MEAM 520

Inverse Kinematics

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Lecture 7: September 27, 2012

Philip Dames

Ryan Wilson

Denise Wong

Pick up graded paper part of Homework 1 from the TAs

- Philip has last names from A to G
- Ryan has last names from H to Q
- Denise has last names from R to Z

Homework 2 due today by 5:00 p.m.

Homework 2:
Manipulator Kinematics and DH Parameters

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

September 18, 2012

This assignment is due on **Thursday, September 27 (updated)**, by 5:00 p.m. sharp. You should aim to turn the paper part in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. The code must be emailed according to the instructions at the end of this document. Late submissions of either or both parts will be accepted until 5:00 p.m. on Friday, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

Written Problems *(30 points)*

The first set of problems are written, including two from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions when provided. Write in pencil, show your work clearly, $\boxed{\text{box your answers}}$, and staple your pages together.

1. Custom problem – Kinematics of Baxter *(5 points)*
   Rethink Robotics recently released a new robot named Baxter. Watch YouTube videos of Baxter (e.g., http://www.youtube.com/watch?v=rjPFqkFyrOY) to learn about its kinematics. Draw a schematic of the serial kinematic chain of Baxter's left arm (the one the woman is touching in the picture above.) Use the book's conventions for how to draw revolute and prismatic joints in 3D.

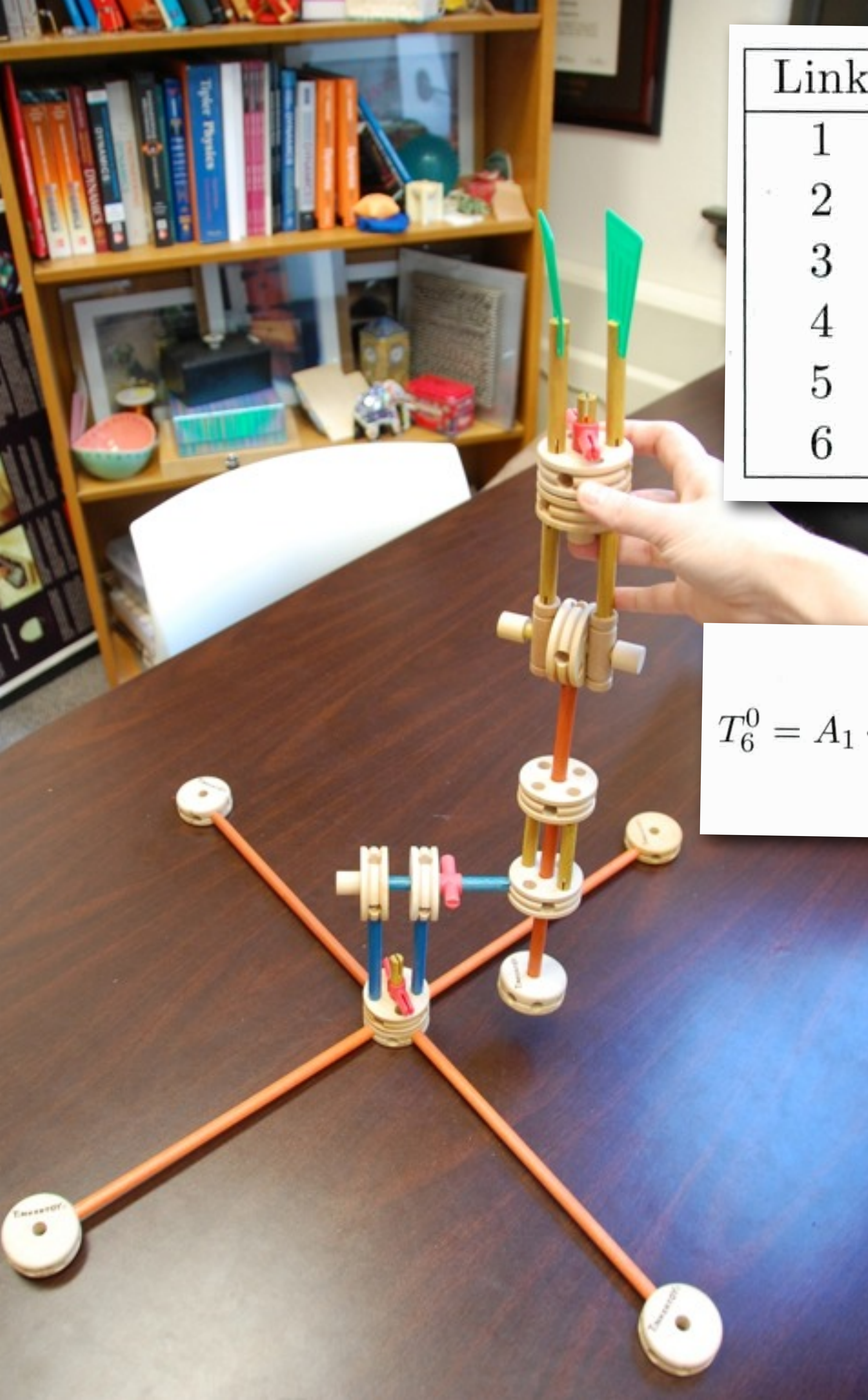
2. SHV 3-7, page 113 – Three-link Cartesian Robot *(10 points)*
   Your solution should include a schematic of the manipulator with appropriately placed coordinate frames, a table of the DH parameters, and the final transformation matrix. Then answer the following question: What are the x, y, and z coordinates of the tip of the robot's end-effector in the base frame (as a function of the robot parameters and the joint coordinates)?

Comment your code.

Do not hack. Get the math to work correctly.

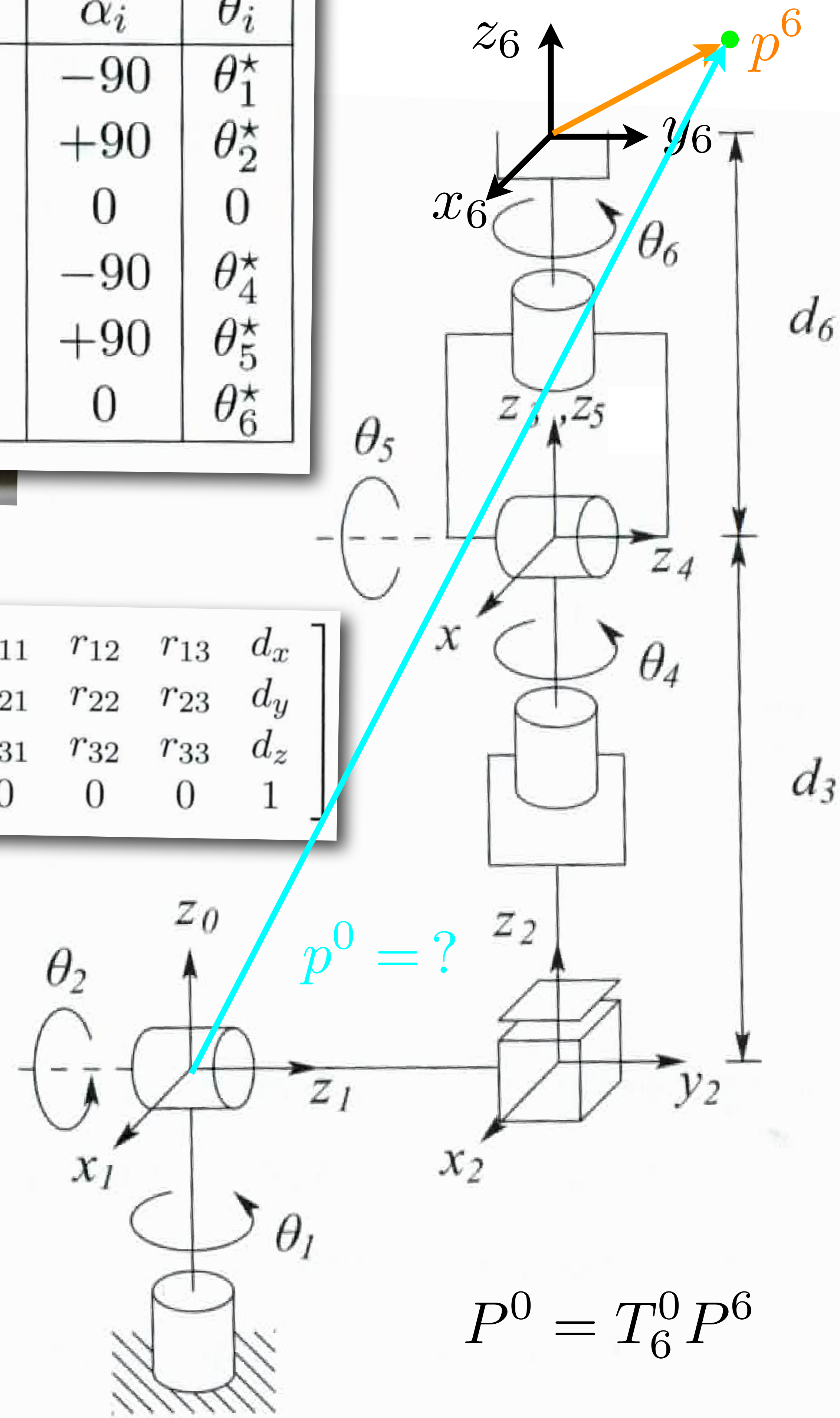
Homework 3 will go out on Tuesday,
due the following Tuesday.

Stanford Manipulator



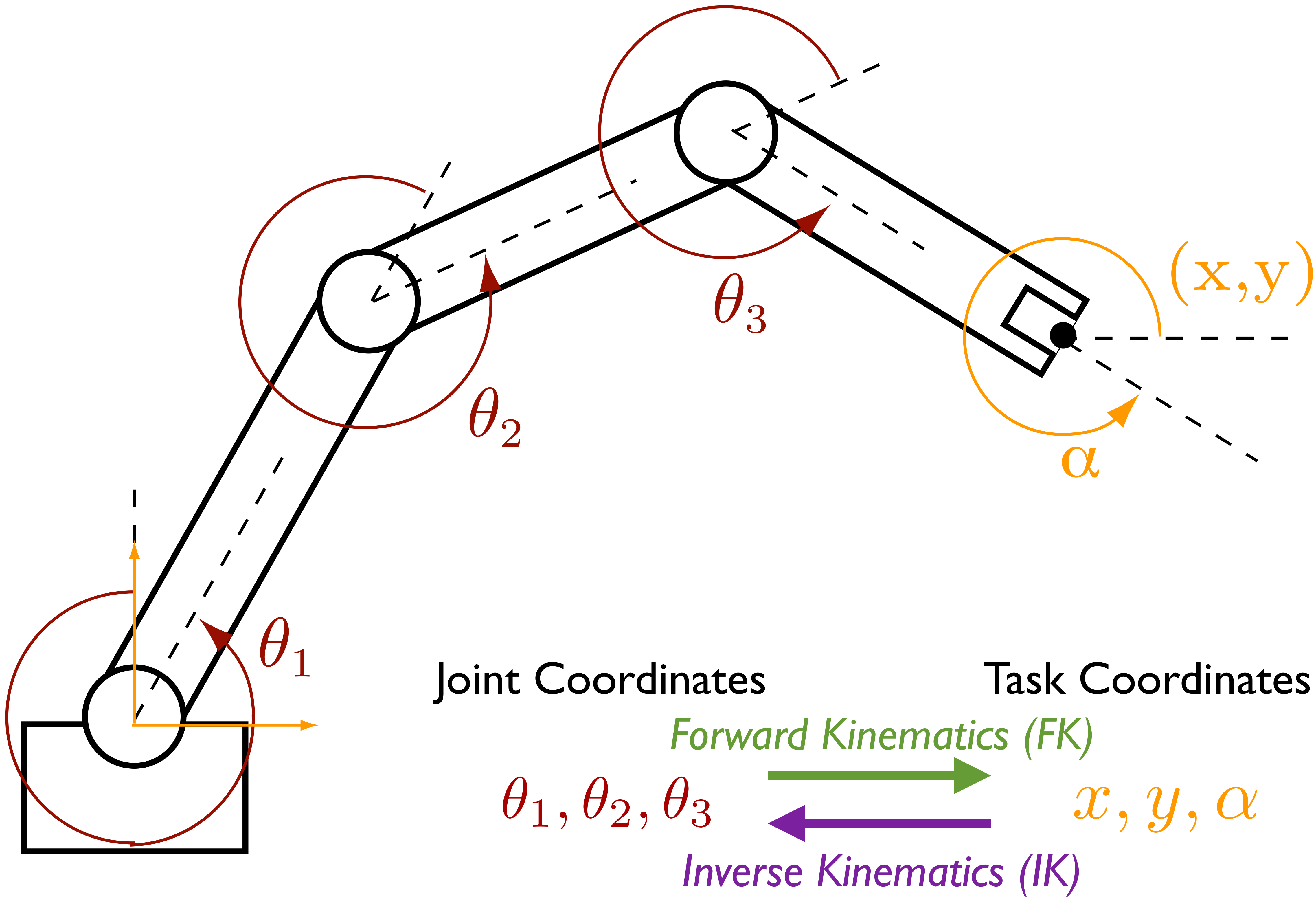
Link	d_i	a_i	α_i	θ_i
1	0	0	-90	$\theta_1^\star$
2	d_2	0	$+90$	$\theta_2^\star$
3	$d_3^\star$	0	0	0
4	0	0	-90	$\theta_4^\star$
5	0	0	$+90$	$\theta_5^\star$
6	d_6	0	0	$\theta_6^\star$

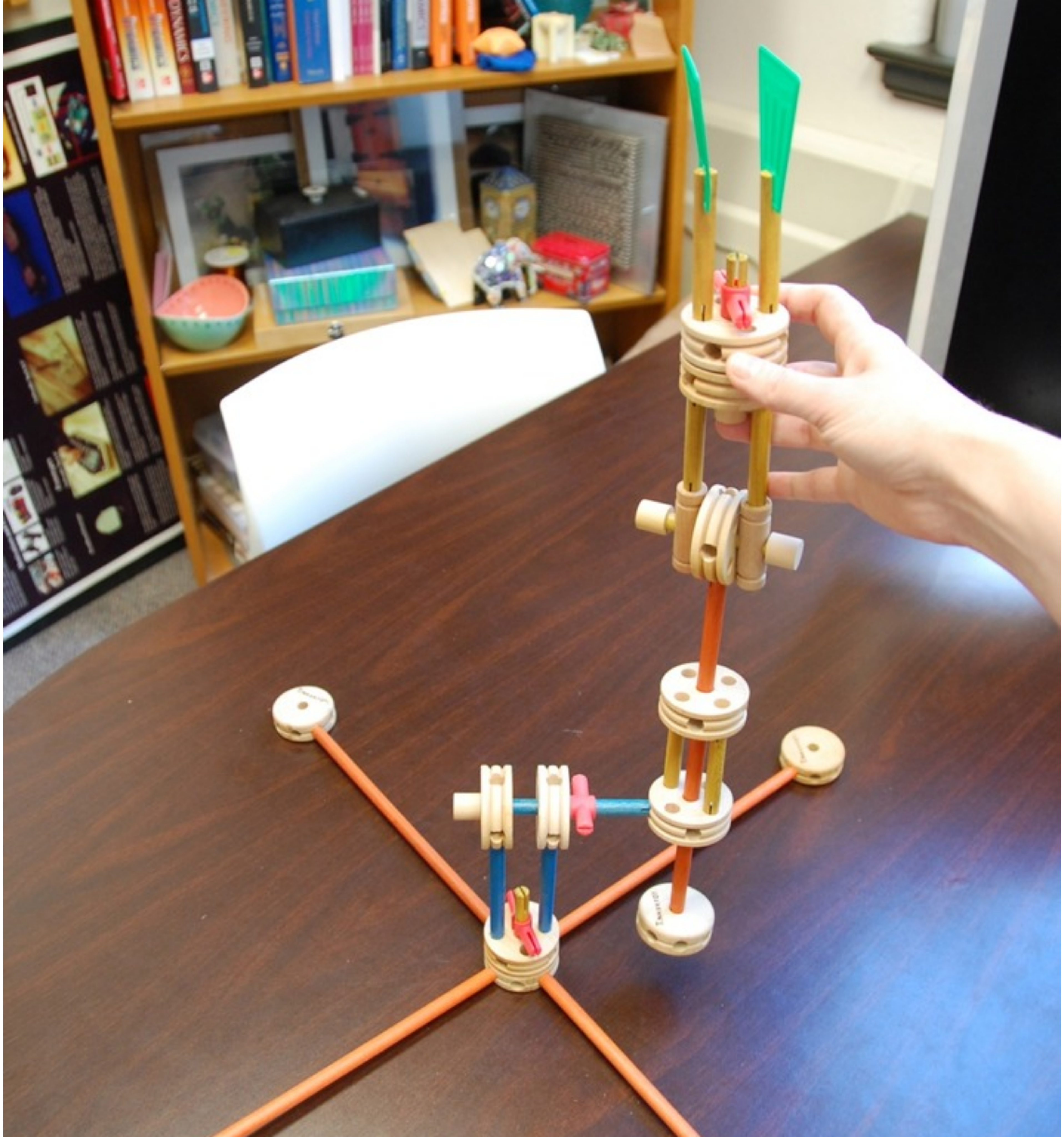
$$T_6^0 = A_1 \cdots A_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$P^0 = T_6^0 P^6$$

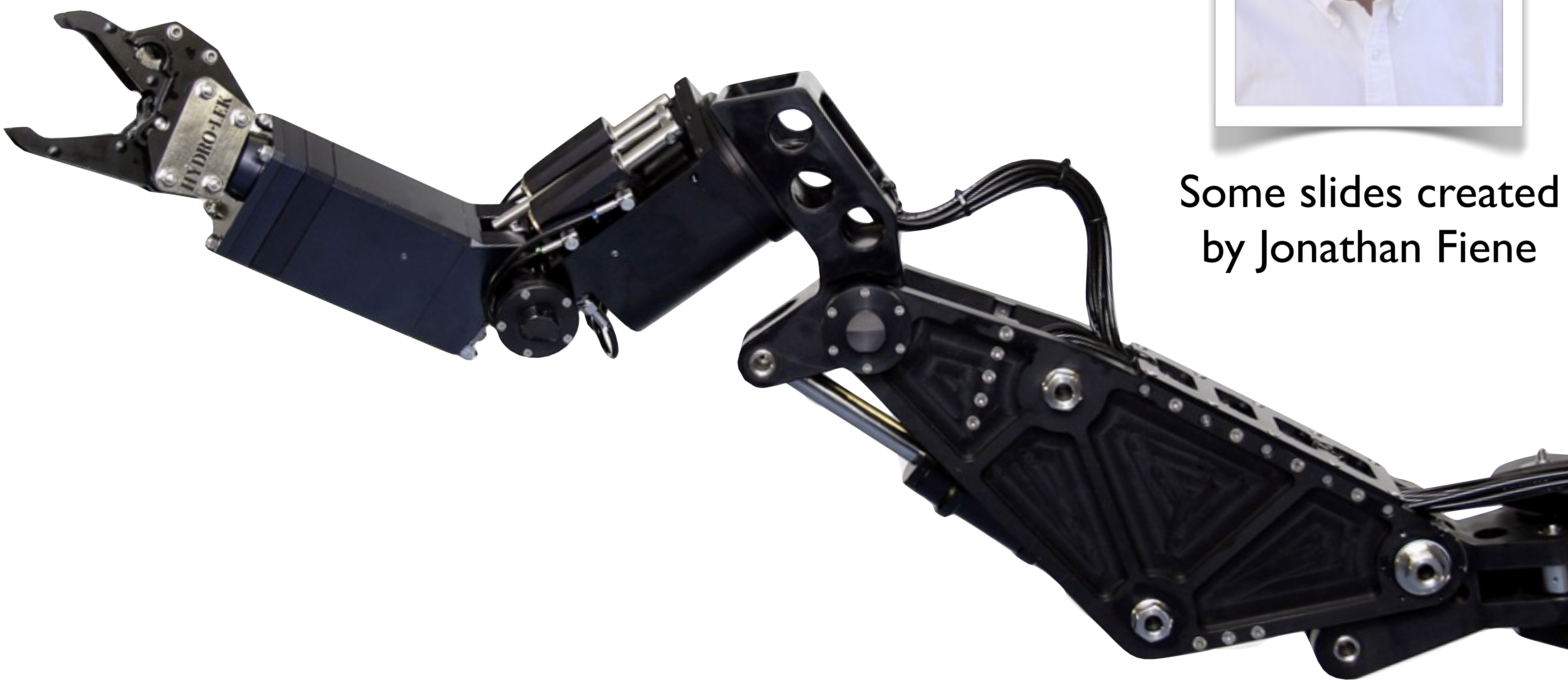
Joint and Task Coordinates

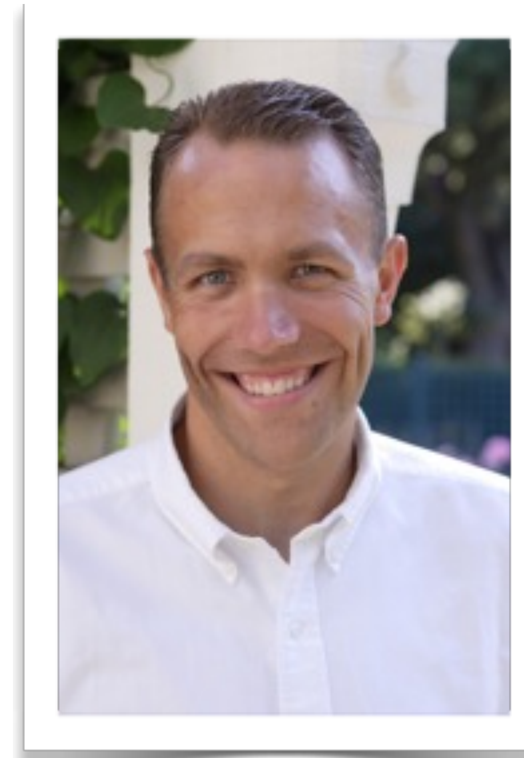




Questions ❓

Inverse Kinematics





Some slides created by Jonathan Fiene

$$\text{given } \mathbf{H} = \begin{bmatrix} \mathbf{R} & o \\ 0 & 1 \end{bmatrix}$$

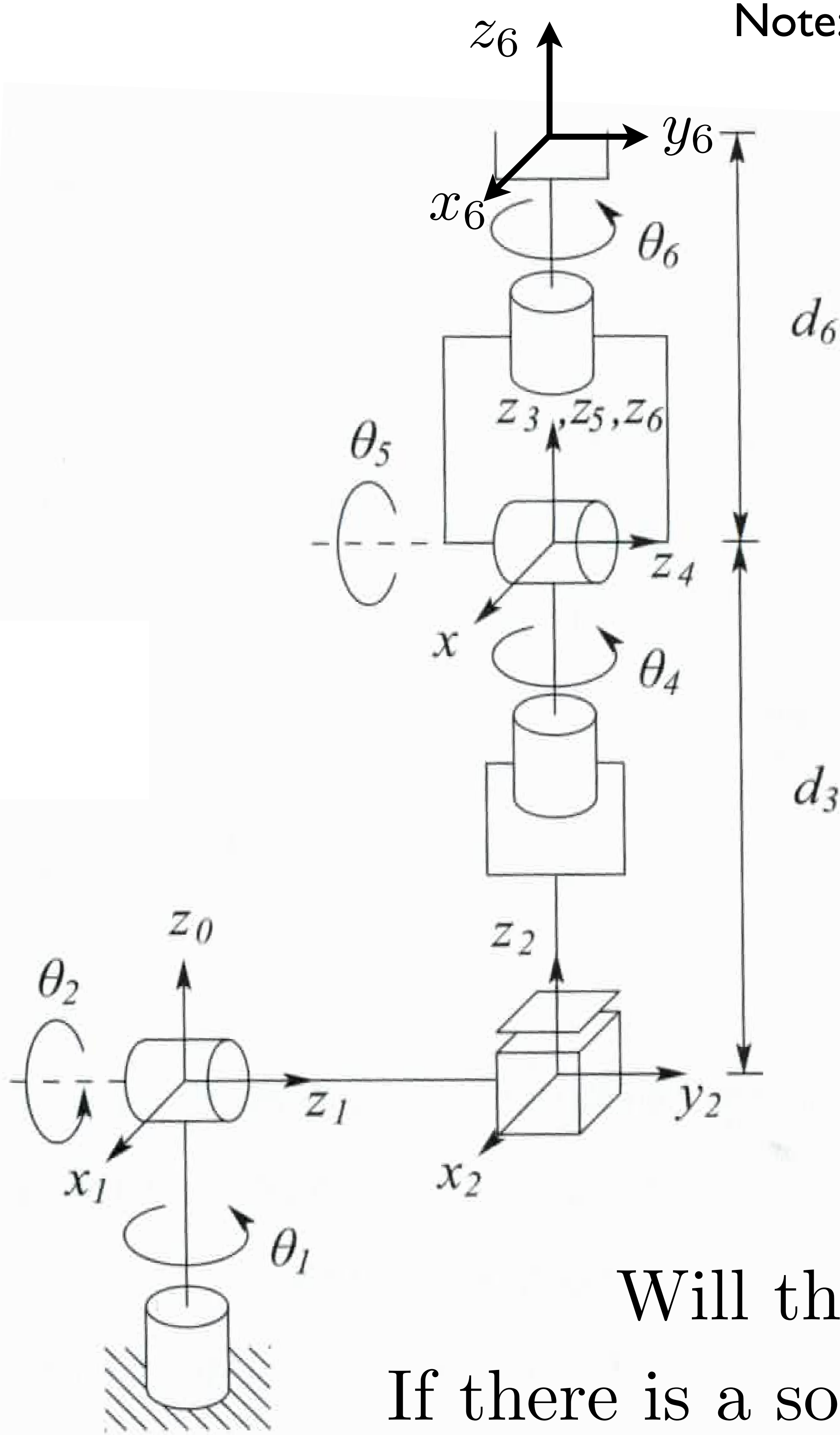
and a certain manipulator with n joints

find $q_1, \ldots, q_n$ such that $\mathbf{T}_n^0(q_1, \ldots, q_n) = \mathbf{H}$

This yields 12 nonlinear equations in n unknown variables.

Yuck.

Note: this slide previously showed the wrong forward kinematics.



$$T_6^0 = A_1 \cdots A_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

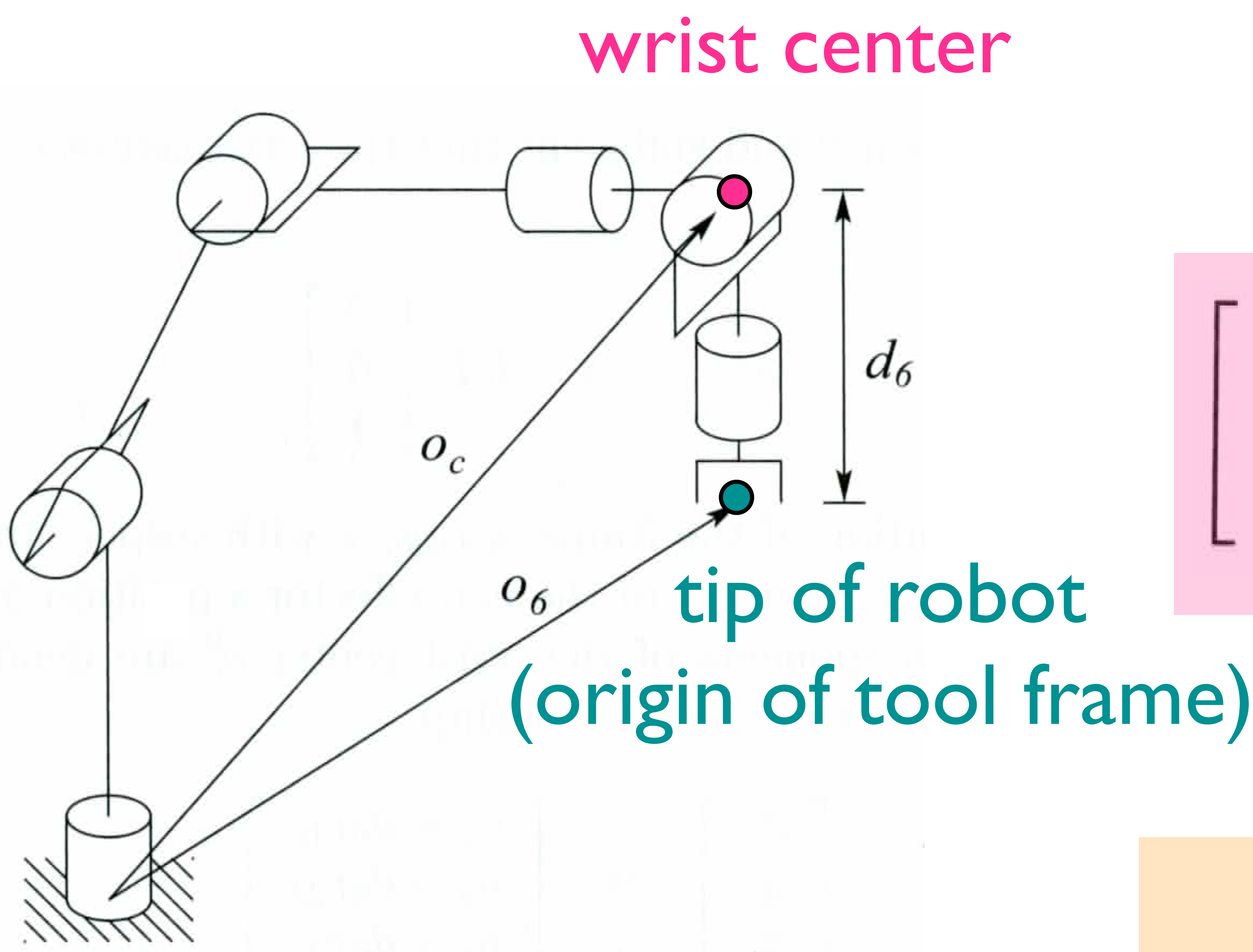
in which

$$\begin{aligned}
r_{11} &= c_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] - d_2(s_4c_5c_6 + c_4s_6) \\
r_{21} &= s_1[c_2(c_4c_5c_6 - s_4s_6) - s_2s_5c_6] + c_1(s_4c_5c_6 + c_4s_6) \\
r_{31} &= -s_2(c_4c_5c_6 - s_4s_6) - c_2s_5c_6 \\
r_{12} &= c_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] - s_1(-s_4c_5s_6 + c_4c_6) \\
r_{22} &= -s_1[-c_2(c_4c_5s_6 + s_4c_6) + s_2s_5s_6] + c_1(-s_4c_5s_6 + c_4c_6) \\
r_{32} &= s_2(c_4c_5s_6 + s_4c_6) + c_2s_5s_6 \\
r_{13} &= c_1(c_2c_4s_5 + s_2c_5) - s_1s_4s_5 \\
r_{23} &= s_1(c_2c_4s_5 + s_2c_5) + c_1s_4s_5 \\
r_{33} &= -s_2c_4s_5 + c_2c_5 \\
d_x &= c_1s_2d_3 - s_1d_2 + +d_6(c_1c_2c_4s_5 + c_1c_5s_2 - s_1s_4s_5) \\
d_y &= s_1s_2d_3 + c_1d_2 + d_6(c_1s_4s_5 + c_2c_4s_1s_5 + c_5s_1s_2) \\
d_z &= c_2d_3 + d_6(c_2c_5 - c_4s_2s_5)
\end{aligned}$$

Will there always be a solution? No.

If there is a solution, will it always be unique? No.

A helpful approach for 6-DOF robots: Kinematic Decoupling



$$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

position

$$R = R_3^0 R_6^3$$

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$$

orientation

Questions ❓

Inverse Position Kinematics

Algebraic vs. Geometric

Algebraic Decomposition

given the forward transform matrix for a manipulator

$$\mathbf{T}_n^0 = \begin{bmatrix} \left[\mathbf{R}_n^0(\mathbf{q})\right]_{3\times 3} & \left[\mathbf{d}_n^0(\mathbf{q})\right]_{3\times 1} \\ [\mathbf{0}]_{1\times 3} & 1 \end{bmatrix}$$

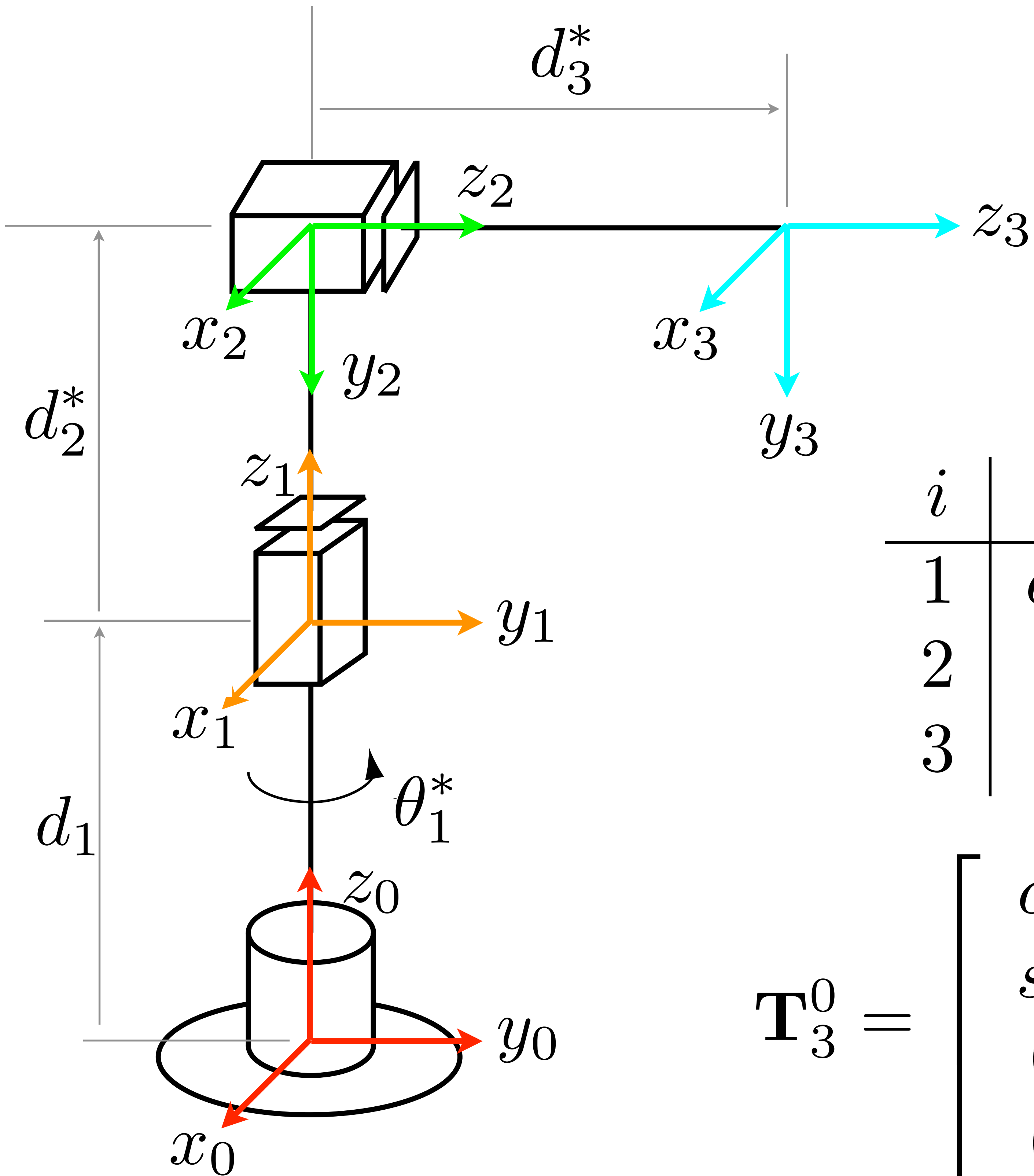
solve the system of 3 equations from the displacement vector

$$\begin{aligned} d_x &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_1 \\ d_y &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_2 \\ d_z &= \left[\mathbf{d}_n^0(\mathbf{q})\right]_3 \end{aligned}$$

to find the joint variables in terms of the end-effector position

$$\mathbf{q} = \begin{bmatrix} q_1(d_x, d_y, d_z) \\ q_2(d_x, d_y, d_z) \\ \vdots \\ q_n(d_x, d_y, d_z) \end{bmatrix}$$

The RPP Cylindrical Robot - Algebraic Approach



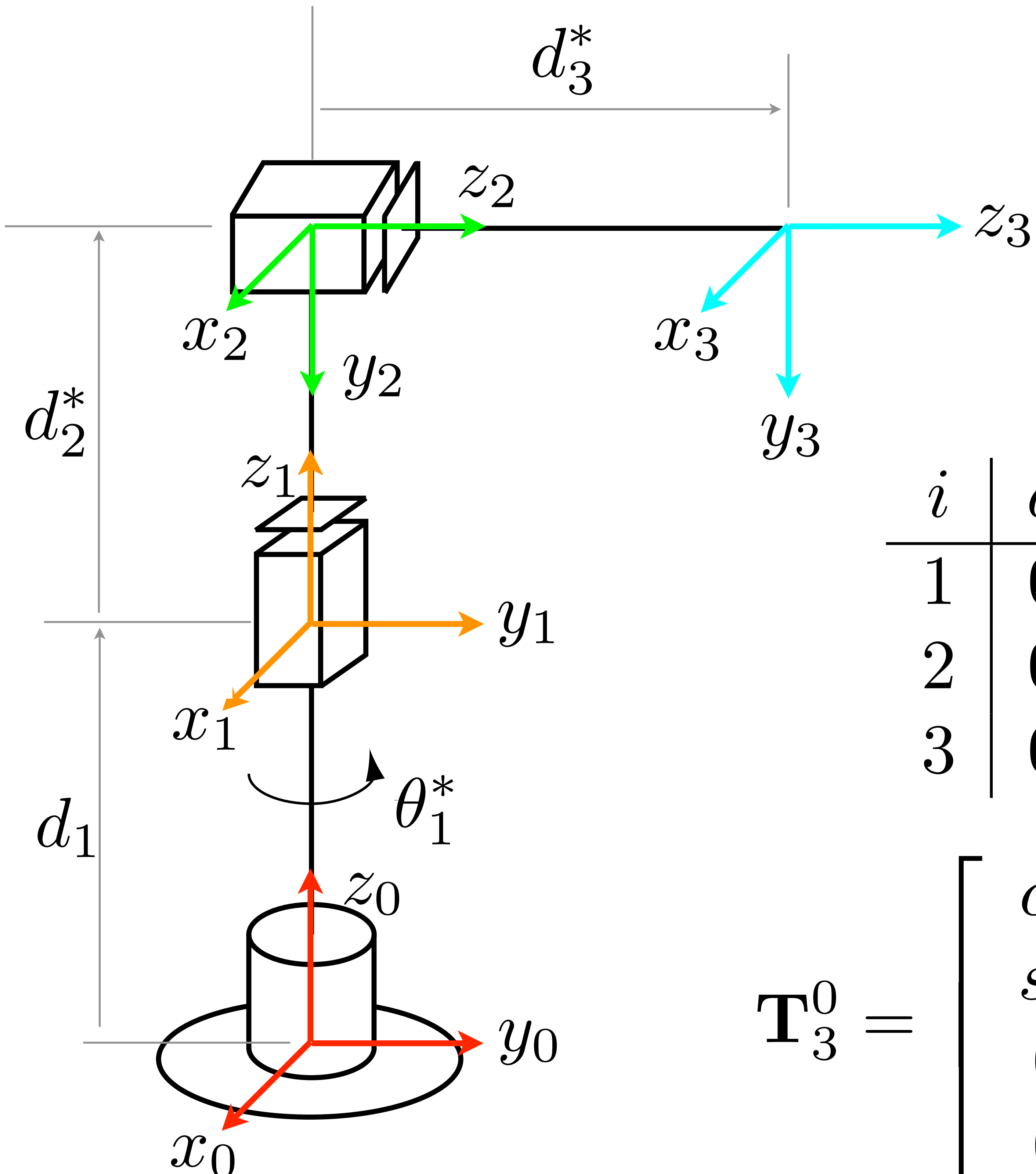
correct?

i	a	α	d	θ
1	a_1	$0°$	d_1	θ_1^*
2	0	$+90°$	d_2^*	$0°$
3	0	$0°$	d_3^*	$90°$

correct?

$$\mathbf{T}_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

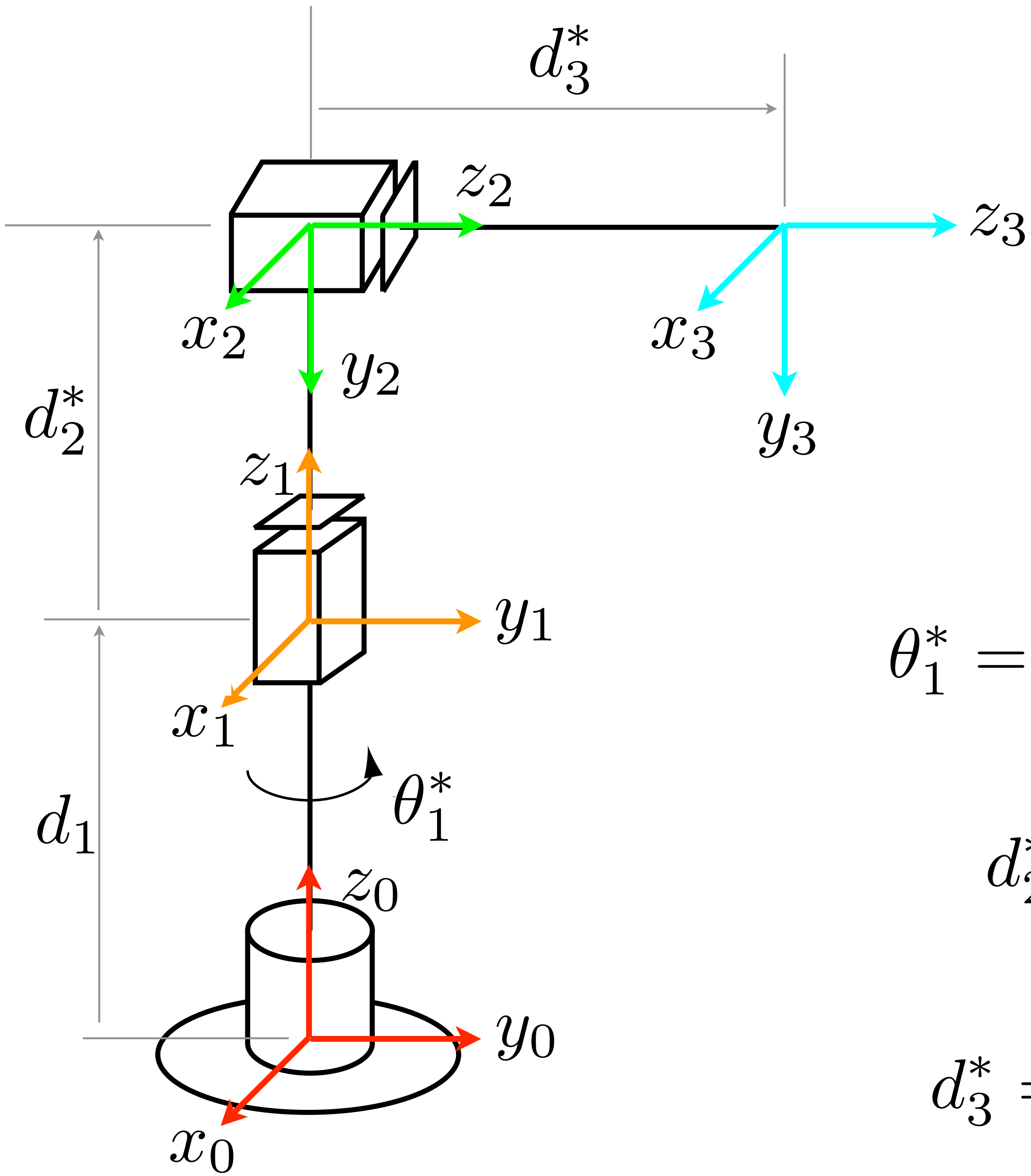
The RPP Cylindrical Robot - Algebraic Approach



i	a	α	d	θ
1	0	$0°$	d_1	θ_1^*
2	0	$-90°$	d_2^*	$0°$
3	0	$0°$	d_3^*	$0°$

$$\mathbf{T}_3^0 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The RPP Cylindrical Robot - Algebraic Approach



$$\theta_1^* = \mathrm{atan2}\left(\frac{-x}{y}\right)$$

$$d_2^* = z - d_1$$

$$d_3^* = \sqrt{x^2 + y^2}$$

Is there always a solution?

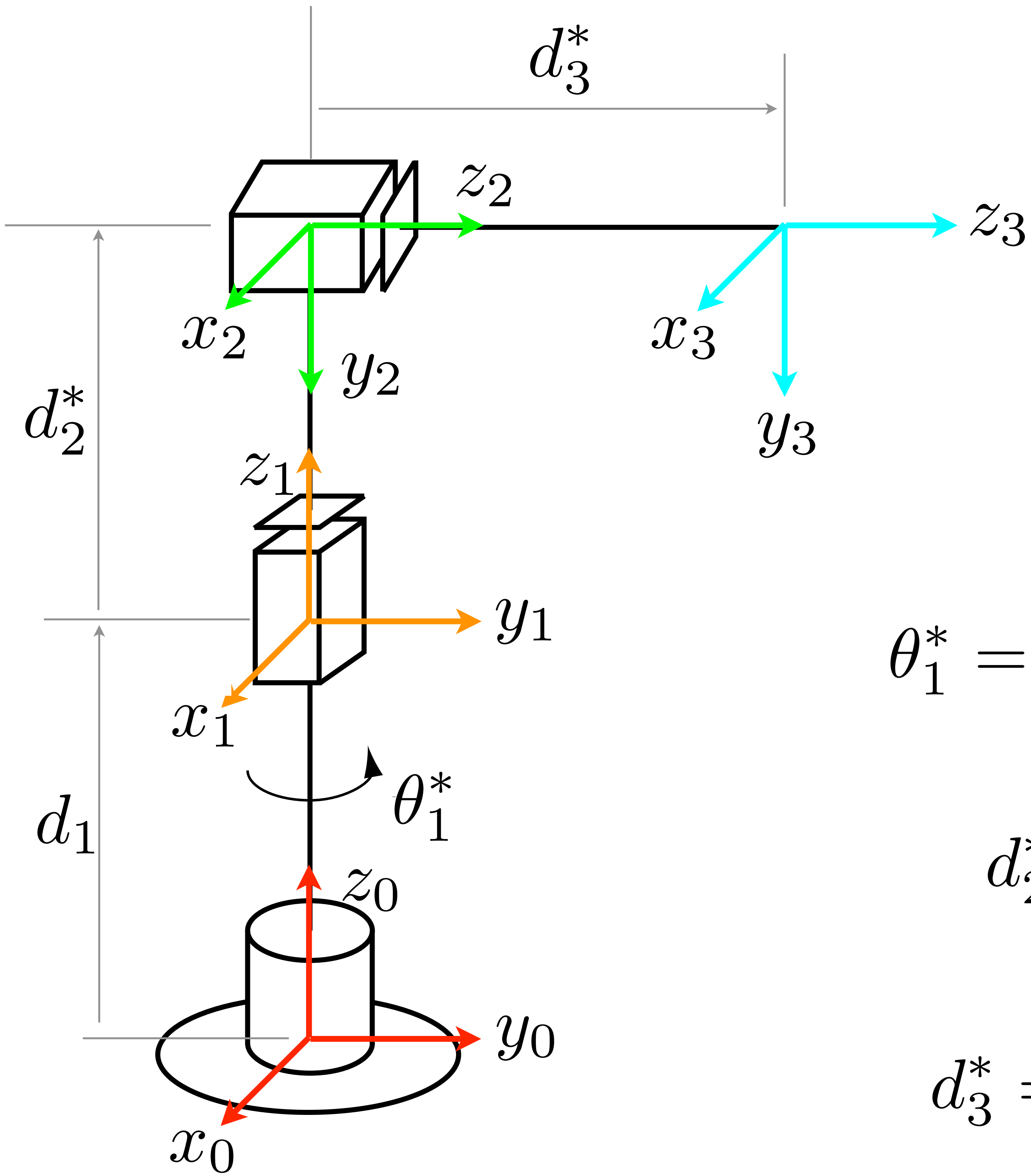
Geometric Analysis

For most simple manipulators, it is easier to use geometry to solve for closed-form solutions to the inverse kinematics

solve for each joint variable q_i
by projecting the manipulator onto the x_{i-1}, y_{i-1} plane

closed-form inverse kinematic solutions are not always possible, and if it is solvable, there are often multiple solutions

The RPP Cylindrical Robot - Geometric Approach



$$\theta_1^* = \operatorname{atan2}\left(\frac{-x}{y}\right)$$

$$d_2^* = z - d_1$$

$$d_3^* = \sqrt{x^2 + y^2}$$

Same answers!

Questions ?

Handing back Homework 1 paper part.
Grades are on Blackboard.
Solutions to both parts on reserve in engineering library.

Grading concerns? Bring your paper to office hours.

Philip Dames

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