MEAM 520

More Inverse Kinematics

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Lecture 8: October 2, 2012

Homework 2:
Manipulator Kinematics and DH Parameters

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

September 18, 2012

This assignment is due on **Thursday, September 27 (updated)**, by 5:00 p.m. sharp. You should aim to turn the paper part in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. The code must be emailed according to the instructions at the end of this document. Late submissions of either or both parts will be accepted until 5:00 p.m. on Friday, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

Written Problems *(30 points)*

The first set of problems are written, including two from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions when provided. Write in pencil, show your work clearly, box your answers, and staple your pages together.

1. Custom problem – Kinematics of Baxter *(5 points)*
   Rethink Robotics recently released a new robot named Baxter. Watch YouTube videos of Baxter (e.g., http://www.youtube.com/watch?v=rjPFqkFyrOY) to learn about its kinematics. Draw a schematic of the serial kinematic chain of Baxter's left arm (the one the woman is touching in the picture above.) Use the book's conventions for how to draw revolute and prismatic joints in 3D.

2. SHV 3-7, page 113 – Three-link Cartesian Robot *(10 points)*
   Your solution should include a schematic of the manipulator with appropriately placed coordinate frames, a table of the DH parameters, and the final transformation matrix. Then answer the following question: What are the x, y, and z coordinates of the tip of the robot's end-effector in the base frame (as a function of the robot parameters and the joint coordinates)?

Solutions to Homework 2

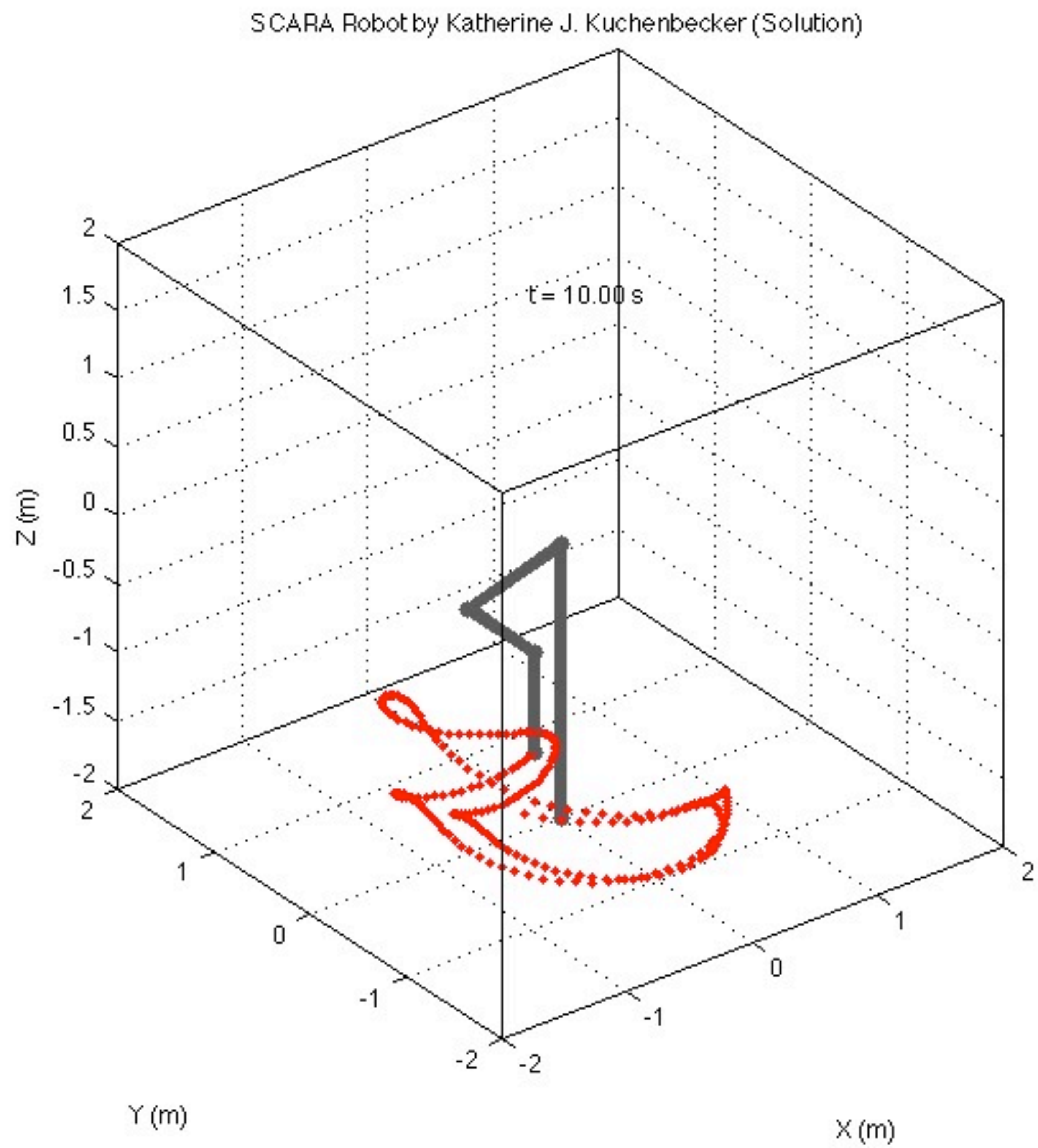
MEAM 520

Introduction to Robotics

University of Pennsylvania

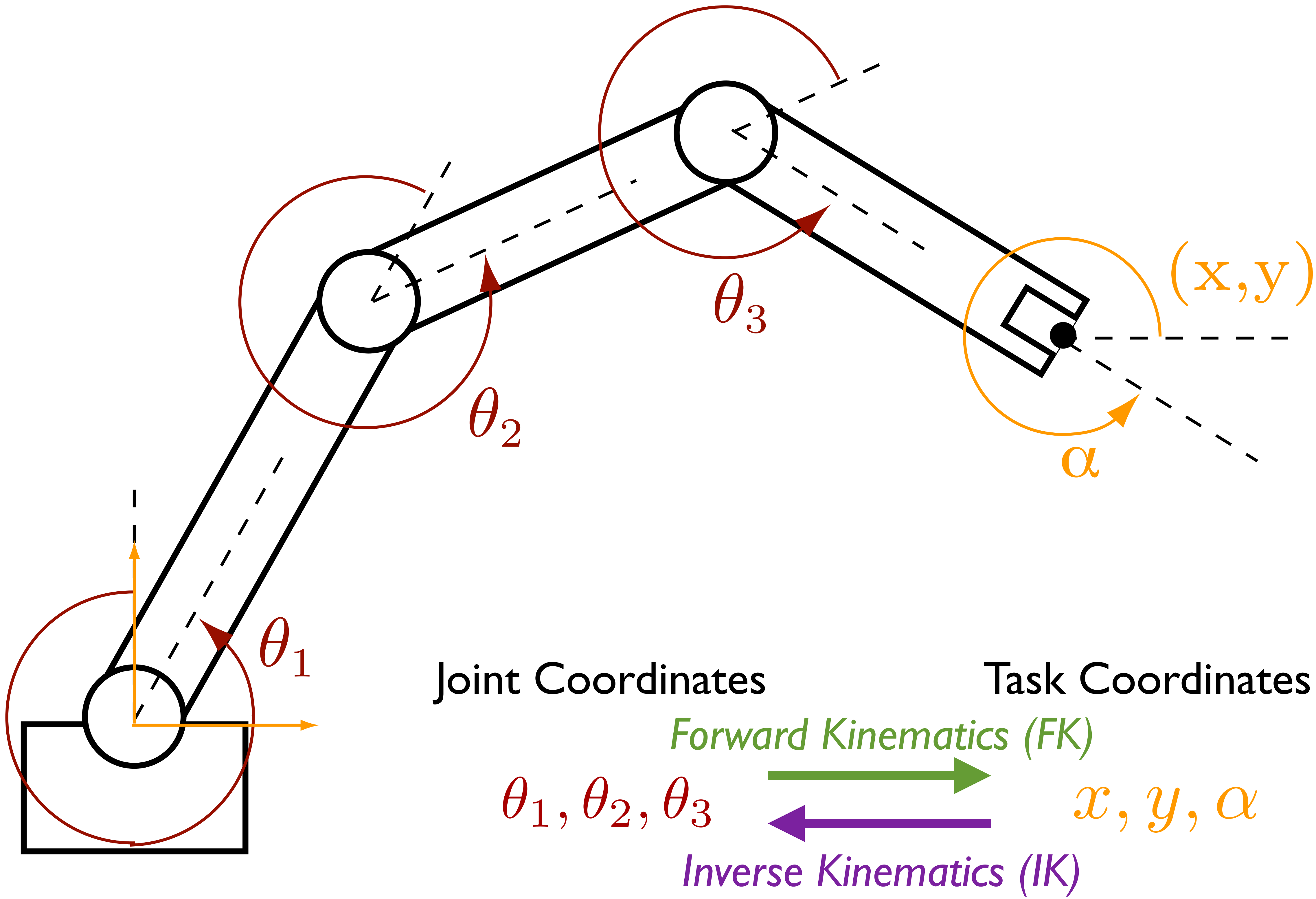
Professor Kuchenbecker

Fall 2012



SCARA Robot by Katherine J. Kuchenbecker (Solution)
t = 10.00 s
Z (m)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Y (m)
2
1
0
-1
-2
X (m)
-2
-1
0
1
2

Joint and Task Coordinates



Due on Tuesday, October 9, by 5:00 p.m.

Homework 3: DH Parameters and Inverse Kinematics

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

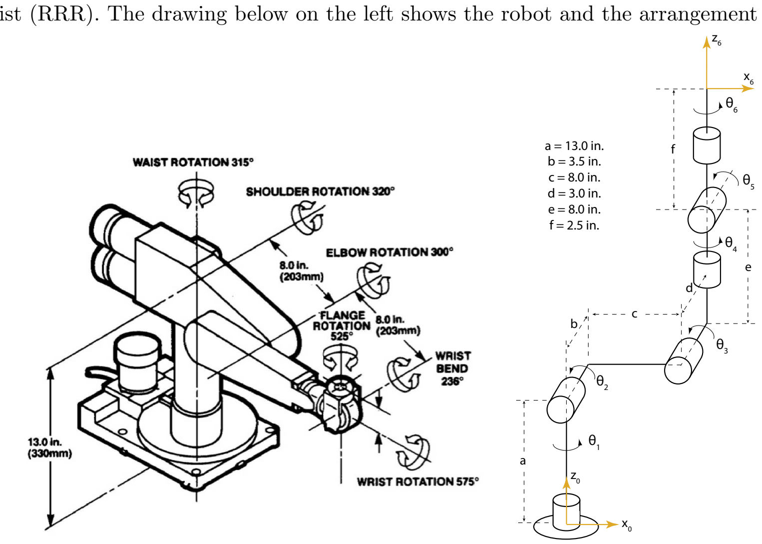
September 30, 2012

This assignment is due on **Tuesday, October 9**, by 5:00 p.m. sharp. You should aim to turn the paper part in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. The code must be emailed according to the instructions at the end of this document. Late submissions of either or both parts will be accepted until 5:00 p.m. on Wednesday, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

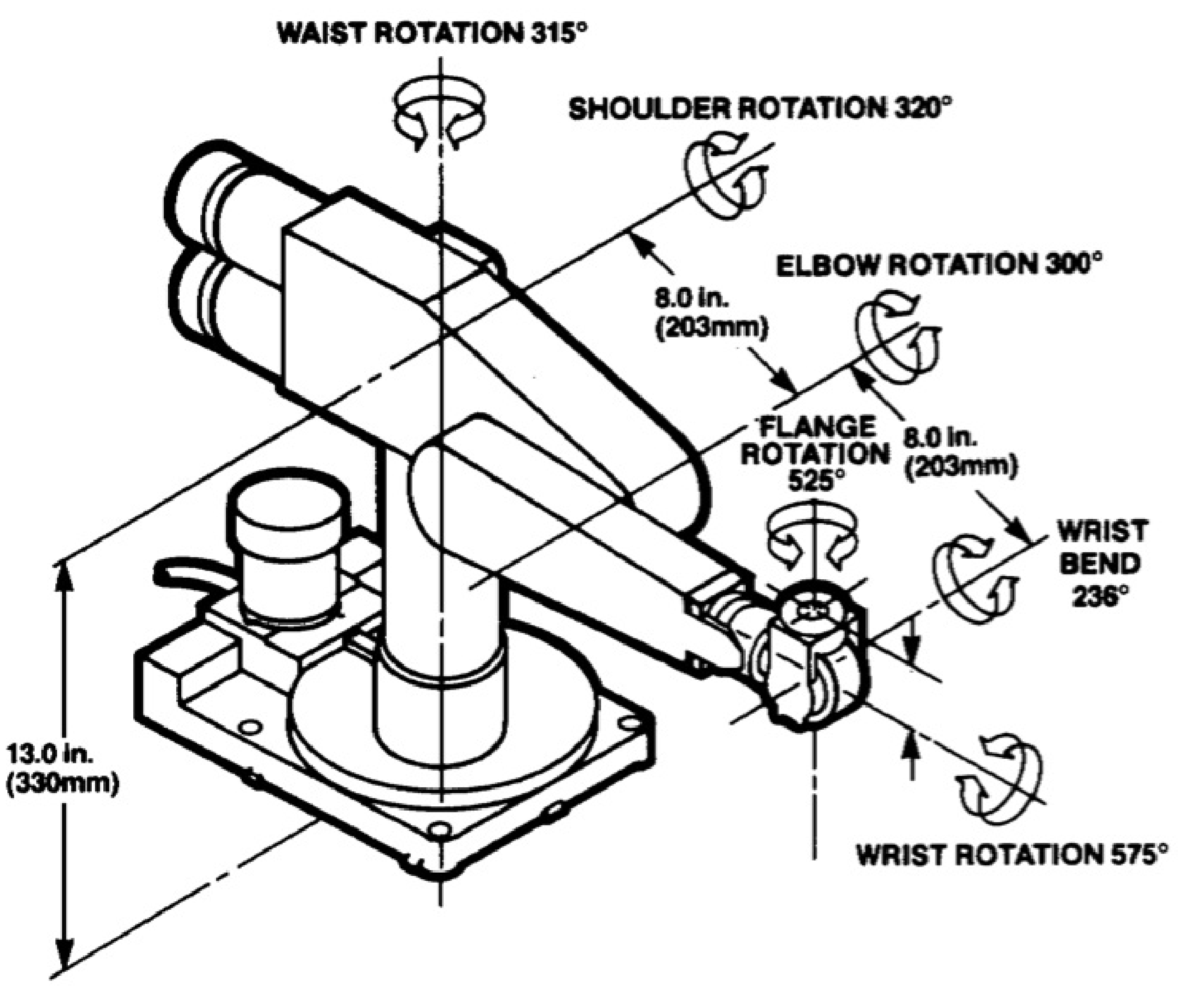
You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

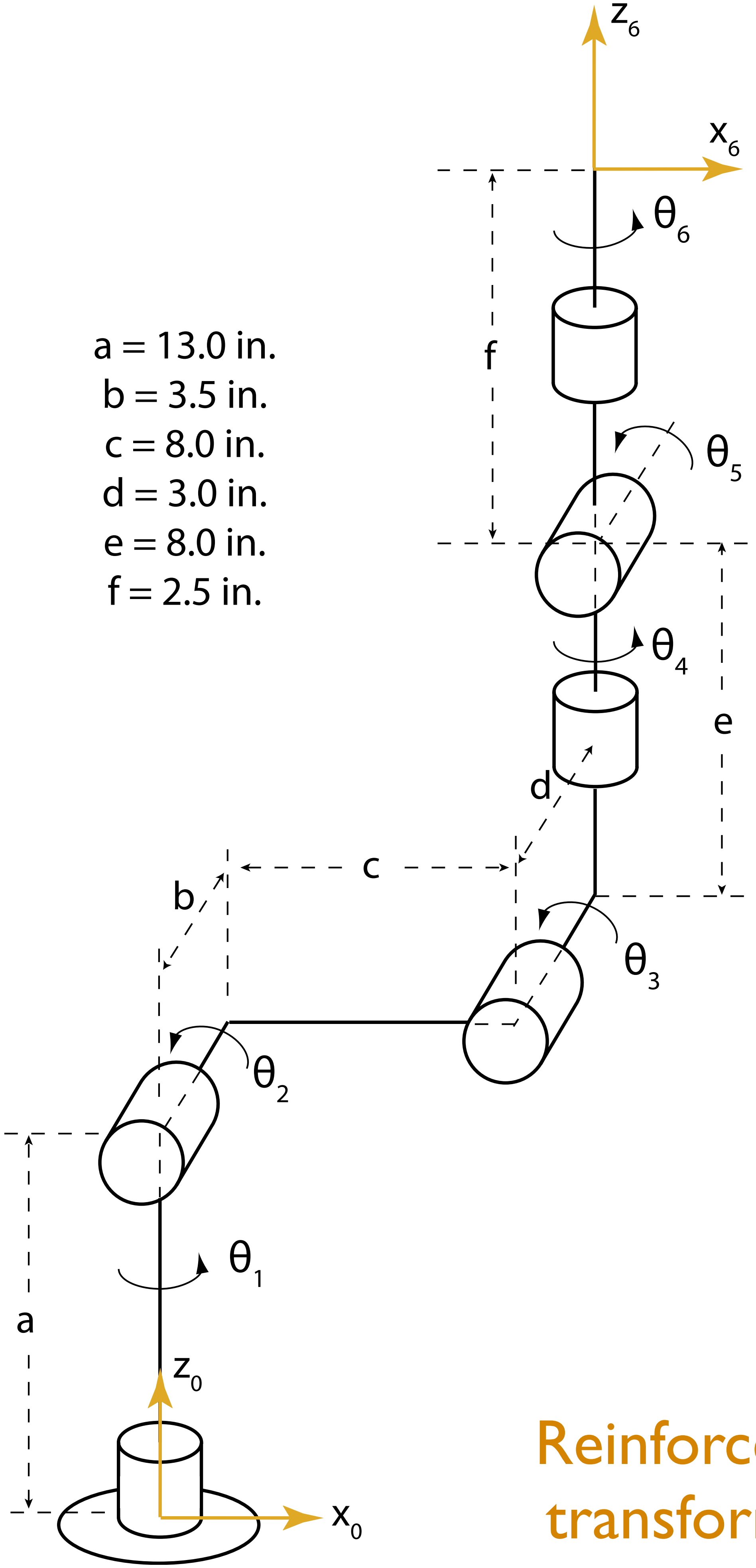
Forward Kinematics of the PUMA 260 *(30 points)*

The first two problems center on the forward kinematics of the PUMA 260. We will be using this robot for the hands-on manipulator labs in this class. It is an articulated robot (RRR) with lateral offsets, plus a spherical wrist (RRR). The drawing below on the left shows the robot and the arrangement of its joints.



The schematic above on the right shows the zero configuration we have chosen for use in this class (a different pose from the drawing at the left). The joint angle arrows show the positive direction for each revolute joint (θ_1 to θ_6). All of the joints are shown at $\theta_i = 0$. The diagram also gives the measurements for the constant dimensions (a to f), all in inches.

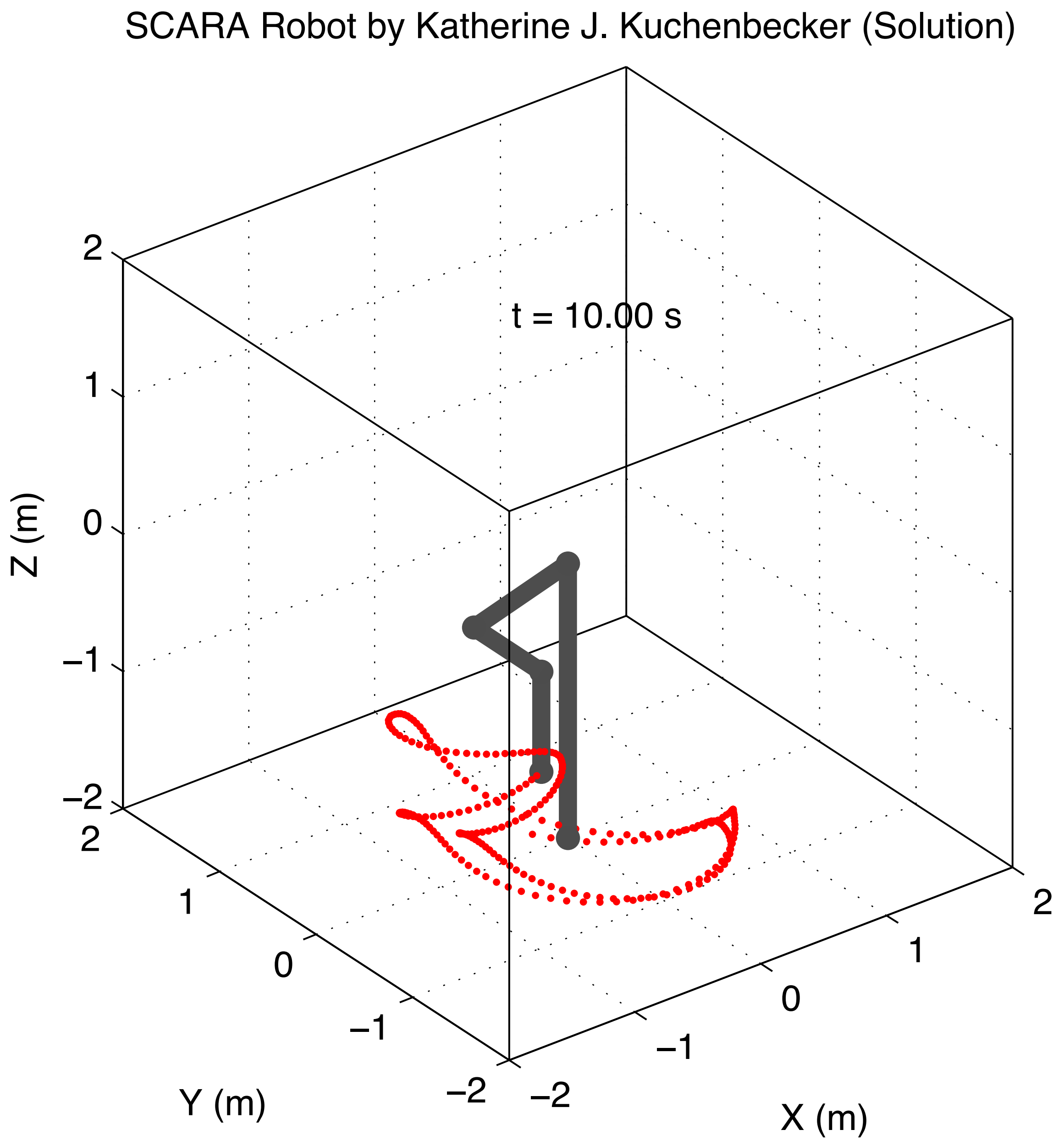

WAIST ROTATION 315°
SHOULDER ROTATION 320°
ELBOW ROTATION 300°
8.0 in. (203mm)
FLANGE ROTATION 525°
8.0 in. (203mm)
WRIST BEND 236°
13.0 in. (330mm)
WRIST ROTATION 575°

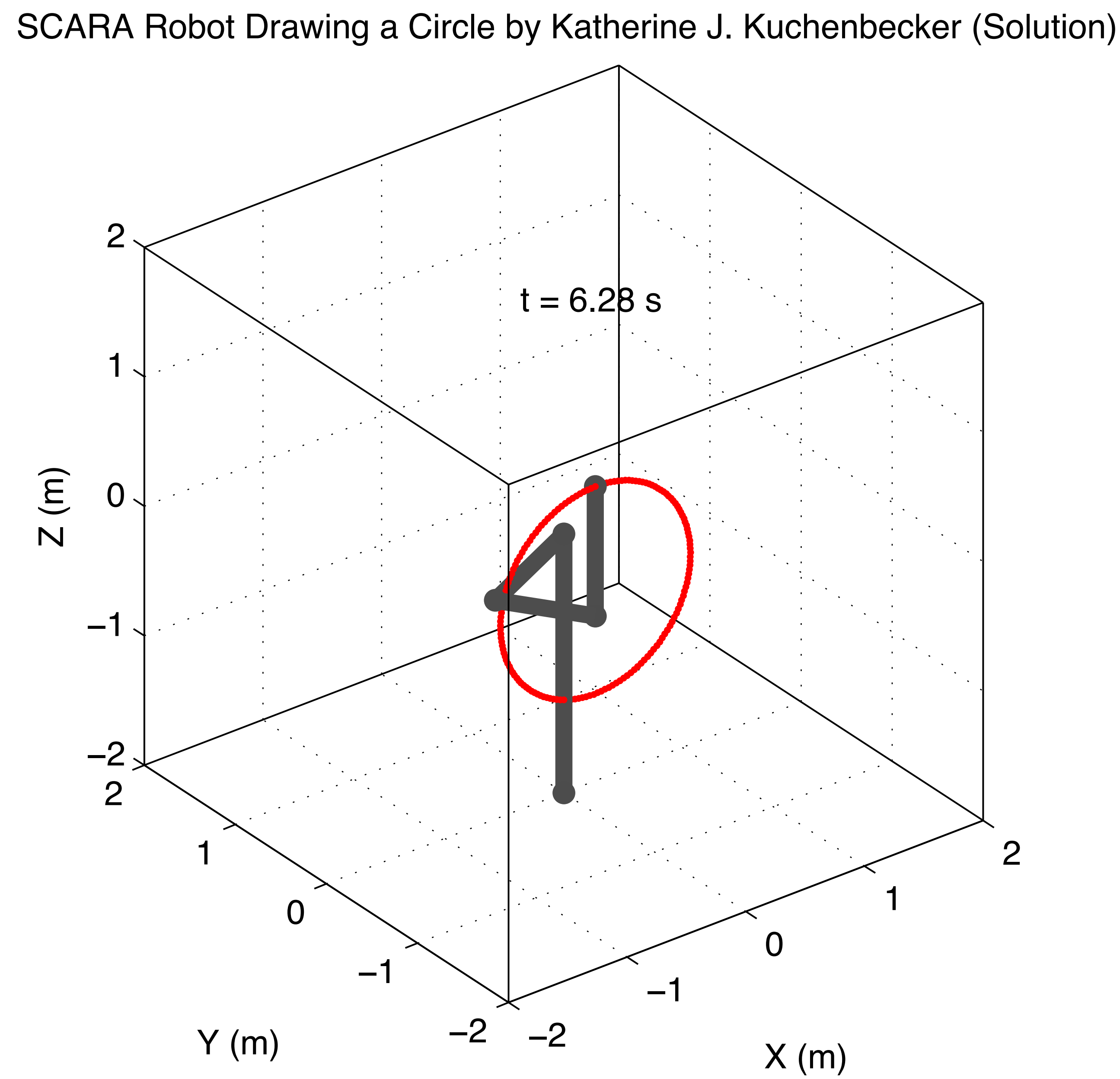


- Assign coordinate frames
- Determine DH parameters
- Update dh_starter.m
  - A = dh_yourpennkey(a, alpha, d, theta);
  - Angles are in degrees, so use sind and cosd
- Update puma_robot_starter.m
  - Calculate all six of the Puma's A matrices using dh_yourpennkey.m
  - Calculate the position of the origin of each frame (o1 through o6) for plotting.

Reinforce and refine your understanding of homogeneous transformations, forward kinematics, and DH parameters

Questions ?





$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\theta_2 = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)$$

$$\theta_1 = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

$$d_3 = -o_z$$

Understand the usefulness of inverse kinematics.

Questions ?

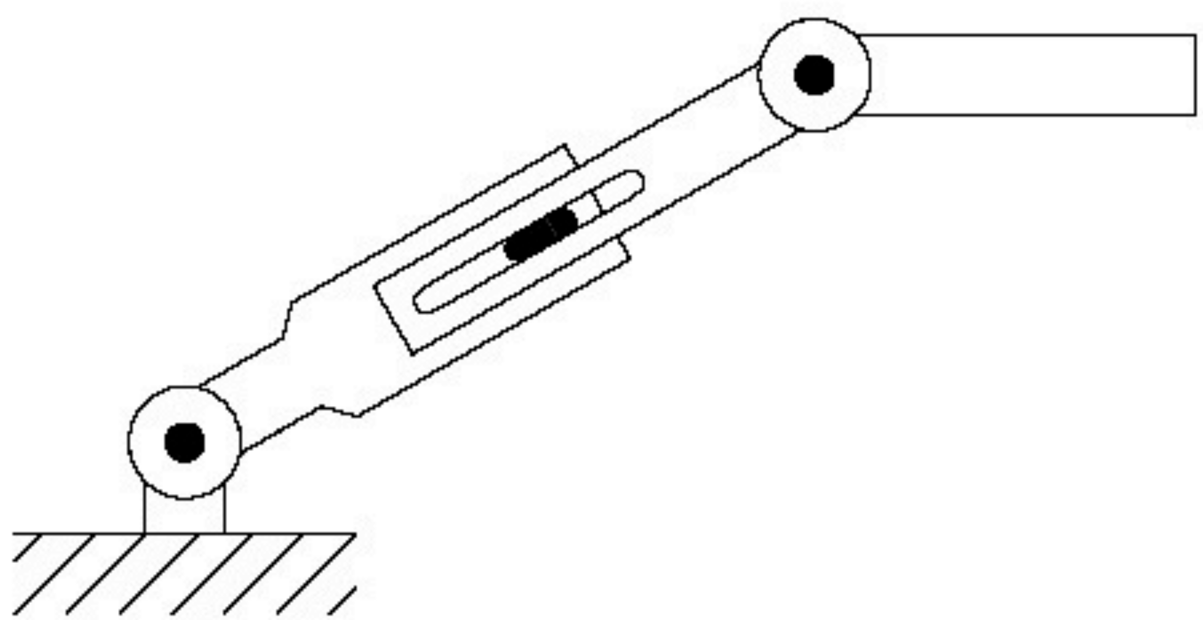


Figure 3.33: Three-link planar robot with prismatic joint.

a. Given a desired position of the end effector, how many solutions are there to the inverse kinematics of the three-link planar arm shown in Figure 3.33? How does the number of solutions depend on the desired position, if at all?

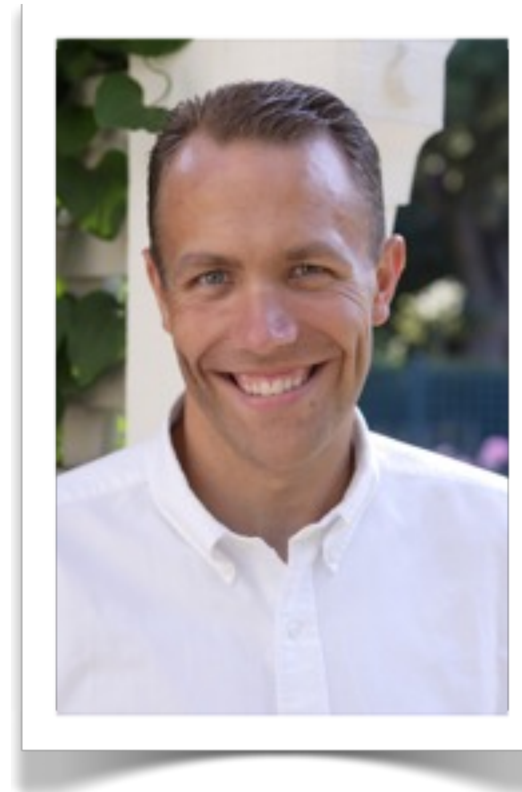
b. If the orientation of the end effector is also specified, how many solutions are there? How does the number of solutions depend on the desired position and orientation, if at all?

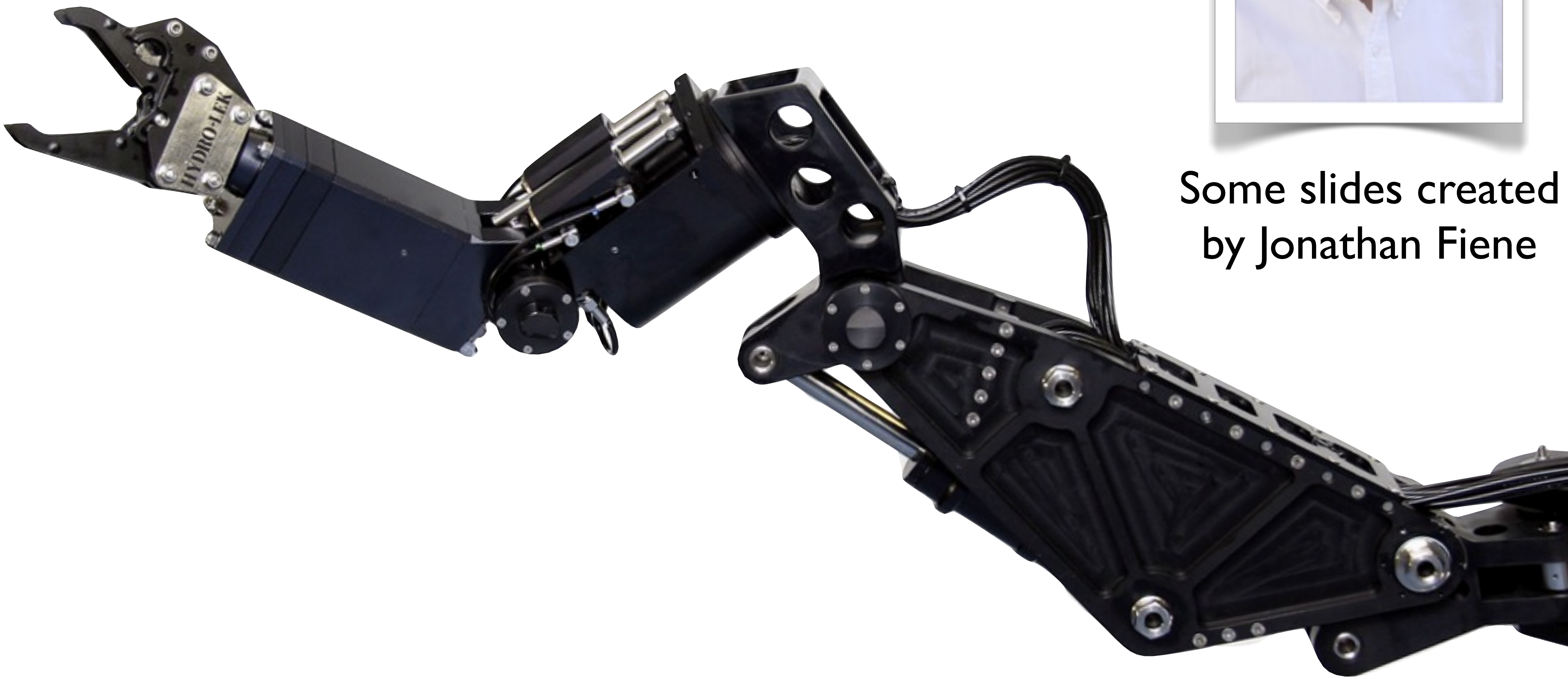
c. Use the geometric approach to find the inverse kinematic solution(s) for the case when both the position and orientation of the end effector are specified as o_x, o_y, and α, remembering the concept of kinematic decoupling.

Learn how to derive the inverse kinematics for a manipulator.

Questions ❓

Inverse Kinematics





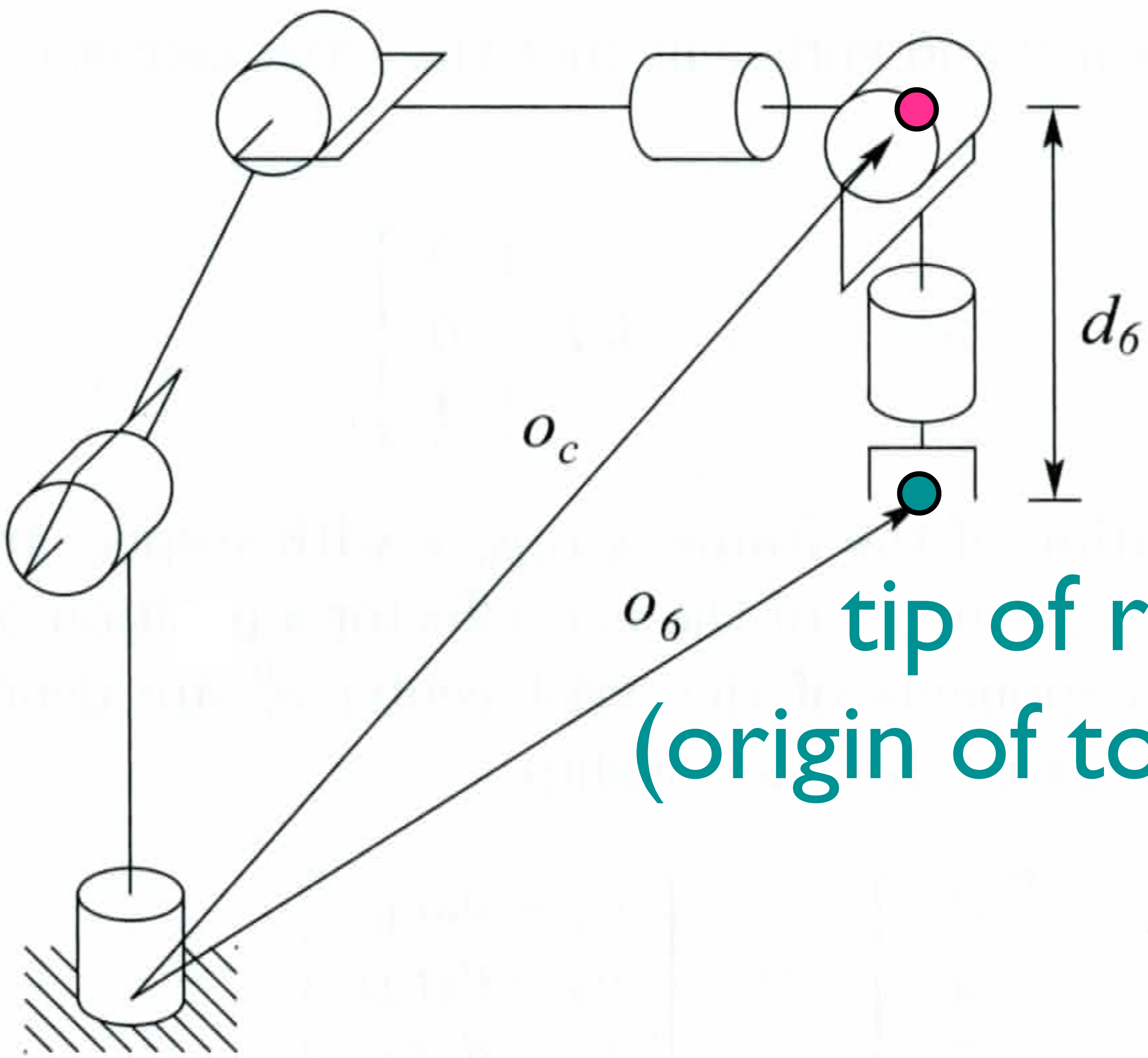
Some slides created by Jonathan Fiene

$$\text{given } \mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$$

and a certain manipulator with n joints

find $q_1, \ldots, q_n$ such that $\mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$

A helpful approach for 6-DOF robots: Kinematic Decoupling



$$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

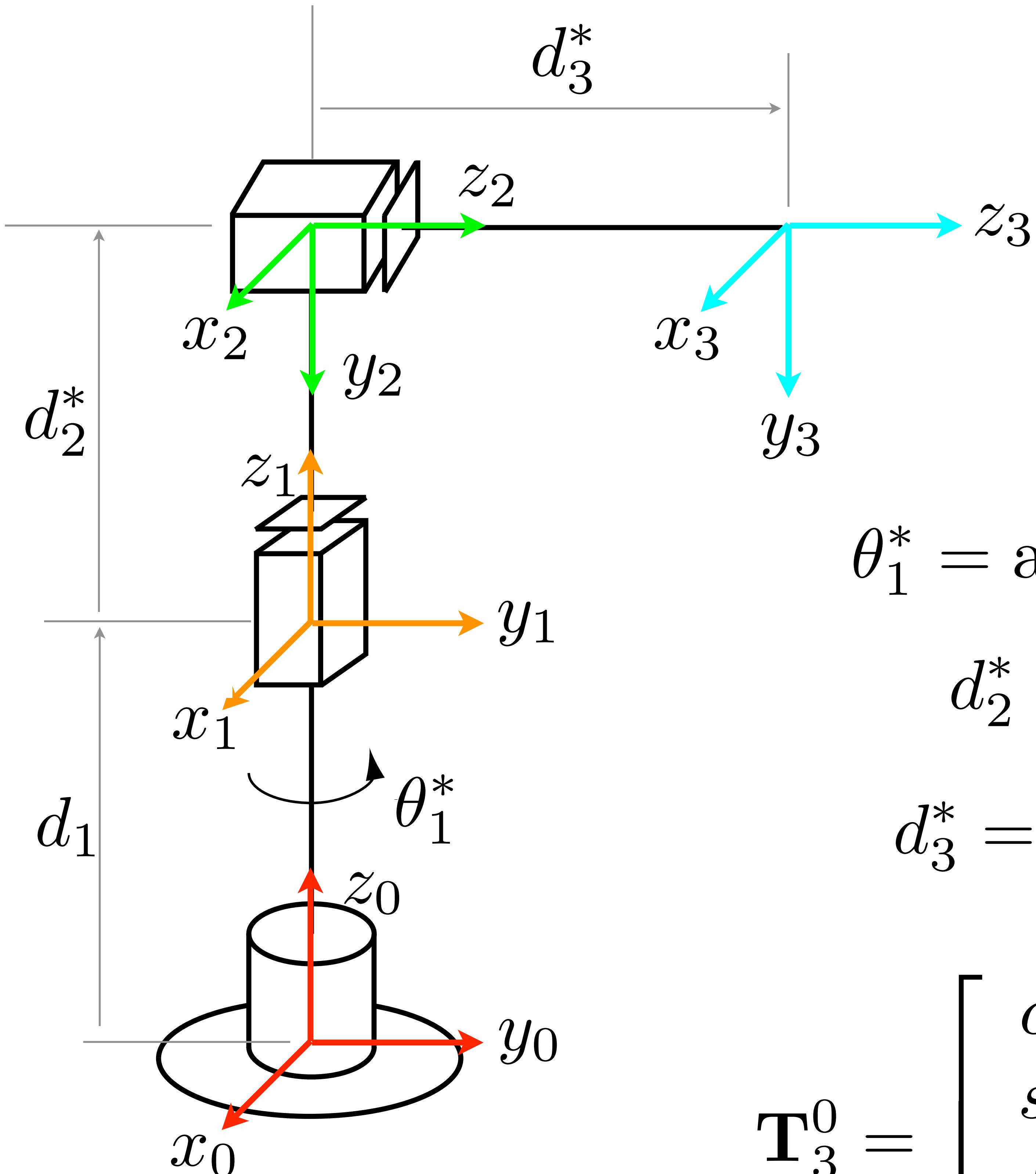
position

$$R = R_3^0 R_6^3$$

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$$

orientation

The RPP Cylindrical Robot – Inverse Kinematics



$$\theta_1^* = \mathrm{atan2}\left(\frac{-x}{y}\right)$$

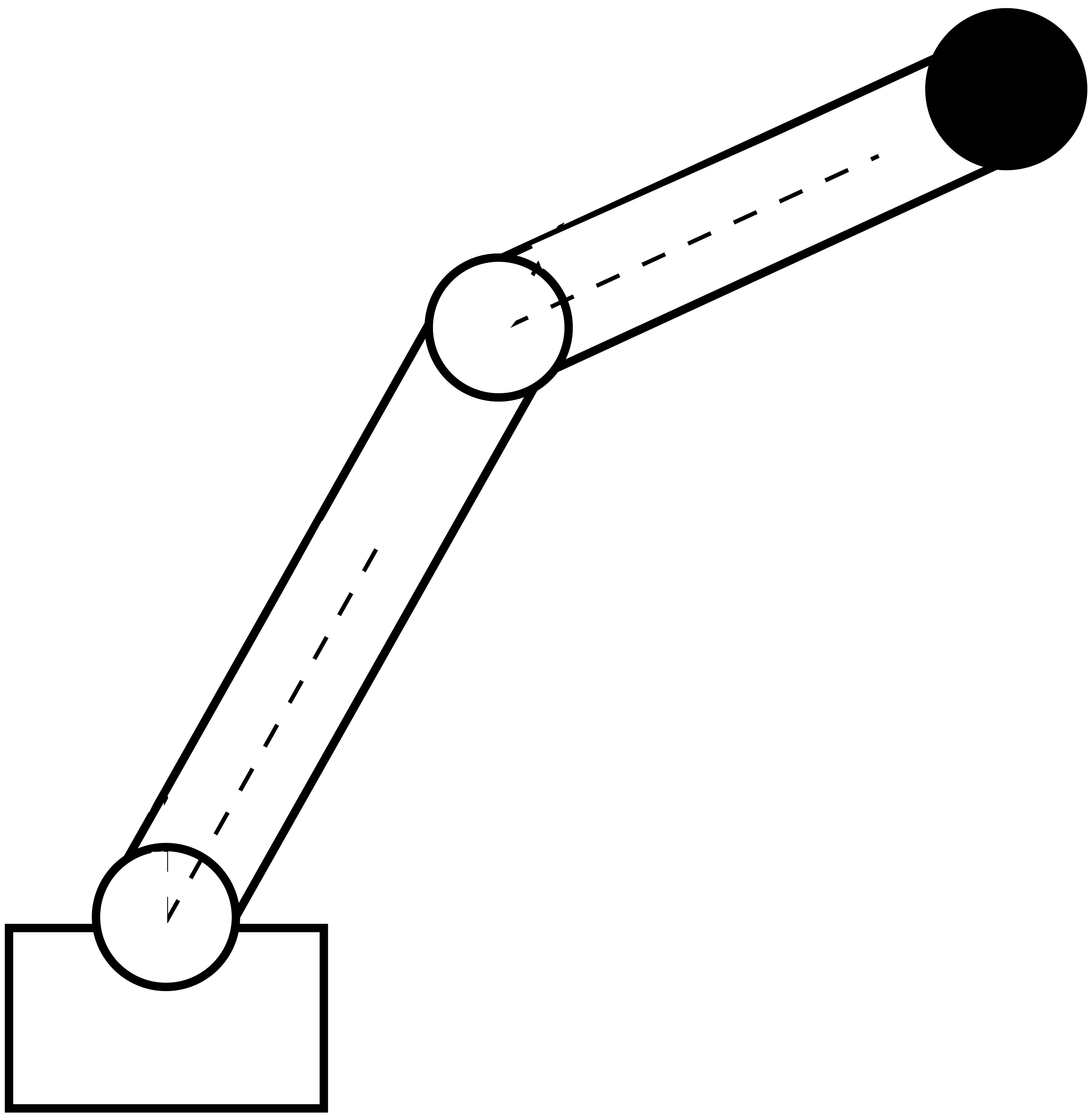
$$d_2^* = z - d_1$$

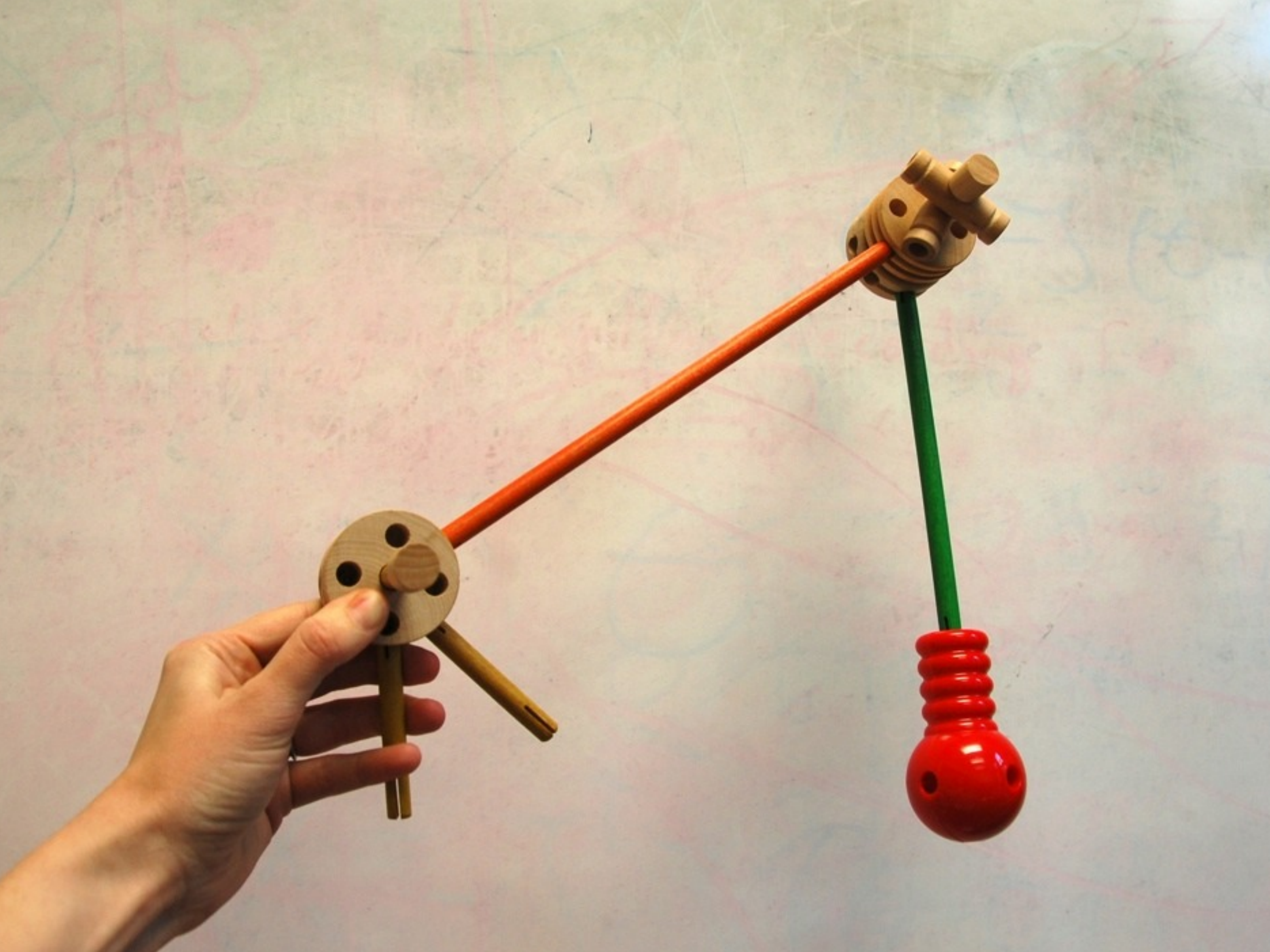
$$d_3^* = \sqrt{x^2 + y^2}$$

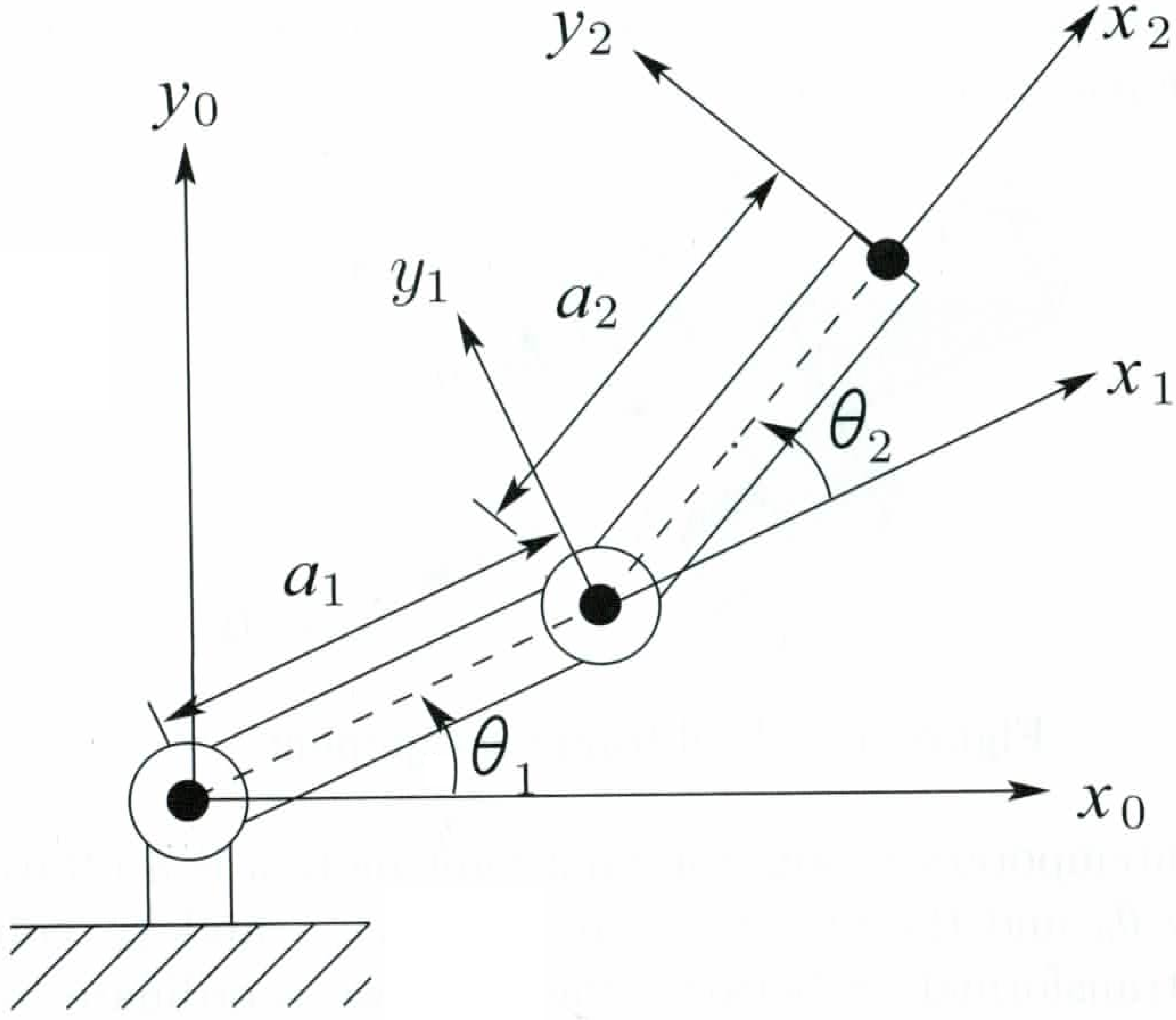
How could I check my answers?

$$\mathbf{T}_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A More Complicated Example – Planar RR Robot





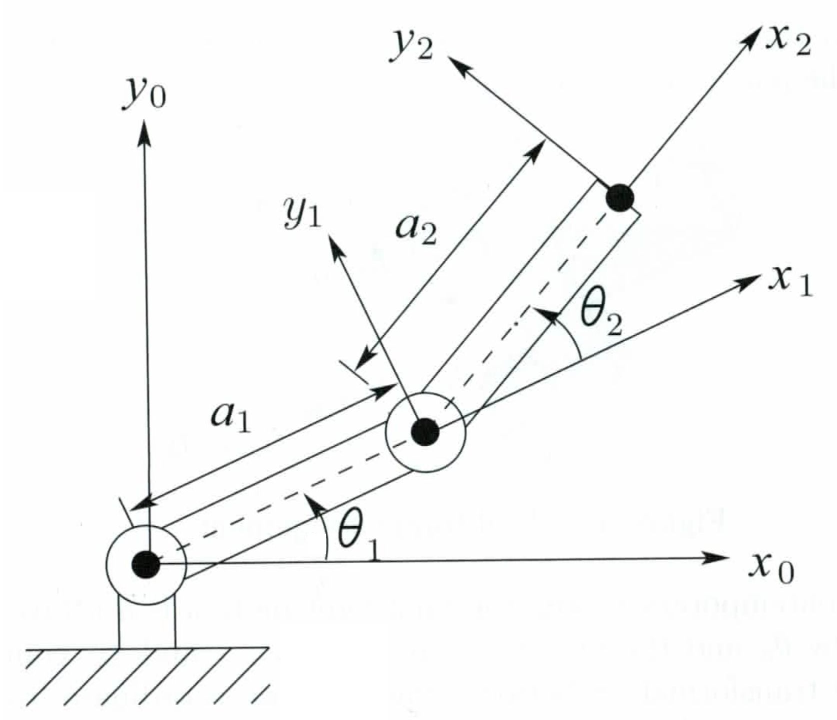


y_2
x_2
y_0
y_1
a_2
x_1
θ_2
a_1
θ_1
x_0

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_1^0 &= A_1 \\ T_2^0 &= A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

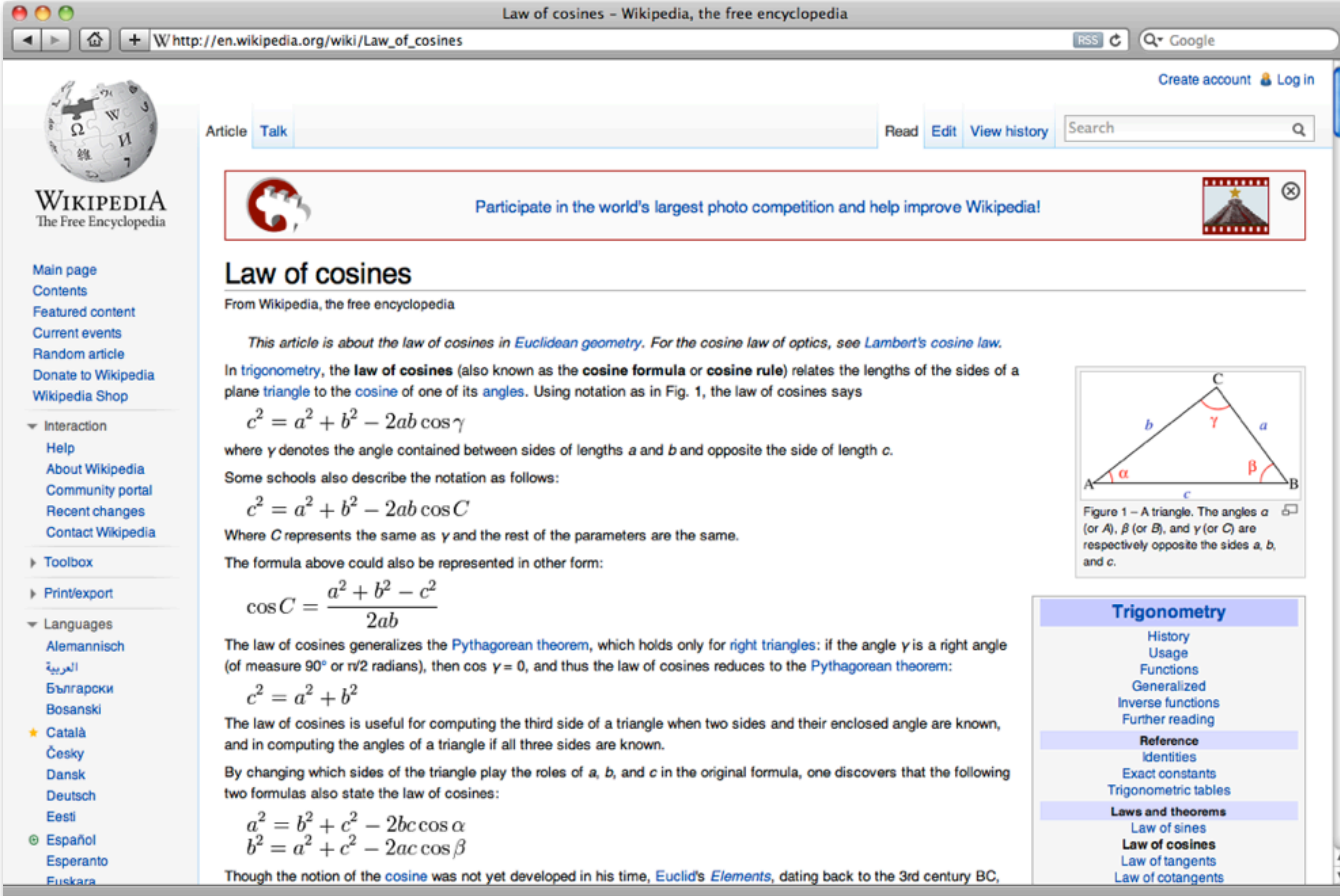


$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 \cos\theta_1^* + a_2 \cos(\theta_1^* + \theta_2^*) \\ a_1 \sin\theta_1^* + a_2 \sin(\theta_1^* + \theta_2^*) \\ 0 \end{bmatrix}$$

Given desired o_x and o_y coordinates,

$$\theta_1^* = \ ?$$

$$\theta_2^* = \ ?$$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Law of cosines

From Wikipedia, the free encyclopedia

This article is about the law of cosines in Euclidean geometry. For the cosine law of optics, see Lambert's cosine law.

In trigonometry, the **law of cosines** (also known as the **cosine formula** or **cosine rule**) relates the lengths of the sides of a plane triangle to the cosine of one of its angles. Using notation as in Fig. 1, the law of cosines says

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

where γ denotes the angle contained between sides of lengths a and b and opposite the side of length c.

Some schools also describe the notation as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Where C represents the same as γ and the rest of the parameters are the same.

The formula above could also be represented in other form:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The law of cosines generalizes the Pythagorean theorem, which holds only for right triangles: if the angle γ is a right angle (of measure 90° or $\pi/2$ radians), then $\cos\gamma = 0$, and thus the law of cosines reduces to the Pythagorean theorem:

$$c^2 = a^2 + b^2$$

The law of cosines is useful for computing the third side of a triangle when two sides and their enclosed angle are known, and in computing the angles of a triangle if all three sides are known.

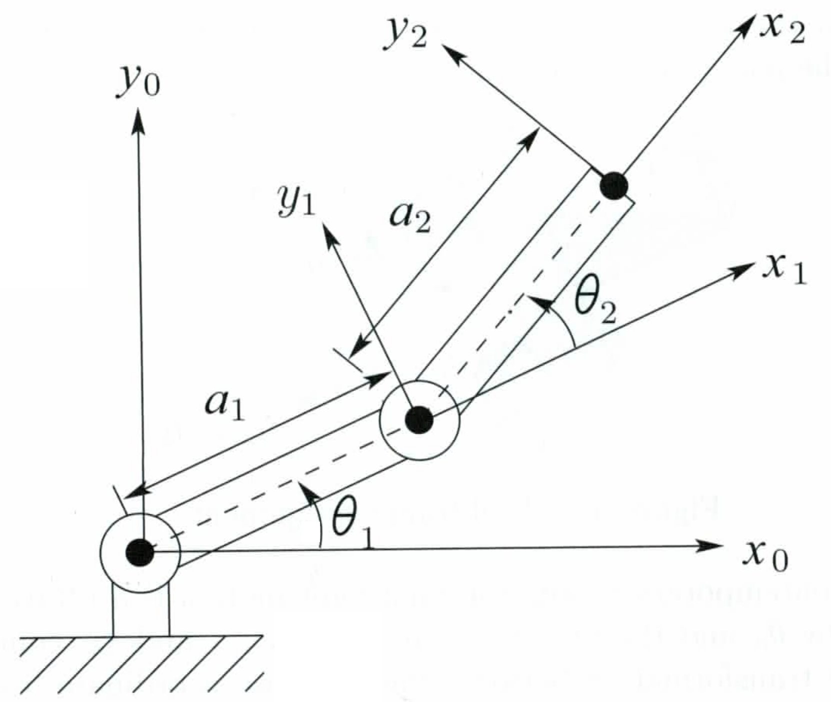
By changing which sides of the triangle play the roles of a, b, and c in the original formula, one discovers that the following two formulas also state the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

Though the notion of the cosine was not yet developed in his time, Euclid's *Elements*, dating back to the 3rd century BC,

Figure 1 – A triangle. The angles α (or A), β (or B), and γ (or C) are respectively opposite the sides a, b, and c.

Trigonometry
History
Usage
Functions
Generalized
Inverse functions
Further reading
Reference
Identities
Exact constants
Trigonometric tables
Laws and theorems
Law of sines
Law of cosines
Law of tangents
Law of cotangents



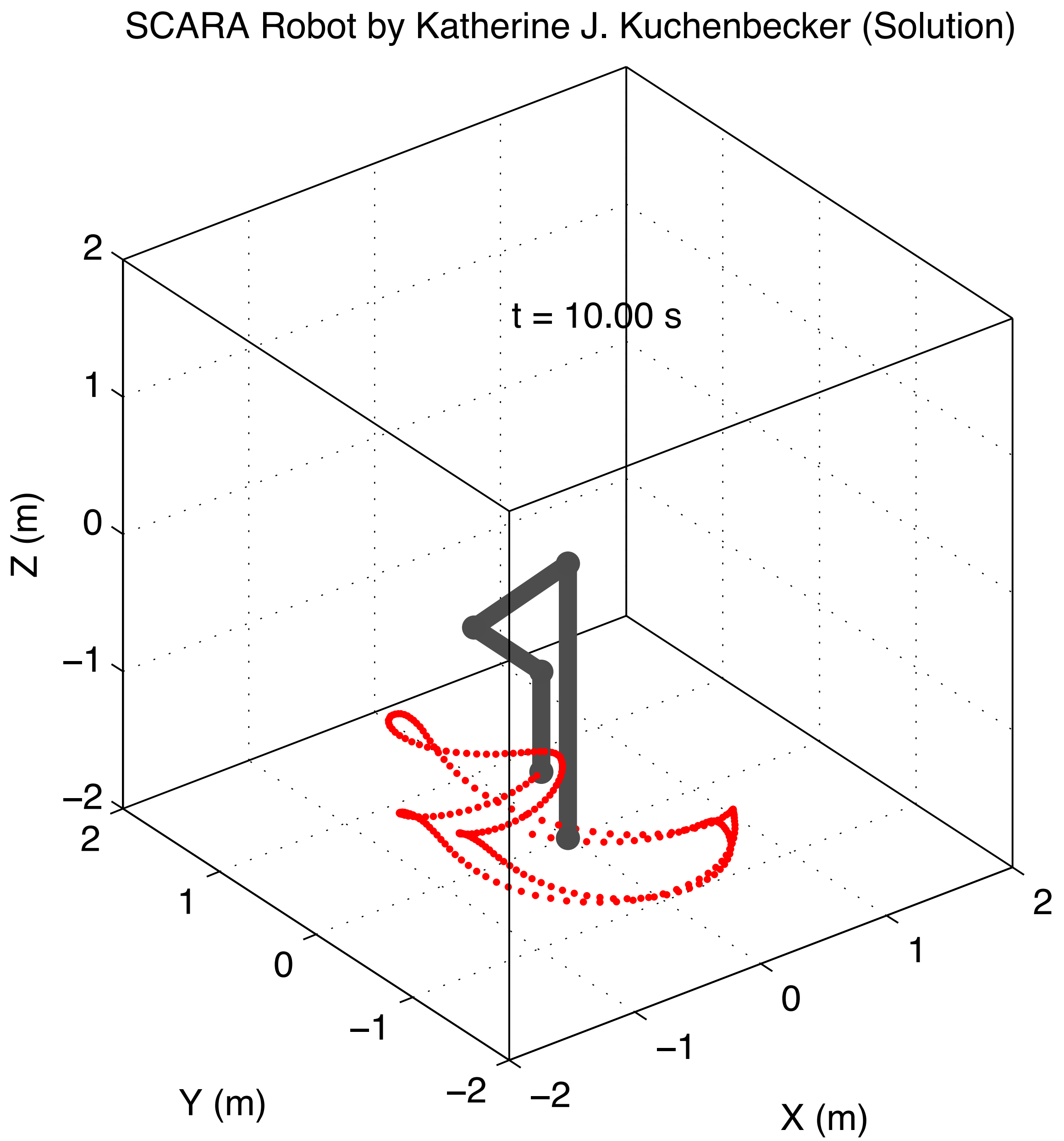
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 \cos\theta_1^* + a_2 \cos(\theta_1^* + \theta_2^*) \\ a_1 \sin\theta_1^* + a_2 \sin(\theta_1^* + \theta_2^*) \\ 0 \end{bmatrix}$$

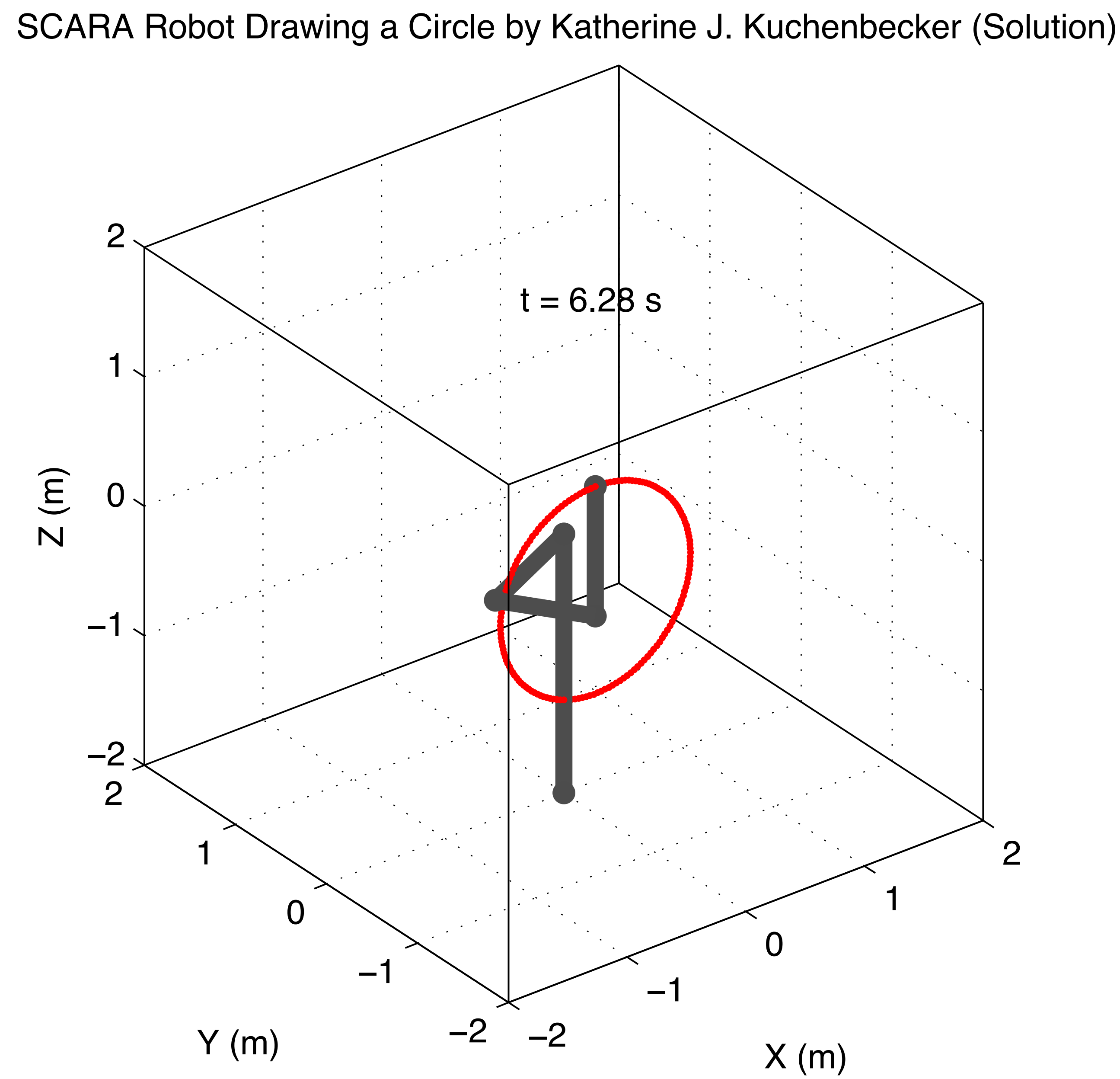
$$\cos\theta_2^* = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

Do these answers look familiar?

$$\theta_2^* = \operatorname{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2^*}}{\cos\theta_2^*}\right)$$

$$\theta_1^* = \operatorname{atan2}\left(\frac{o_y}{o_x}\right) - \operatorname{atan2}\left(\frac{a_2\sin\theta_2^*}{a_1 + a_2\cos\theta_2^*}\right)$$





$$\cos\theta_2 = \frac{o_x^2 + o_y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$\theta_2 = \mathrm{atan2}\left(\frac{\pm\sqrt{1-\cos^2\theta_2}}{\cos\theta_2}\right)$$

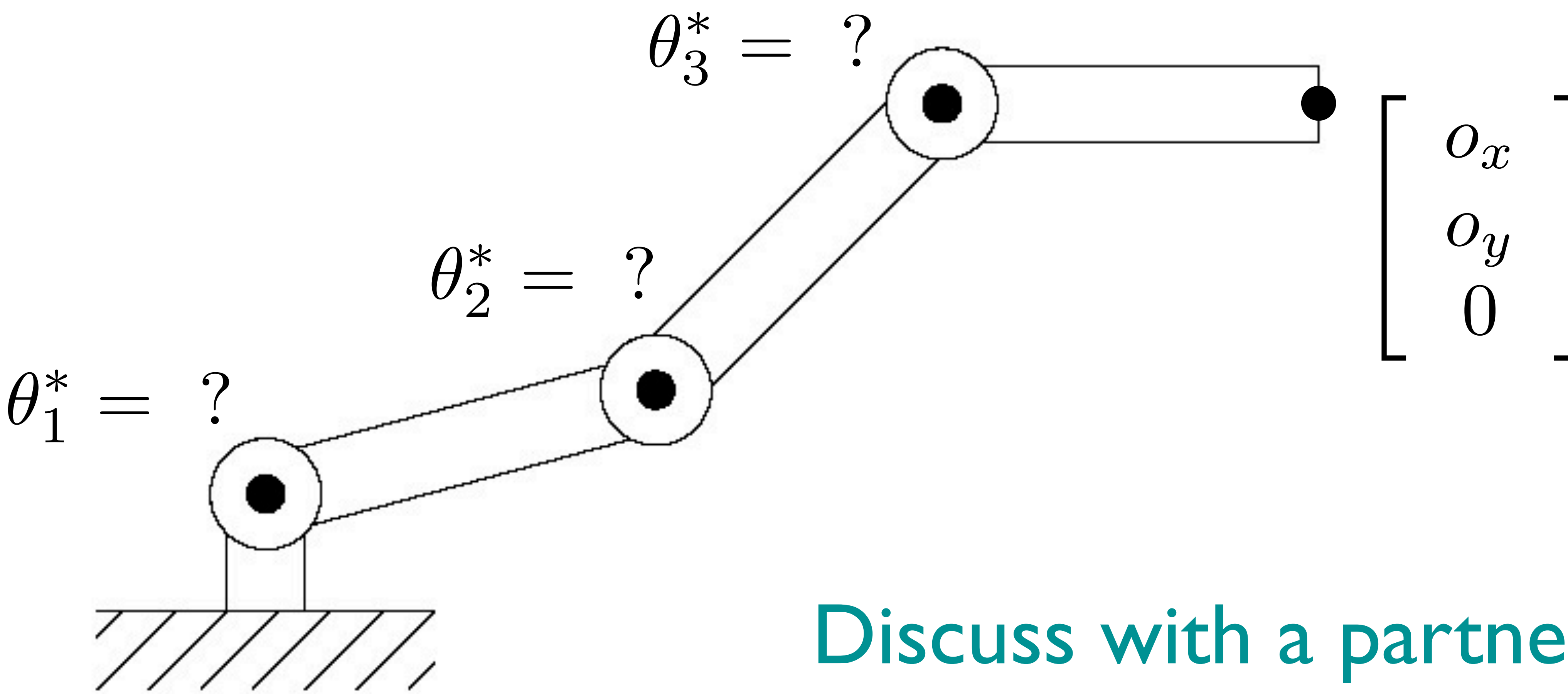
$$\theta_1 = \mathrm{atan2}\left(\frac{o_y}{o_x}\right) - \mathrm{atan2}\left(\frac{a_2\sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

$$d_3 = -o_z$$

Understand the usefulness of inverse kinematics.

Questions ?

An Even More Complicated Example – Planar RRR Robot

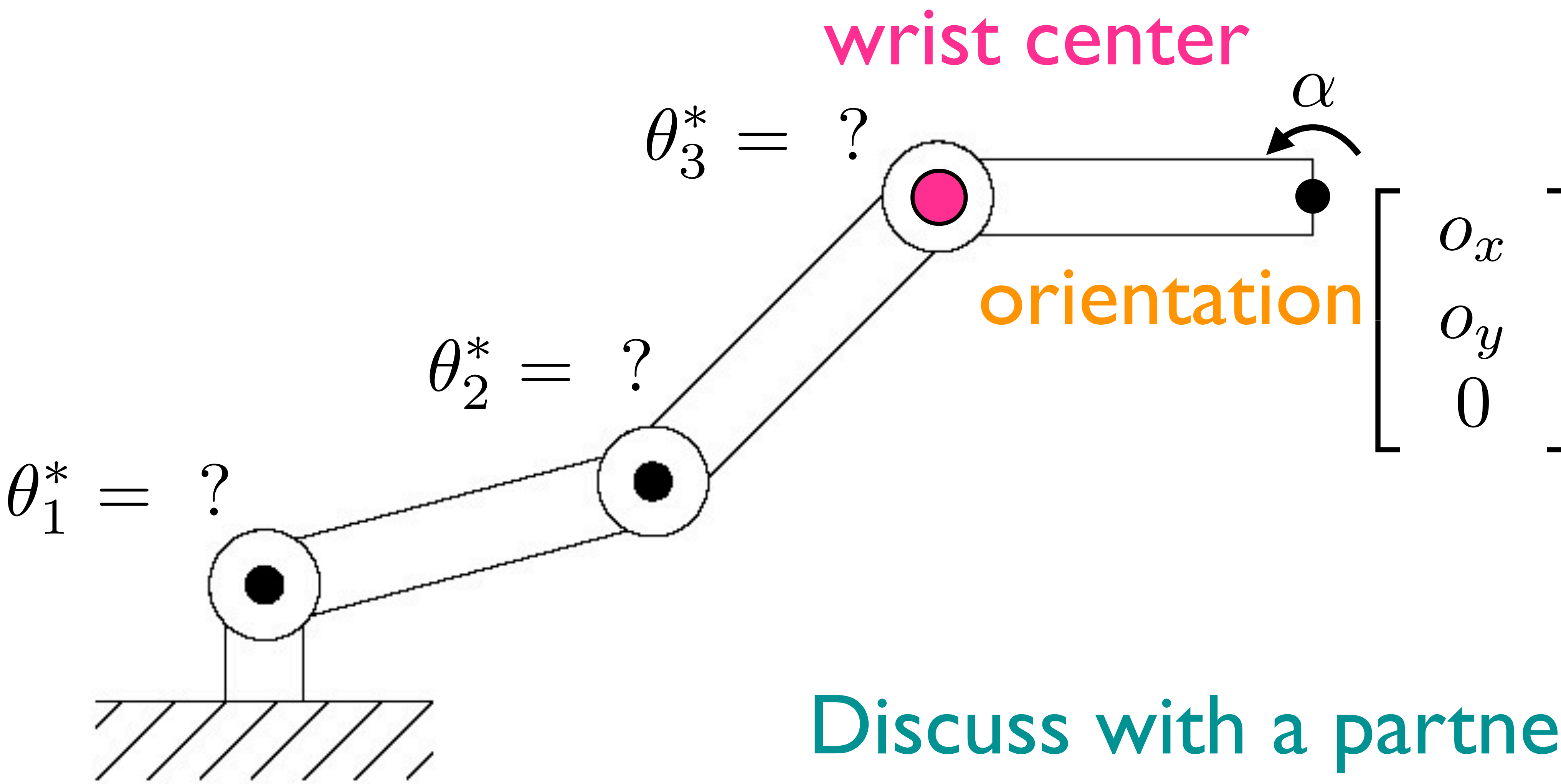


Discuss with a partner.

Given a desired position of the end effector, how many solutions are there to the inverse kinematics for this robot?

Infinitely many solutions if the target position is in the workspace
1 solution if the target position is on the workspace boundary
0 solutions if the target position is outside the workspace

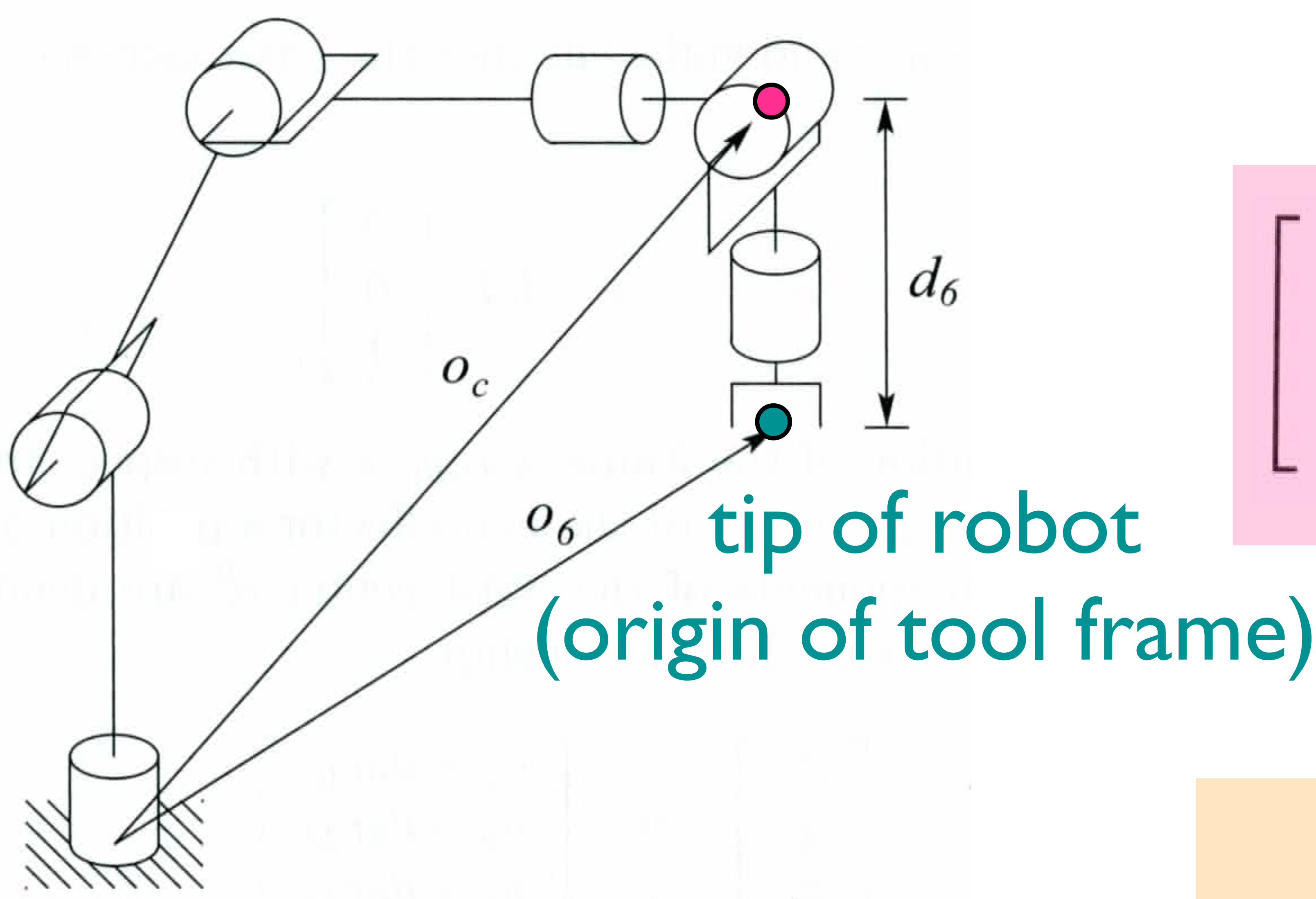
An Even More Complicated Example – Planar RRR Robot



Discuss with a partner.

If the orientation of the end effector is also specified, how many inverse kinematics solutions are there?

A helpful approach for 6-DOF robots: Kinematic Decoupling



$$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

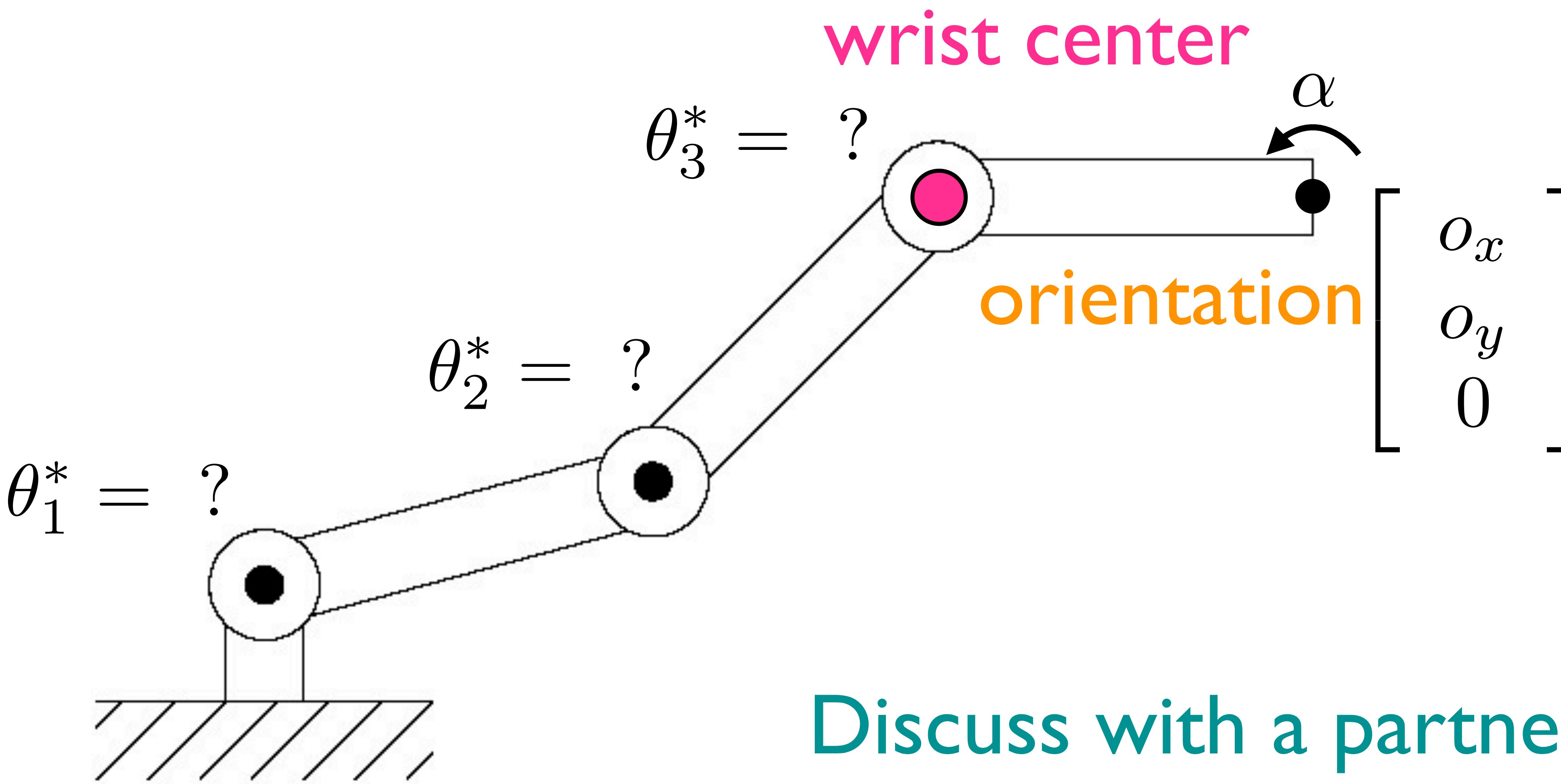
position

$$R = R_3^0 R_6^3$$

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$$

orientation

An Even More Complicated Example – Planar RRR Robot



Discuss with a partner.

If the orientation of the end effector is also specified, how many inverse kinematics solutions are there?

2 solutions if the wrist center is inside the 2-link workspace
1 solution if the wrist center is on the 2-link workspace boundary
0 solutions if the wrist position is outside the 2-link workspace
Infinitely many solutions if the wrist center is the origin

Questions ?