCTOBER 26 (2:30PM)

FOR MORE INFORMATION VISIT: WWW.CARBONDANCETHEATRE.ORG

BRYN MAWR



MASCHER SPACE



PennDesign



THE HACKTORY



DanceUP
Dance/USA Philadelphia

GRASP
LABORATORY





Science per Forms

October 25, 27 & 28, 2012
Neighborhood House Theater, Christ Church
Philadelphia, PA
\$25 general admission \$20 seniors \$15 students and
dancepass holders

Showtimes:

Thursday 10/25 @ 7:30PM

Saturday 10/27 @ 7:30PM

Sunday 10/28 @ 2:30PM

[for tickets click here](#)

More information on Carbon Dance Theatre's



CARBON DANCE THEATRE creates projects that empower performers and entertain audiences
by creating work that is rooted in classical ballet and infused with the collaborative process of theatre.

meredith@carbondancetheatre.com | 2920 Cambridge Street Philadelphia, PA 19130 | graphic design: tori lawrence | www.torilawrence.com



p01-ik — meam520@seas.upenn.edu (40 messages)

MAILBOXES

- Inbox
 - kuchenbe@seas.upenn.edu
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 - hw02
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From	Subject	Date Received	Time	Attachments	Count
qiong@seas.upenn.edu	ik	Oct 16, 2012	5:01 PM	1 item	1 item
→ shengj@seas.upenn.edu	Project 1: Team04	Oct 16, 2012	1:14 AM	1 item	1 item
↩ shengj@seas.upenn.edu	Project 1: Team04 (Resubmit)	Today	4:44 AM	1 item	1 item
jonghak@seas.upenn.edu	PUMA IK : team19	Oct 16, 2012	4:58 PM	6 items	6 items
Dalton Banks	PUMA IK: Team 00	Oct 16, 2012	5:00 PM	2 items	2 items
yuluo@seas.upenn.edu	PUMA IK: Team 01	Oct 16, 2012	2:13 PM	5 items	5 items
Thomas Koletschka	PUMA IK: Team 02	Oct 16, 2012	4:51 PM	1 item	1 item
Jonathan Balloch	PUMA IK: Team 05	Oct 16, 2012	5:02 PM	9 items	9 items
chao qu	PUMA IK: Team 06	Oct 16, 2012	3:10 PM	15 items	15 items
chao qu	PUMA IK: Team 06	Oct 16, 2012	8:33 PM	16 items	16 items
amirand@seas.upenn.edu	PUMA IK: Team 07	Oct 16, 2012	4:59 PM	5 items	5 items
Alex Sher	PUMA IK: Team 08	Oct 16, 2012	4:25 PM	4 items	4 items
→ Zhongqi Yue	PUMA IK: Team 08 Resubmission	Oct 16, 2012	5:21 PM	4 items	4 items
Vivian Chu	PUMA IK: Team 09	Oct 16, 2012	4:58 PM	8 items	8 items
Michael Lo	PUMA IK: Team 10	Oct 16, 2012	3:14 PM	1 item	1 item
James Yang	PUMA IK: Team 12	Oct 16, 2012	3:24 PM	1 item	1 item
Shaojun ZHU	PUMA IK: Team 13	Oct 16, 2012	5:07 PM	4 items	4 items
Jesse E. Morzel	PUMA IK: Team 14	Oct 16, 2012	4:58 PM	4 items	4 items
Collin Boots	PUMA IK: Team 15	Oct 16, 2012	4:03 PM	1 item	1 item
Dieter Neckermann	PUMA IK: Team 16	Oct 16, 2012	4:10 PM	4 items	4 items
Sawyer Brooks	PUMA IK: Team 17	Oct 16, 2012	5:03 PM	6 items	6 items
siyuanz@seas.upenn.edu	PUMA IK: Team 20	Oct 16, 2012	12:23 PM	6 items	6 items
Romaine Waite	PUMA IK: Team 21	Oct 16, 2012	4:58 PM	1 item	1 item
Tyler Barkin	PUMA IK: Team 21	Oct 16, 2012	6:50 PM	1 item	1 item
Hardik Gupta	PUMA IK: Team 22	Oct 16, 2012	4:39 PM	4 items	4 items
Alex Jose	PUMA IK: Team 23	Oct 16, 2012	4:21 PM	9 items	9 items
Hang	PUMA IK: Team 24	Oct 16, 2012	4:41 PM	7 items	7 items
Annie Mroz	PUMA IK: Team 25	Oct 16, 2012	4:31 PM	4 items	4 items
Takashi Furuya	PUMA IK: Team 26	Oct 16, 2012	3:45 PM	5 items	5 items
吴迪	PUMA IK: Team 27	Oct 16, 2012	2:29 PM	1 item	1 item
Charlie Xu	PUMA IK: Team 28	Oct 16, 2012	12:11 AM	4 items	4 items
qiangeng@seas.upenn.edu	PUMA IK: Team 28	Oct 16, 2012	3:23 PM	4 items	4 items
Brian C. Lee	PUMA IK: Team 29	Oct 16, 2012	4:44 PM	3 items	3 items
Brian C. Lee	PUMA IK: Team 29	Oct 16, 2012	4:52 PM	3 items	3 items
golas@seas.upenn.edu	PUMA IK: Team 30	Oct 16, 2012	4:28 PM	3 items	3 items
Robert Parajon	PUMA IK: Team 31	Oct 16, 2012	10:01 AM	4 items	4 items
Gabrielle Merritt	PUMA IK: Team 32	Oct 16, 2012	4:52 PM	3 items	3 items
Fiona Strain	PUMA IK: Team 33	Oct 16, 2012	2:42 PM	4 items	4 items
Leslie Callaghan	PUMA IK: Team 34	Oct 16, 2012	3:53 PM	4 items	4 items
tianjud@seas.upenn.edu	Fwd: test_puma_ik_team11 (Latest code)	Oct 14, 2012	8:24 PM	4 items	4 items



Project I : PUMA Light Painting



MEAM.Design : MEAM520-12C-P01-Sim

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Now that you did your [inverse kinematics](#) solution, it's time to do light painting. This assignment is due by **5:00 p.m. on Thursday, October 25**. Your team must submit this assignment and get it to work correctly before you will be allowed to do the next part of the project (working with the robot). Submissions after the deadline will be penalized, but not as harshly as for individual homework assignments.

Your task is to write a MATLAB program that moves the PUMA's LED around in space to create a lovely light painting (long exposure image).

You should use our [PUMA simulator \(v1\)](#) to test your light painting code. As shown at right, it creates an animation of the PUMA and leaves colored markers in the air so you can see how your creation looks. After you download the simulator, run `demo.m` to see how it works. Read `pumasim_manual_v1.pdf` to learn more about the simulator's interface. Please post on Piazza if you are confused about any aspect of the simulator or if you find any bugs.

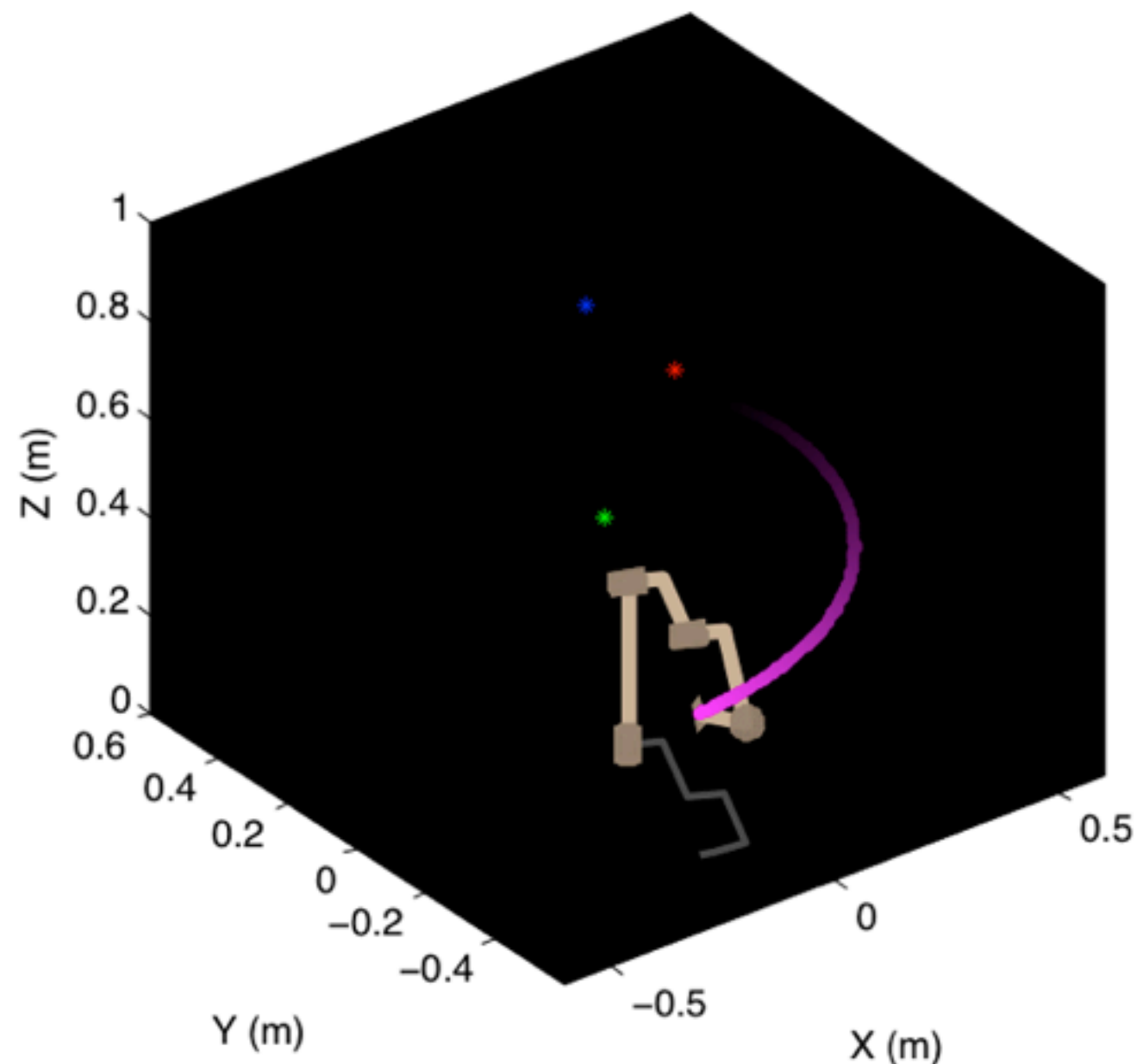
Submission

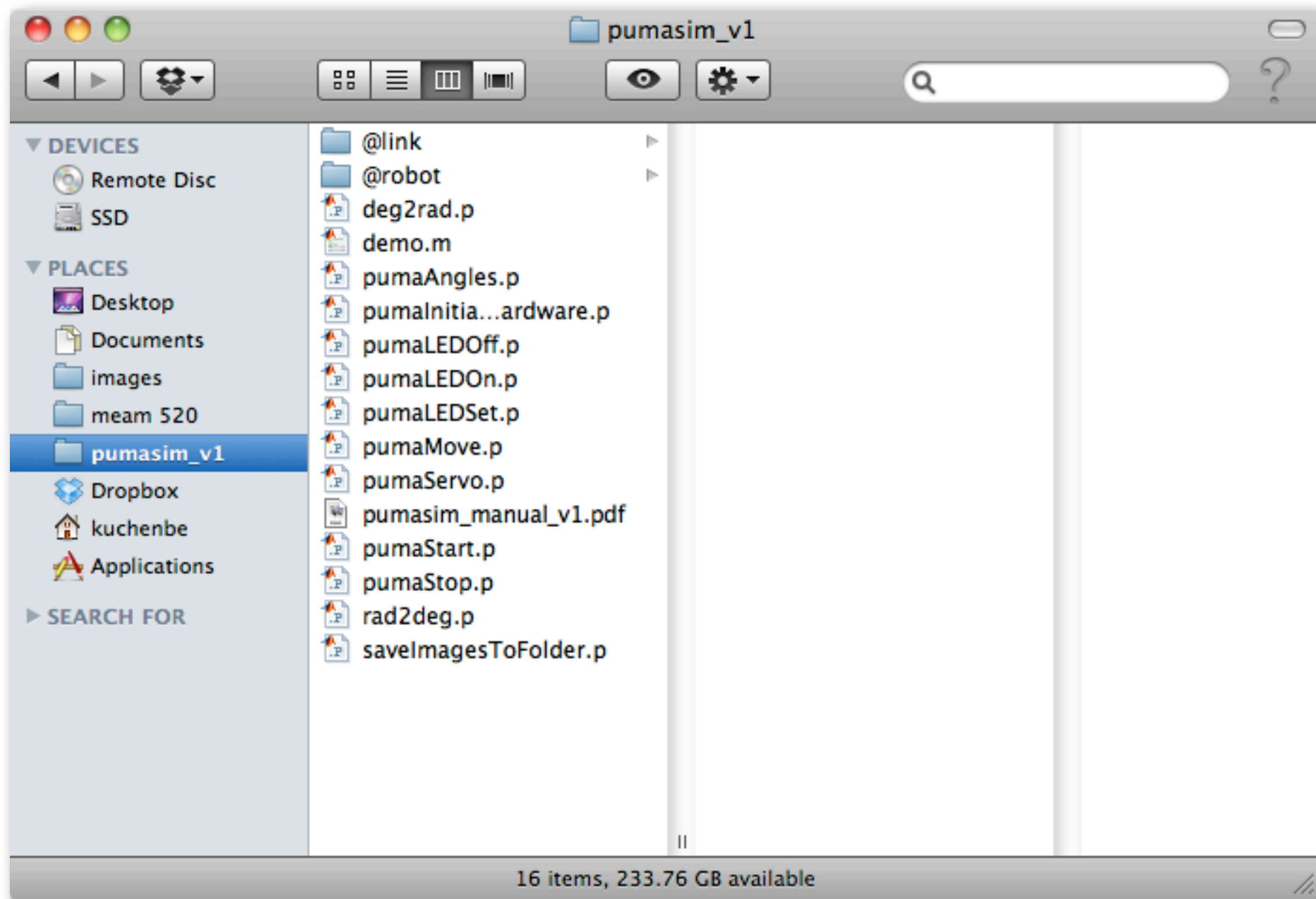
1. Start an email to meam520@seas.upenn.edu
2. Make the subject *PUMA Simulation: Team 00*, replacing 00 with your team number.
3. Attach all of your correctly named MATLAB files to the email. It should be

`puma_light_painting_teamXX.m`, where XX is your team number, plus any additional files you may have created, also named according to this convention.

4. In the body of the email, explain the status of your submission. If you are submitting a new version of your IK with this assignment, state that in the email.
5. Send the email.
6. Wait for a response from the teaching team about whether your code is ready to run on the robot.

Press ENTER to make the robot move with the LED off





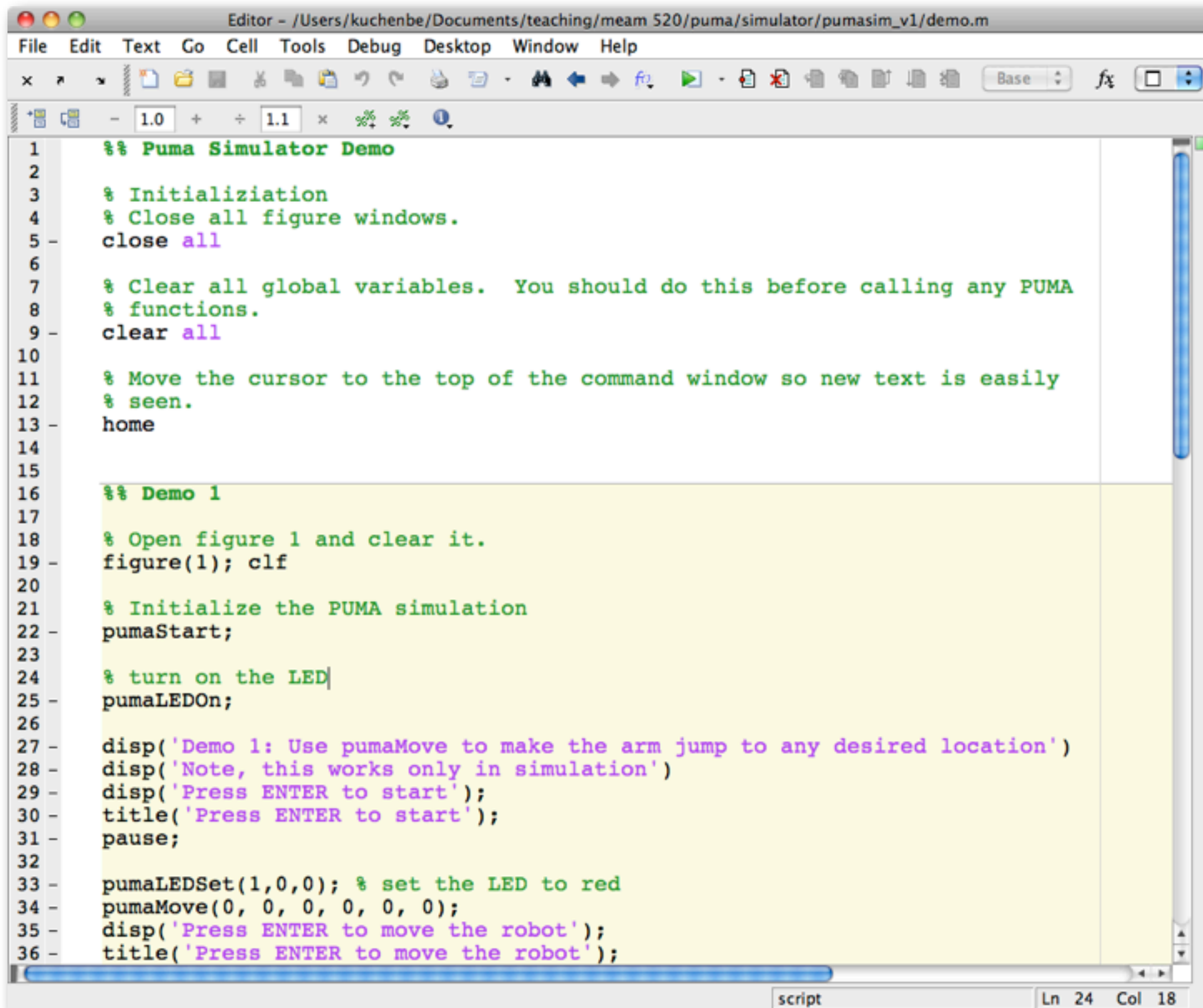
RUNNING THE PUMA

The basic work flow when using the PUMA 260 arm is

1. Make sure the Emergency Stop (**E-stop**) button is engaged (pressed down).
2. Call `pumaStart('Hardware', 'on', 'Delay', 10)`, where the number following delay is the minimum allowable time, in milliseconds, between calls to `pumaServo`. This may be set to any value above 0.5ms. This will display a warning that the PUMA will return to the home position. Ensure that the workspace is clear and manually move the PUMA closer to the home position if you think it may hit the table or an object.
3. Type 'y' or 'yes' then hit Enter to continue.
4. Release the **E-stop** by pulling up on the button, at which time the PUMA will return to the home position.
5. In a separate MATLAB process, call `startFrameBuffer` to begin capturing video from the webcam.
6. Make a light painting, using `pumaServo` to command the robot to move. Remember to call `pumaLEDOn` to enable the LED and use `pumaLEDSet` to select the color.
7. Call `stopFrameBuffer` to finish capturing video.
8. Return the PUMA to the home position, if possible.
9. Call `pumaStop` to disable the controller.
10. Engage the **E-stop** by pressing the button down.
11. Use `makeVideoAndImage` to create long-exposure picture and video. Optionally, you may wish to save the image files to a different location using `saveImagesToFolder` before starting a new video.

There are several other things to keep in mind:

- Do *NOT* use the clear all command once the PUMA has been initialized before calling `pumaStop`, otherwise MATLAB will crash.
- Test the video capture before running your whole light painting.



The image shows a MATLAB Editor window with the following details:

- Title Bar:** Editor - /Users/kuchenbe/Documents/teaching/meam 520/puma/simulator/pumasim_v1/demo.m
- Menu Bar:** File Edit Text Go Cell Tools Debug Desktop Window Help
- Toolbar:** Includes icons for file operations (new, open, save, print), editing (undo, redo, cut, copy, paste), and execution (run, stop, step-through). It also features a 'Base' dropdown and a 'fx' icon.
- Code Editor:** Contains a MATLAB script with line numbers 1 through 36. The script is divided into two sections: initialization and a demo function. The demo function includes comments and commands for opening a figure, initializing the PUMA simulation, turning on the LED, and moving the robot arm.
- Status Bar:** Located at the bottom right, it displays 'script' and 'Ln 24 Col 18'.

```
1 %% Puma Simulator Demo
2
3 % Initialization
4 % Close all figure windows.
5 close all
6
7 % Clear all global variables. You should do this before calling any PUMA
8 % functions.
9 clear all
10
11 % Move the cursor to the top of the command window so new text is easily
12 % seen.
13 home
14
15 %% Demo 1
16
17 % Open figure 1 and clear it.
18 figure(1); clf
19
20 % Initialize the PUMA simulation
21 pumaStart;
22
23 % turn on the LED
24 pumaLEDOn;
25
26 disp('Demo 1: Use pumaMove to make the arm jump to any desired location')
27 disp('Note, this works only in simulation')
28 disp('Press ENTER to start');
29 title('Press ENTER to start');
30 pause;
31
32 pumaLEDSet(1,0,0); % set the LED to red
33 pumaMove(0, 0, 0, 0, 0, 0);
34 disp('Press ENTER to move the robot');
35 title('Press ENTER to move the robot');
```

Confirmed Midterm Date

Thursday, November 8, in class

~ email KJK if you have a severe conflict ~



Velocity Kinematics



Slides created by
Jonathan Fiene

How do the velocities of the joints affect the linear and angular velocity of the end-effector?

These quantities are related by the **Jacobian**, a matrix that generalizes the notion of an ordinary derivative of a scalar function.

Jacobians are useful for planning and executing smooth trajectories, determining singular configurations, executing coordinated anthropomorphic motion, deriving dynamic equations of motion, and transforming forces and torques from the end-effector to the manipulator joints.

explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$v_n^0 = J_v \dot{q}$$

$$\omega_n^0 = J_\omega \dot{q}$$

How do we calculate the position Jacobian?

$$\dot{\mathbf{p}} = \mathbf{J}_p(q) \dot{\mathbf{q}}$$

endpoint
velocity

Jacobian
matrix

joint
velocity

$$\mathbf{J}_p = \begin{bmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \cdots & \frac{\delta x}{\delta q_n} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \cdots & \frac{\delta y}{\delta q_n} \\ \frac{\delta z}{\delta q_1} & \frac{\delta z}{\delta q_2} & \cdots & \frac{\delta z}{\delta q_n} \end{bmatrix}$$

Prismatic

$$J_{v_i} = z_{i-1}$$

Revolute

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$

$$v_n^0 = J_v \dot{q}$$

What joint velocities should I choose to cause a desired end-effector velocity?
(inverse velocity kinematics)

$$\dot{q} = J_v^{-1} v_n^0$$

This works only when the Jacobian is square and invertible (non-singular).

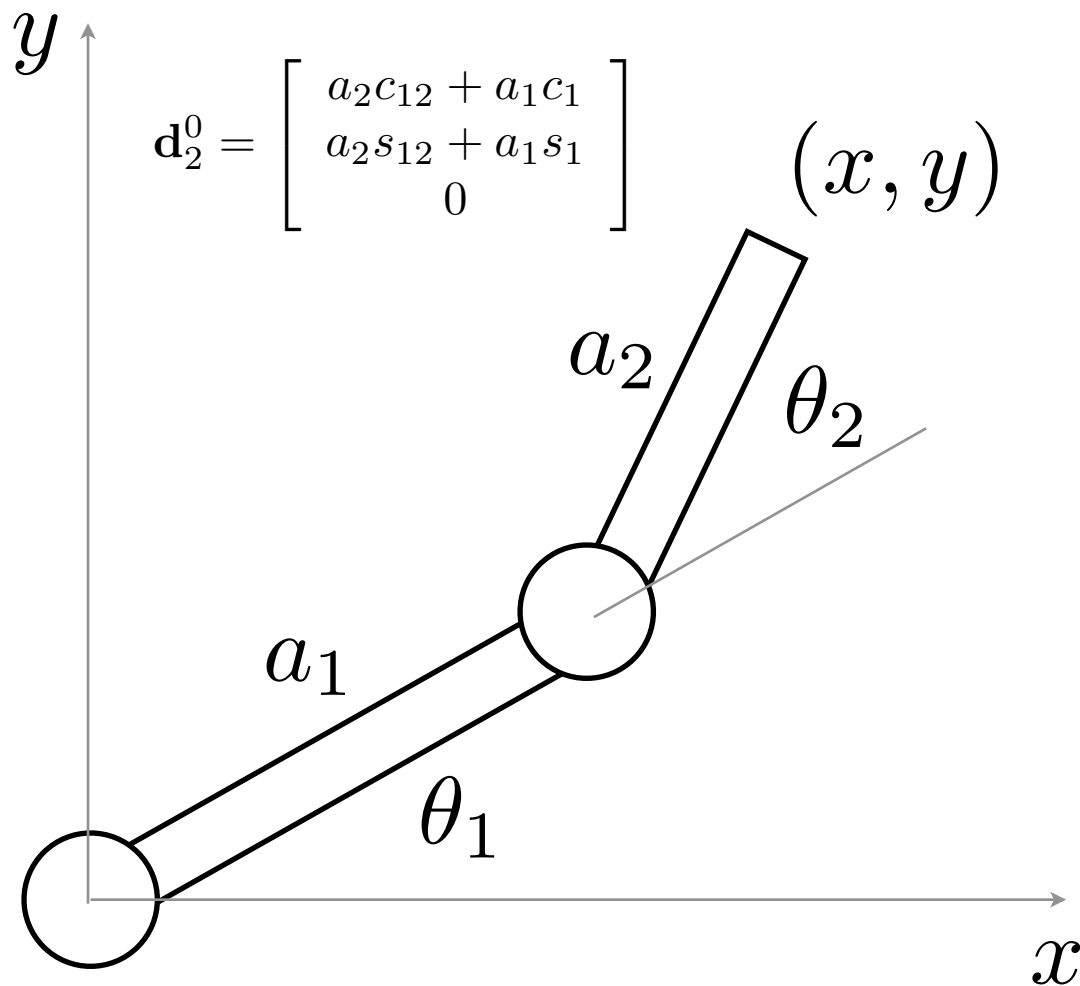
SHV 4.11 explains what to do when the Jacobian is not square:
rank test (v is in range of J)
use J^+ (right pseudoinverse of J)
when the robot has extra joints, there are many solutions

Singularities are points in the configuration space where infinitesimal motion in a certain direction is not possible and the manipulator loses one or more degrees of freedom

Mathematically, singularities exist at any point in the workspace where the Jacobian matrix loses rank.

a matrix is singular if and only if its determinant is zero:

$$\det(\mathbf{J}) = 0$$



$$\mathbf{d}_2^0 = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

$$\det(\mathbf{J}) = ?$$

$$\det(\mathbf{J}) = a_1 a_2 (c_1 s_{12} - s_1 c_{12})$$

When does $\det(\mathbf{J}) = 0$?

$$\det(\mathbf{J}) = 0 \text{ when } \theta_2 = 0$$

Is that the only time?

No... $\det(\mathbf{J}) = 0$ when $\theta_2 = \dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots$

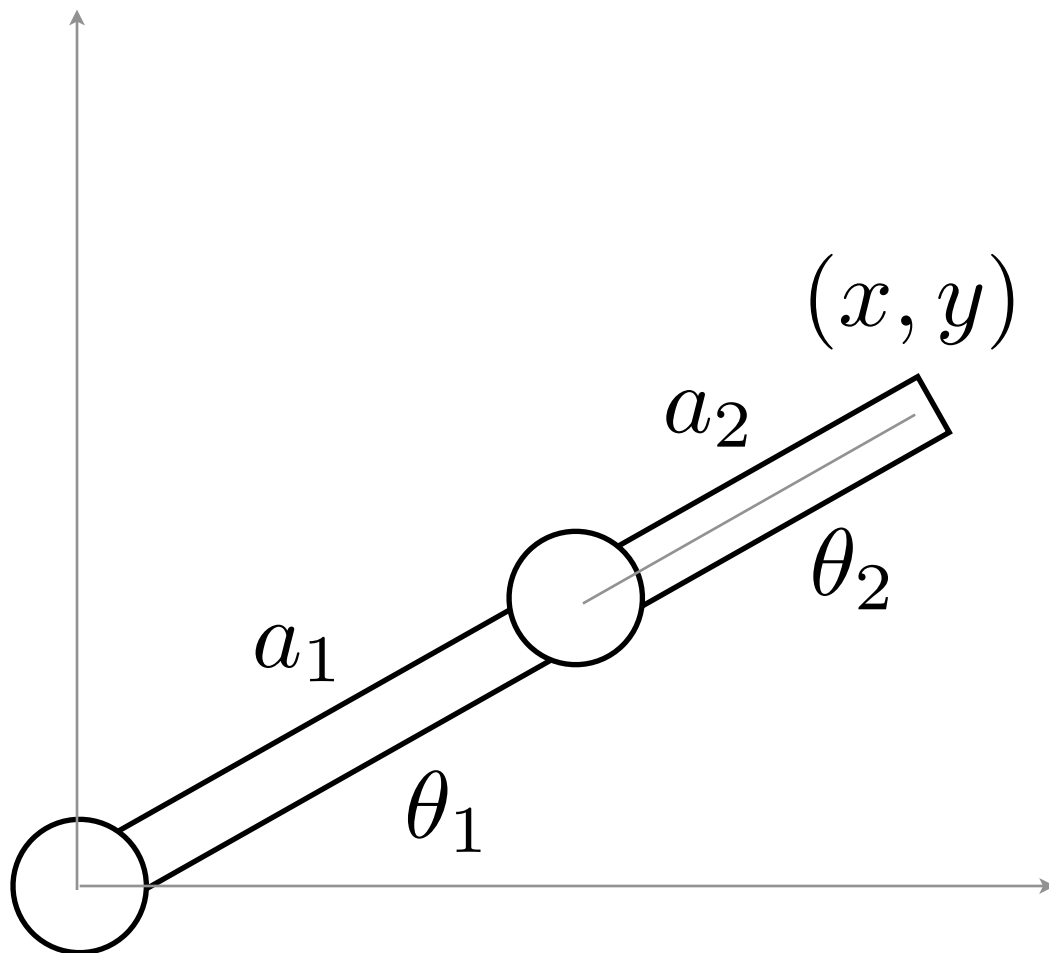
Any other times? $\det(\mathbf{J}) = 0$ when $a_1 = 0$ or $a_2 = 0$

For $\theta_2 = 0$

The Jacobian collapses to have linearly dependent rows

$$\mathbf{J}_{\theta_2=0} = \begin{bmatrix} -a_1 s_1 - a_2 s_1 & -a_2 s_1 \\ a_1 c_1 + a_2 c_1 & a_2 c_1 \end{bmatrix}$$

This means that actuating either joint causes motion in the same direction



Questions ?

explore how **changes** in joint values affect the end-effector movement

could have **N joints**, but only **six** end-effector velocity terms (xyzpts)

The **Jacobian** matrix lets us calculate how joint velocities translate into end-effector velocities (depends on configuration)

look at it in two parts - position and orientation

$$v_n^0 = J_v \dot{q}$$

$$\omega_n^0 = J_\omega \dot{q}$$

How do we calculate the orientation Jacobian?

And now, angular velocity

$$\omega = \mathbf{J}_\omega(q) \dot{\mathbf{q}}$$

↑
final frame
angular
velocity

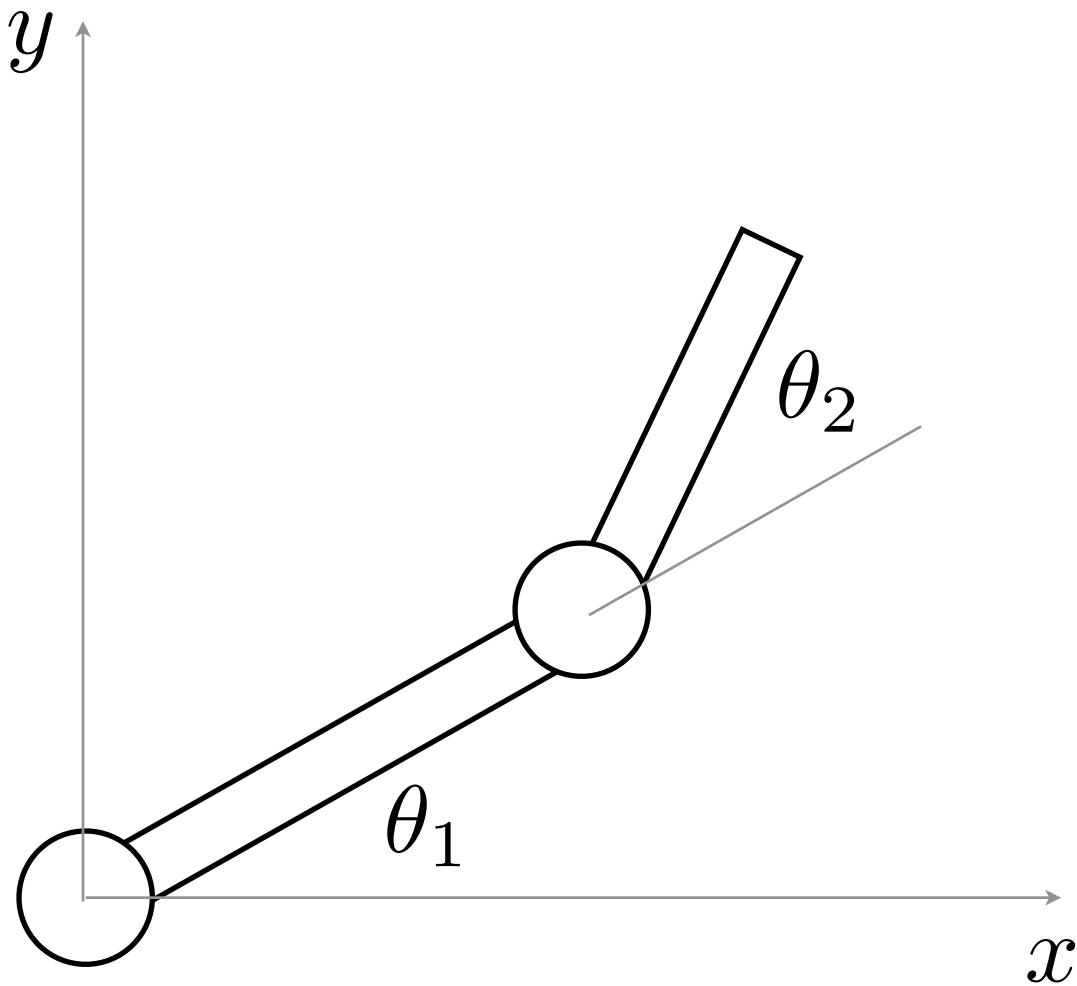
↑
joint
velocities

$$\omega_{i,j}^k$$

this is the angular velocity of frame **j**
with respect to frame **i**,
expressed in frame **k**

SHV 4.1 gives a good explanation of angular velocity for fixed-axis rotation. SHV 4.2-4.5 go into greater detail.

The Angular Velocity of Connected Rigid Bodies



$$\omega_{0,1}^0 = 0 \hat{x}_0 + 0 \hat{y}_0 + \dot{\theta}_1 \hat{z}_0$$

$$\omega_{1,2}^1 = 0 \hat{x}_1 + 0 \hat{y}_1 + \dot{\theta}_2 \hat{z}_1$$

$$\omega_{1,2}^0 = \mathbf{R}_1^0 \omega_{1,2}^1$$

$$\begin{aligned} \omega_{0,2}^0 &= \omega_{0,1}^0 + \mathbf{R}_1^0 \omega_{1,2}^1 \\ &= 0 \hat{x}_0 + 0 \hat{y}_0 + (\dot{\theta}_1 + \dot{\theta}_2) \hat{z}_0 \end{aligned}$$

$$\omega_{0,n}^0 = \sum_{i=1}^n \mathbf{R}_{i-1}^0 \omega_{i-1,i}^{i-1}$$

$$\omega_{0,n}^0 = \sum_{i=1}^n (\mathbf{R}_{i-1}^0 \hat{\mathbf{z}}) \dot{\theta}_i$$

note: this holds for revolute joints only (by definition, a prismatic joint cannot create angular velocity)

And now, for that Jacobian!

$$\omega_{0,n}^0 = \sum_{i=1}^n \rho_i (\mathbf{R}_{i-1}^0 \hat{\mathbf{z}}) \dot{\theta}_i \quad \rho_i = \begin{array}{l} 0 \text{ for prismatic} \\ 1 \text{ for revolute} \end{array}$$

$$\omega_{0,n}^0 = \begin{bmatrix} \rho_1 \hat{\mathbf{z}} & \rho_2 \mathbf{R}_1^0 \hat{\mathbf{z}} & \rho_2 \mathbf{R}_2^0 \hat{\mathbf{z}} & \dots & \rho_n \mathbf{R}_{n-1}^0 \hat{\mathbf{z}} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

$$\omega = \mathbf{J}_\omega(q) \dot{\mathbf{q}}$$

(6 x n) Jacobian
a.k.a. manipulator Jacobian
a.k.a. geometric Jacobian

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_p \\ \mathbf{J}_\omega \end{bmatrix}$$

(3 x n) cartesian velocity Jacobian

(3 x n) angular velocity Jacobian

The Jacobian is easily constructed from the
manipulator's forward kinematics.

What do you need from the forward kinematics?

4.6.3 Combining the Linear and Angular Velocity Jacobians

As we have seen in the preceding section, the upper half of the Jacobian J_v is given as

$$J_v = [J_{v_1} \cdots J_{v_n}] \quad (4.56)$$

in which the i^{th} column J_{v_i} is

$$J_{v_i} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for revolute joint } i \\ z_{i-1} & \text{for prismatic joint } i \end{cases} \quad (4.57)$$

The lower half of the Jacobian is given as

$$J_\omega = [J_{\omega_1} \cdots J_{\omega_n}] \quad (4.58)$$

in which the i^{th} column J_{ω_i} is

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{for revolute joint } i \\ 0 & \text{for prismatic joint } i \end{cases} \quad (4.59)$$

Singularities are points in the configuration space where infinitesimal motion in a certain direction is not possible and the manipulator loses one or more degrees of freedom

Mathematically, singularities exist at any point in the workspace where the Jacobian matrix loses rank.

a matrix is singular if and only if its determinant is zero:

$$\det(\mathbf{J}) = 0$$

$$\xi = J(q)\dot{q}$$

(6 × 1) body velocity \nearrow
 \uparrow (6 × n) Jacobian \nwarrow (n × 1) joint velocities

$$\begin{bmatrix} v_n^0 \\ \omega_n^0 \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \dot{q}$$

For a 6-DOF manipulator with a spherical wrist, we can decouple the determination of singular configurations into two simpler problems.

$$J = [J_{\text{arm}} \mid J_{\text{wrist}}]$$

(the book calls this $J = [J_P \mid J_O]$)

$$J = [J_{\text{arm}} \mid J_{\text{wrist}}] = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array} \right]$$

$$J_{\text{wrist}} = \left[\begin{array}{ccc} z_3 \times (o_6 - o_3) & z_4 \times (o_6 - o_4) & z_5 \times (o_6 - o_5) \\ z_3 & z_4 & z_5 \end{array} \right]$$

if we choose $o_4 = o_5 = o_6$

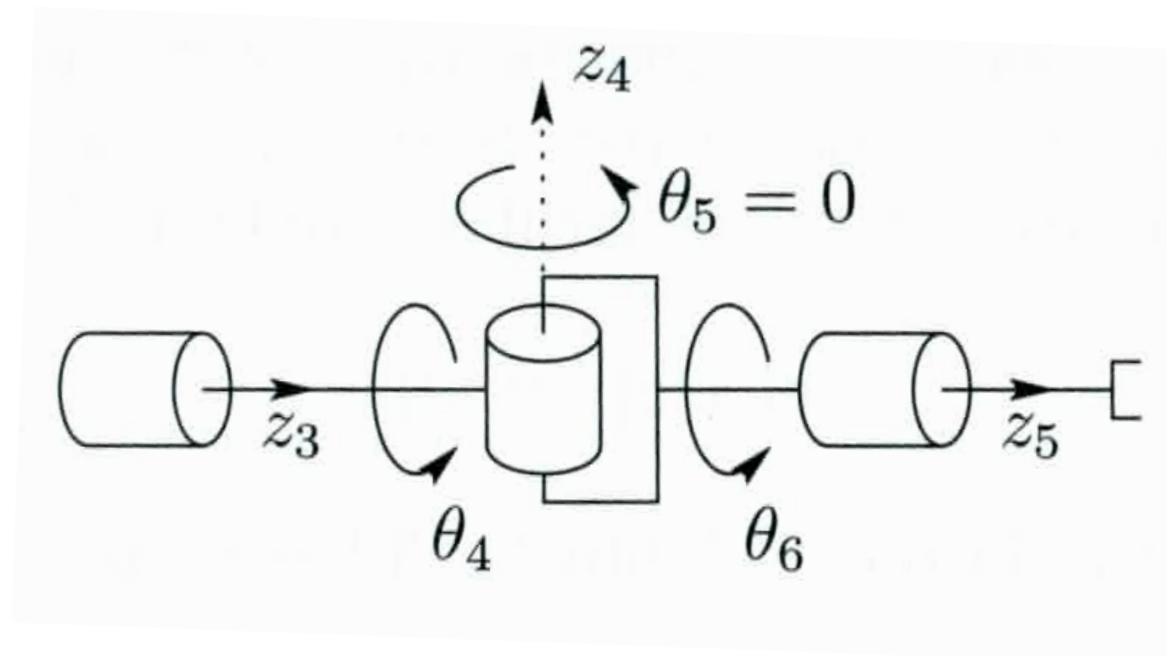
$$J_{\text{wrist}} = \left[\begin{array}{ccc} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{array} \right]$$

$$J = \left[\begin{array}{c|c} J_{11} & 0 \\ \hline J_{21} & J_{22} \end{array} \right] \quad \det(J) = \det(J_{11}) \det(J_{22})$$

$$\det(J) = \det(J_{11}) \det(J_{22})$$

$$J_{22} = \begin{bmatrix} z_3 & z_4 & z_5 \end{bmatrix}$$

Singular when any two wrist axes align



$$z_3 \perp z_4$$

$$z_4 \perp z_5$$

z_3 can become $\parallel z_5$

$\theta_5 = 0, \pi$ are singular configurations

For a specific configuration, the Jacobian scales the input (joint velocities) to the output (body velocity)

$$\xi = J(q)\dot{q}$$

If you put in a joint velocity vector with unit norm, you can calculate in which direction and how fast the robot will translate and rotate.

If the Jacobian is full rank, you can calculate the manipulability ellipsoid.

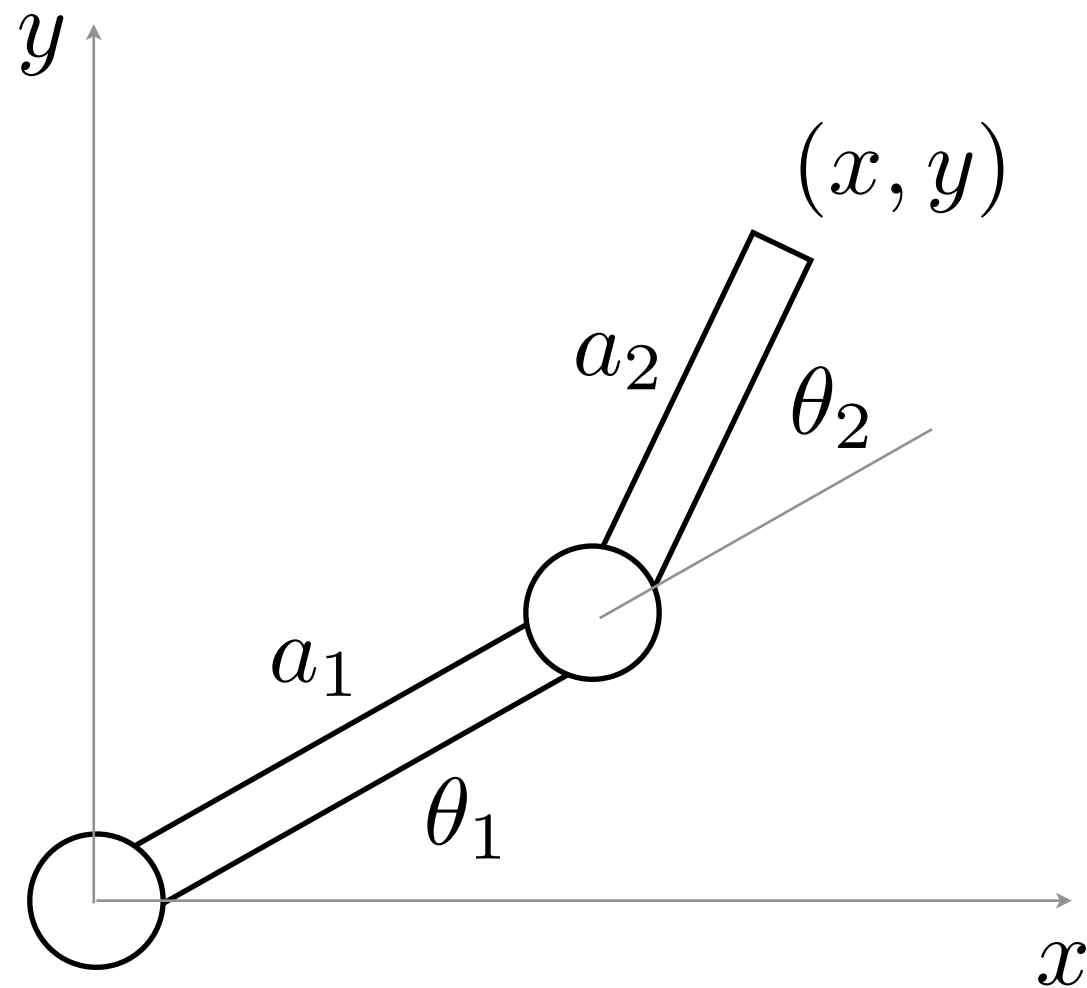
choose $\dot{q} = J^+ \xi$

$$||\dot{q}||^2 = \xi^T (J J^T)^{-1} \xi$$

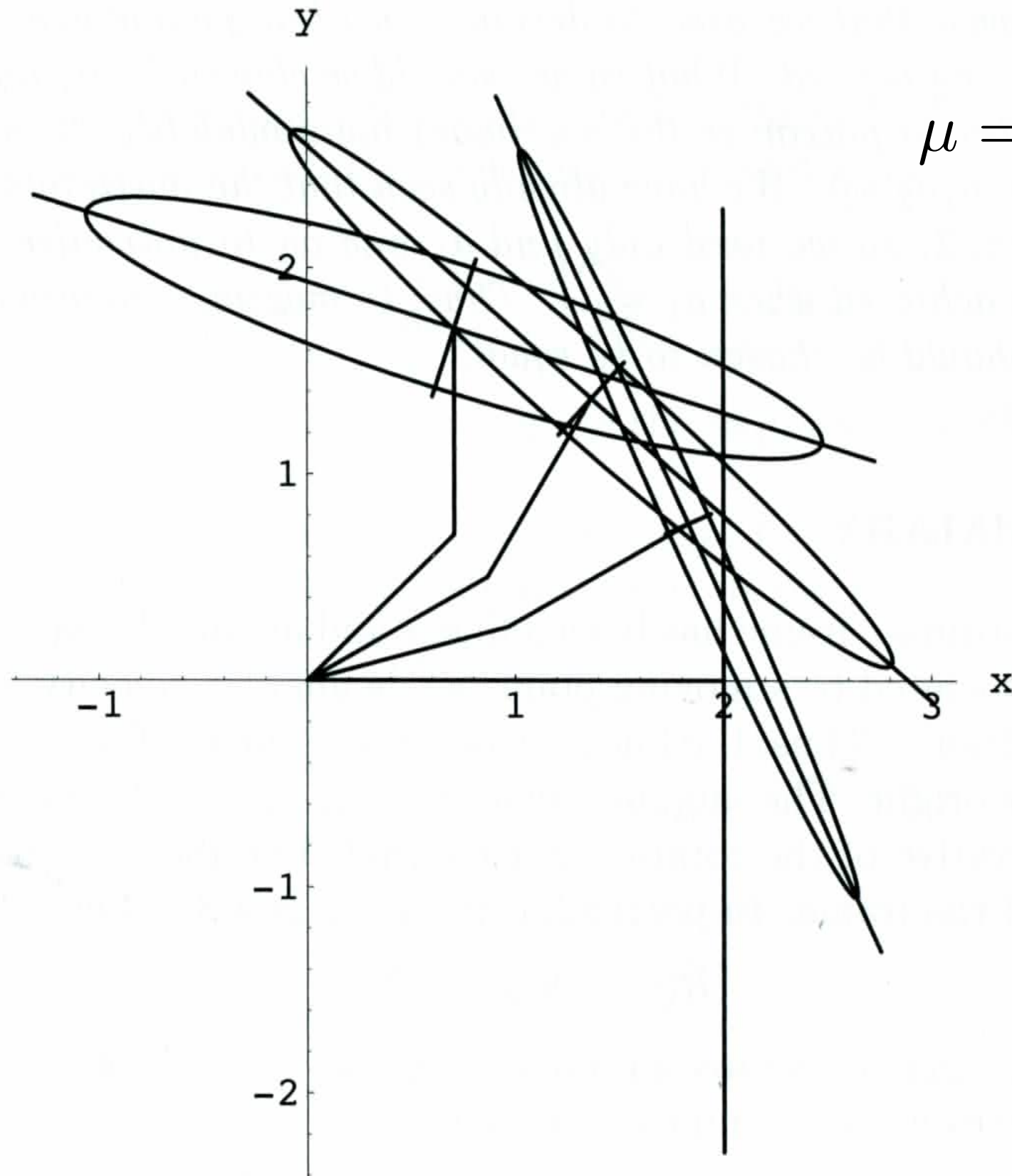
If not redundant, manipulability

$$\mu = |\det(J)|$$

What does the manipulability ellipsoid look like for the planar RR robot?



$$\mu = |\det(J)| = a_1 a_2 |\sin(\theta_2)|$$



Can be used to tell you
where to perform
certain tasks.

Also useful for deciding
how to design a
manipulator.

Soon I will release Homework 4,
an individual assignment on Jacobians

Not sure when it will be due...

Homework 2 and 3 graded