MEAM 520

Denavit-Hartenberg (DH)

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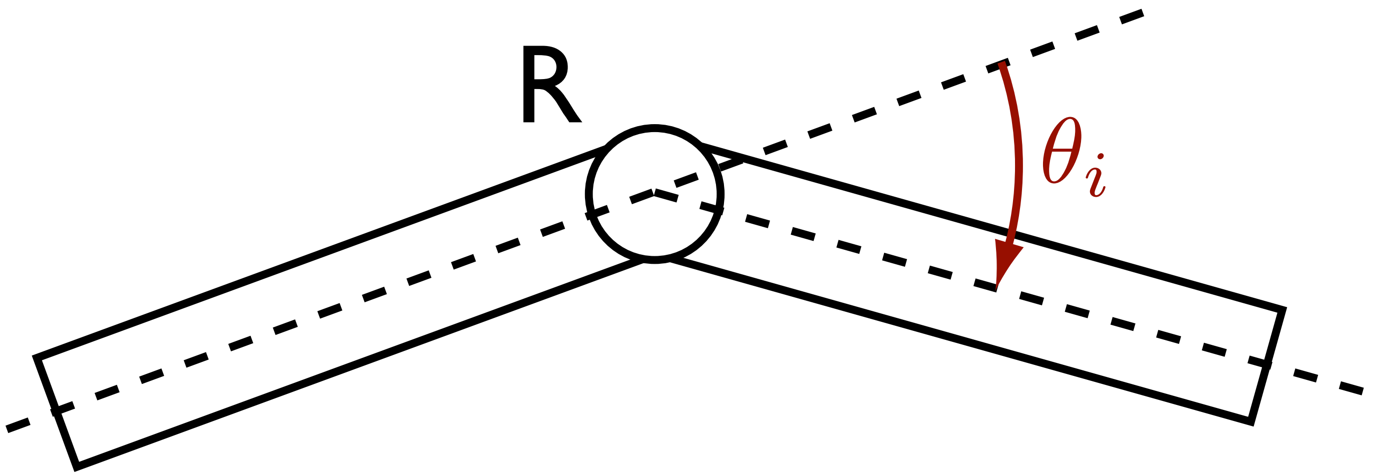
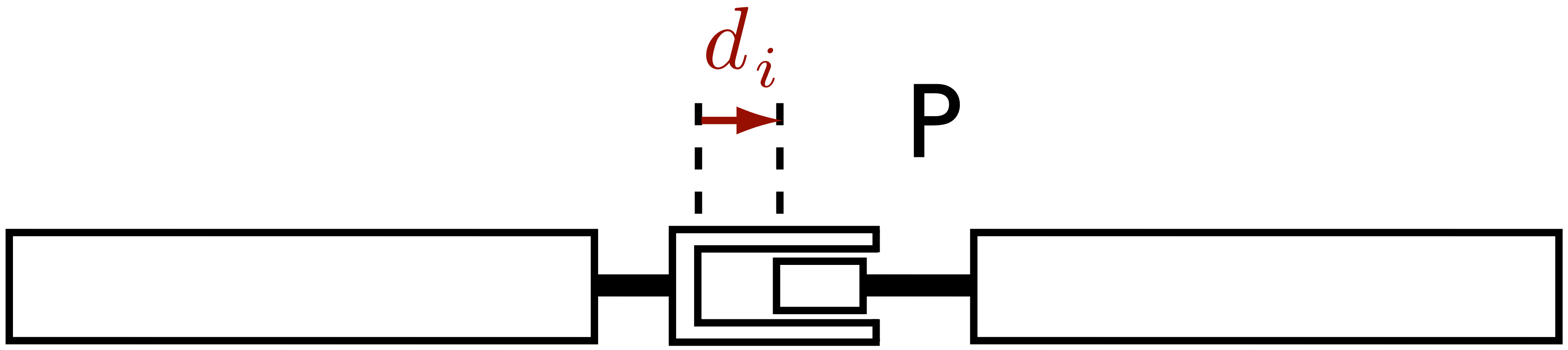
Lecture 5: September 20, 2012

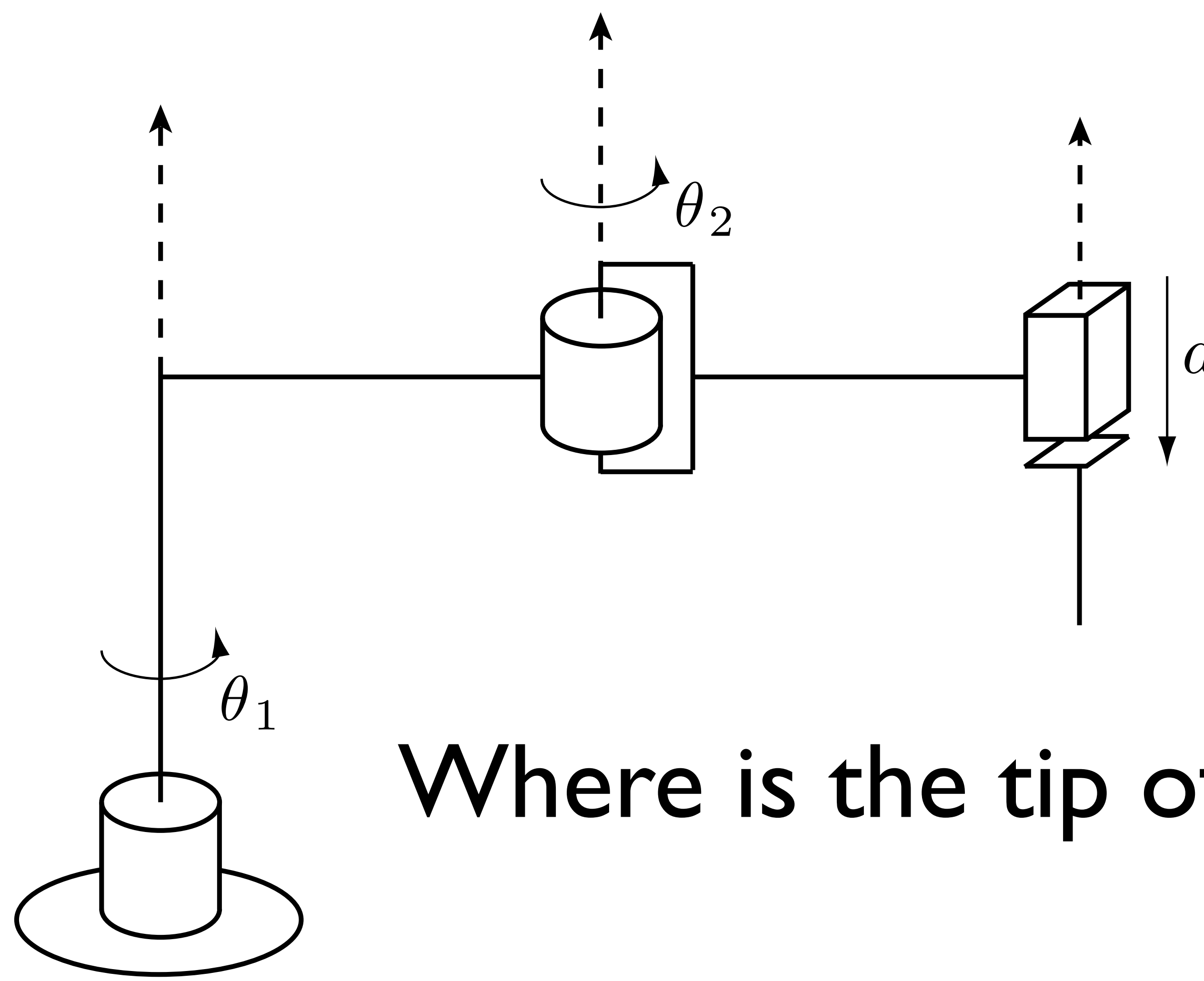




$$\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{d} \\ \mathbf{0} & 1 \end{bmatrix}$$

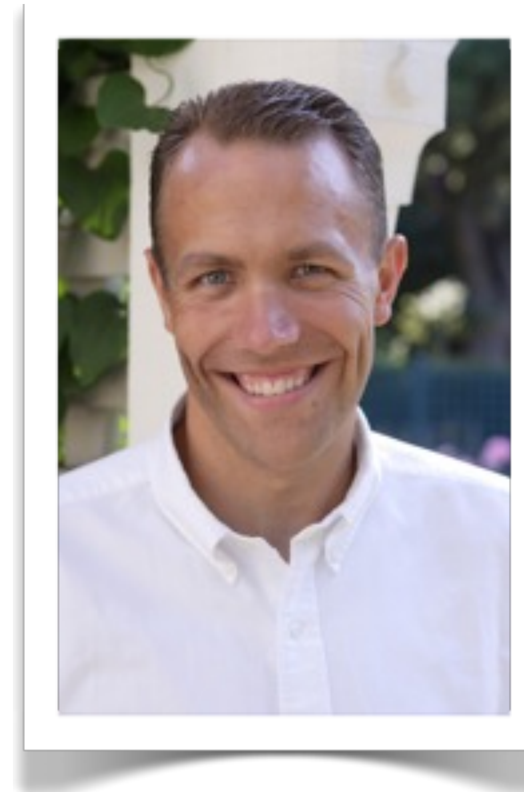
$$\mathbf{P}^0 = \mathbf{H}_1^0 \, \mathbf{P}^1$$

$$\mathbf{H}_2^0 = \mathbf{H}_1^0 \, \mathbf{H}_2^1$$



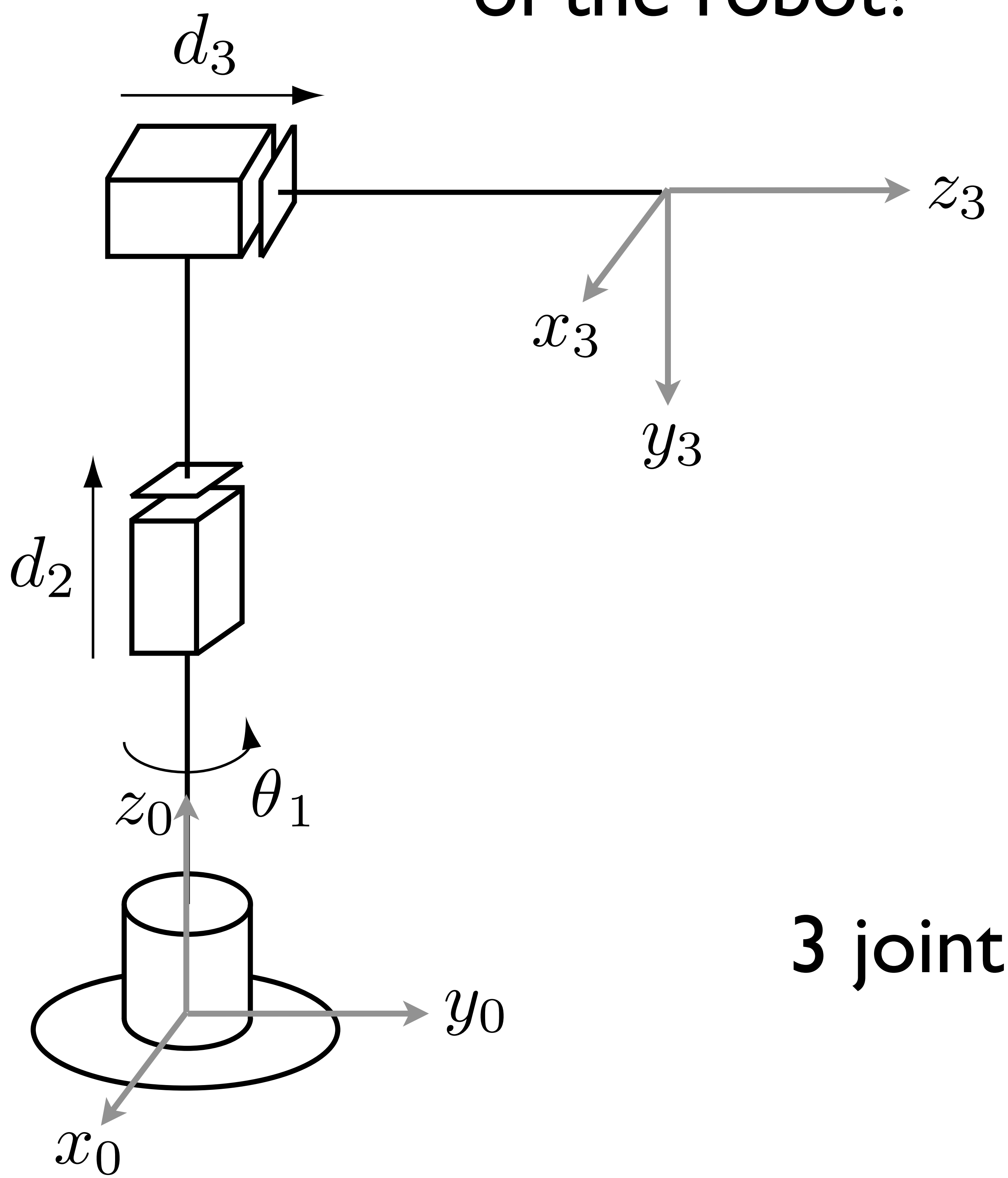
Where is the tip of the robot?

Forward Kinematics



Slides created by Jonathan Fiene

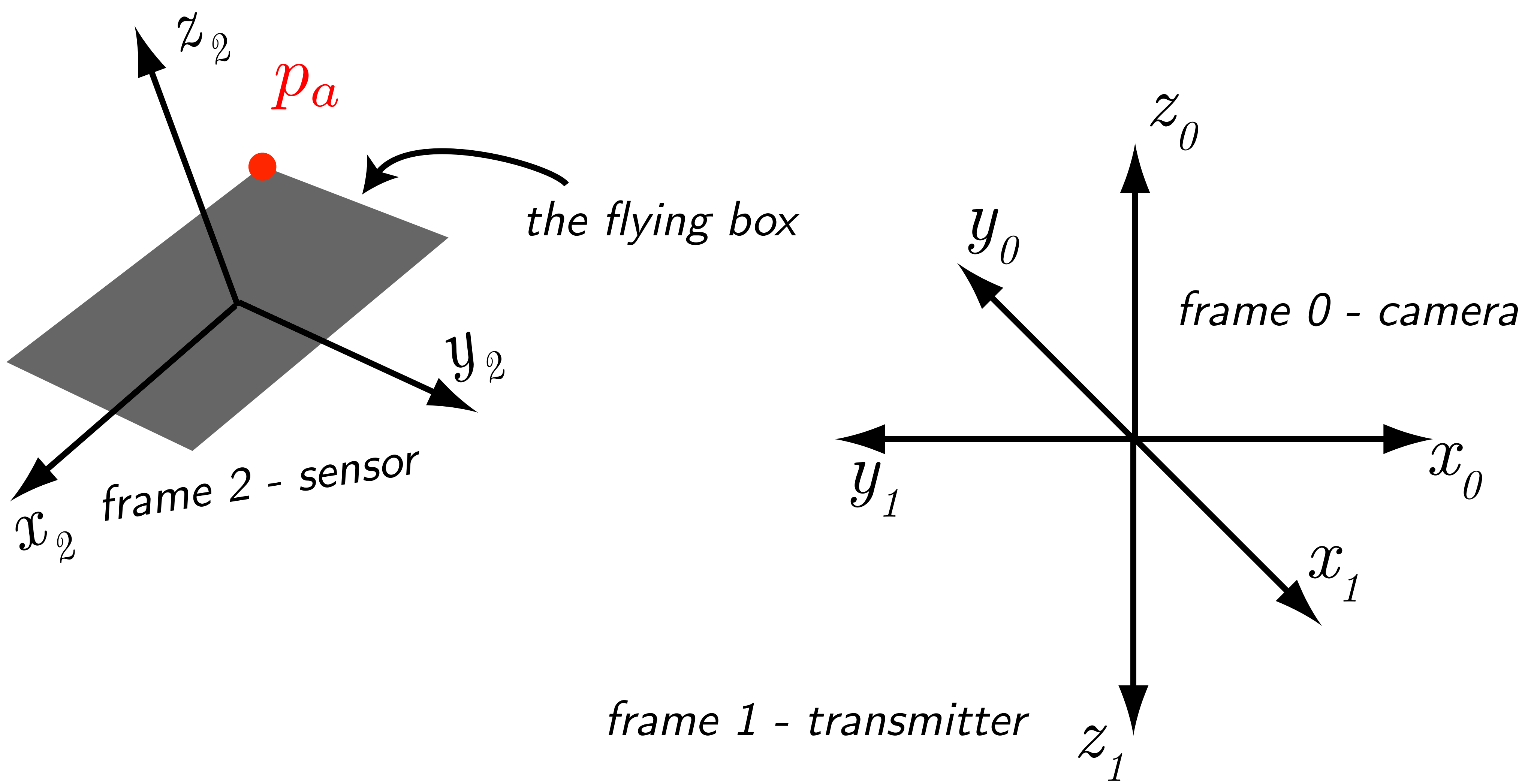
Given (q_1, q_2, q_3), where is the tip of the robot?



4 links

3 joints

3 joint variables (q_1, q_2, q_3)

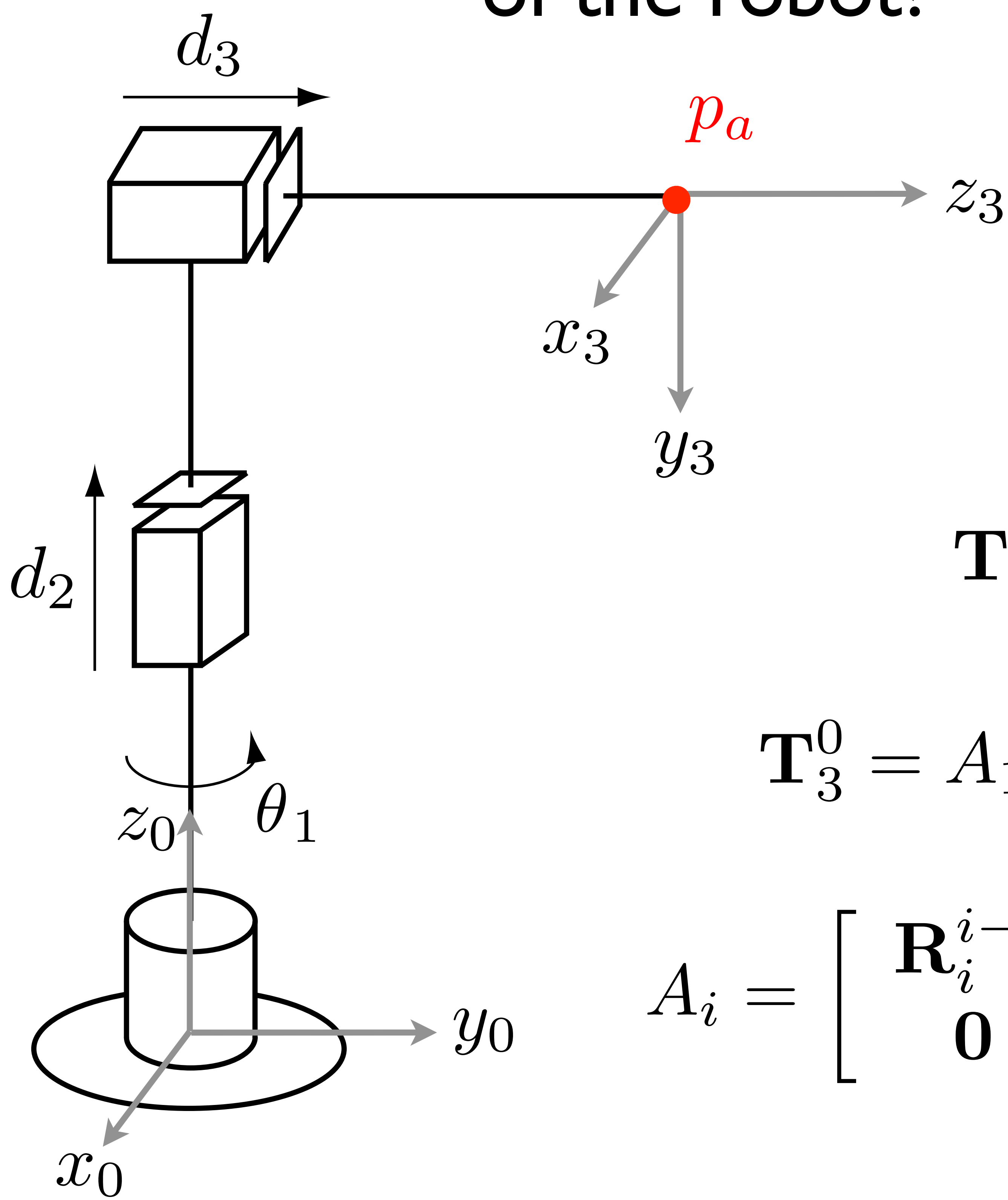


$$\mathbf{P}_a^0 = \mathbf{H}_1^0 \mathbf{H}_2^1 \mathbf{P}_a^2$$

HW1 solutions will go on reserve in library after everyone has turned in the assignment (late additions to the class)

Forward Kinematics of the RPP Cylindrical Robot

Given (q_1, q_2, q_3), where is the tip of the robot?



$$\mathbf{T}_3^0 = \ ?$$

$$\mathbf{T}_3^0 = A_1(q_1)A_2(q_2)A_3(q_3)$$

$$A_i = \begin{bmatrix} \mathbf{R}_i^{i-1} & \mathbf{d}_i^{i-1} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{P}_a^0 = \mathbf{T}_3^0 \mathbf{P}_a^3$$

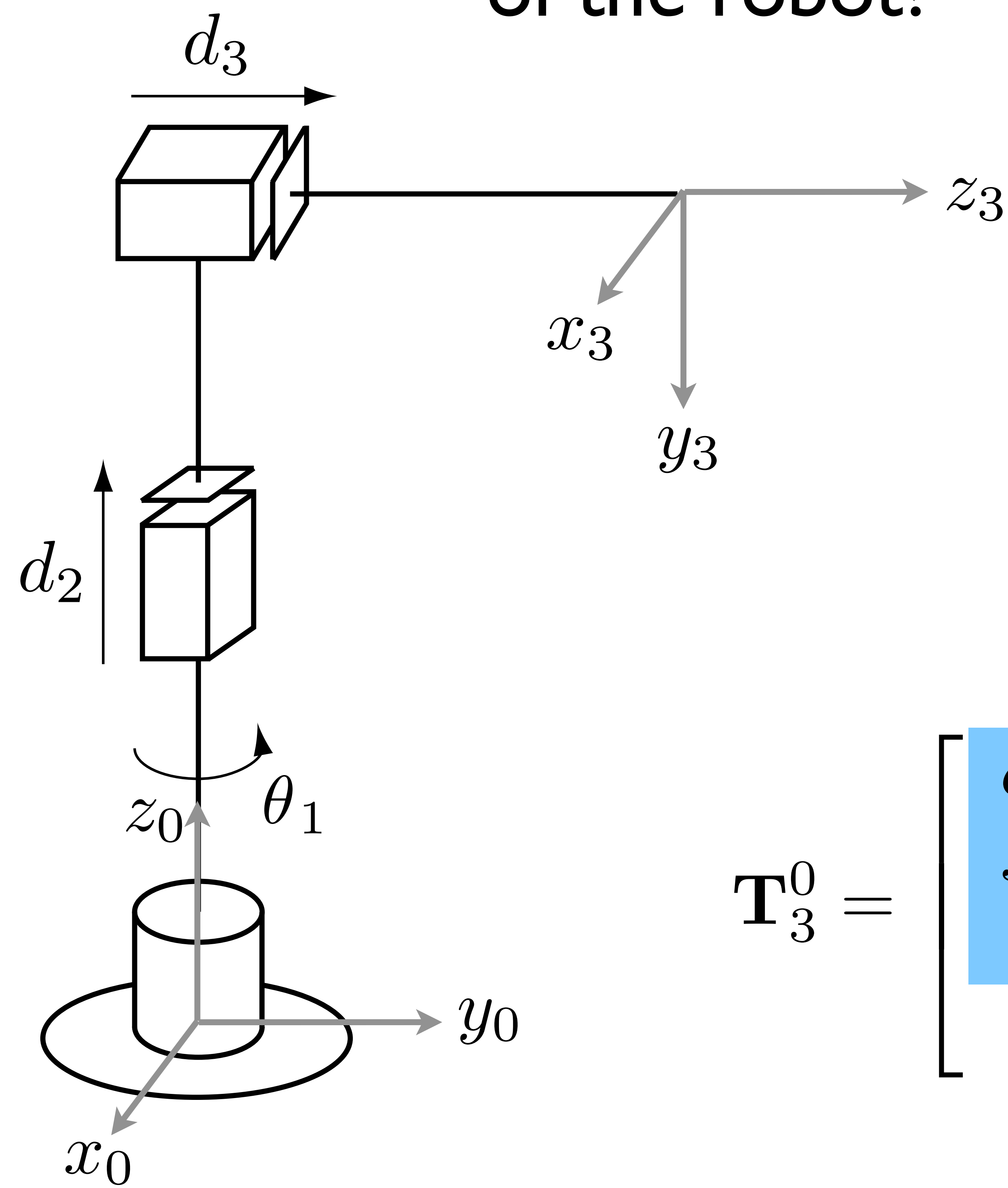
$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics of the RPP Cylindrical Robot

Given (q1, q2, q3), where is the tip of the robot?



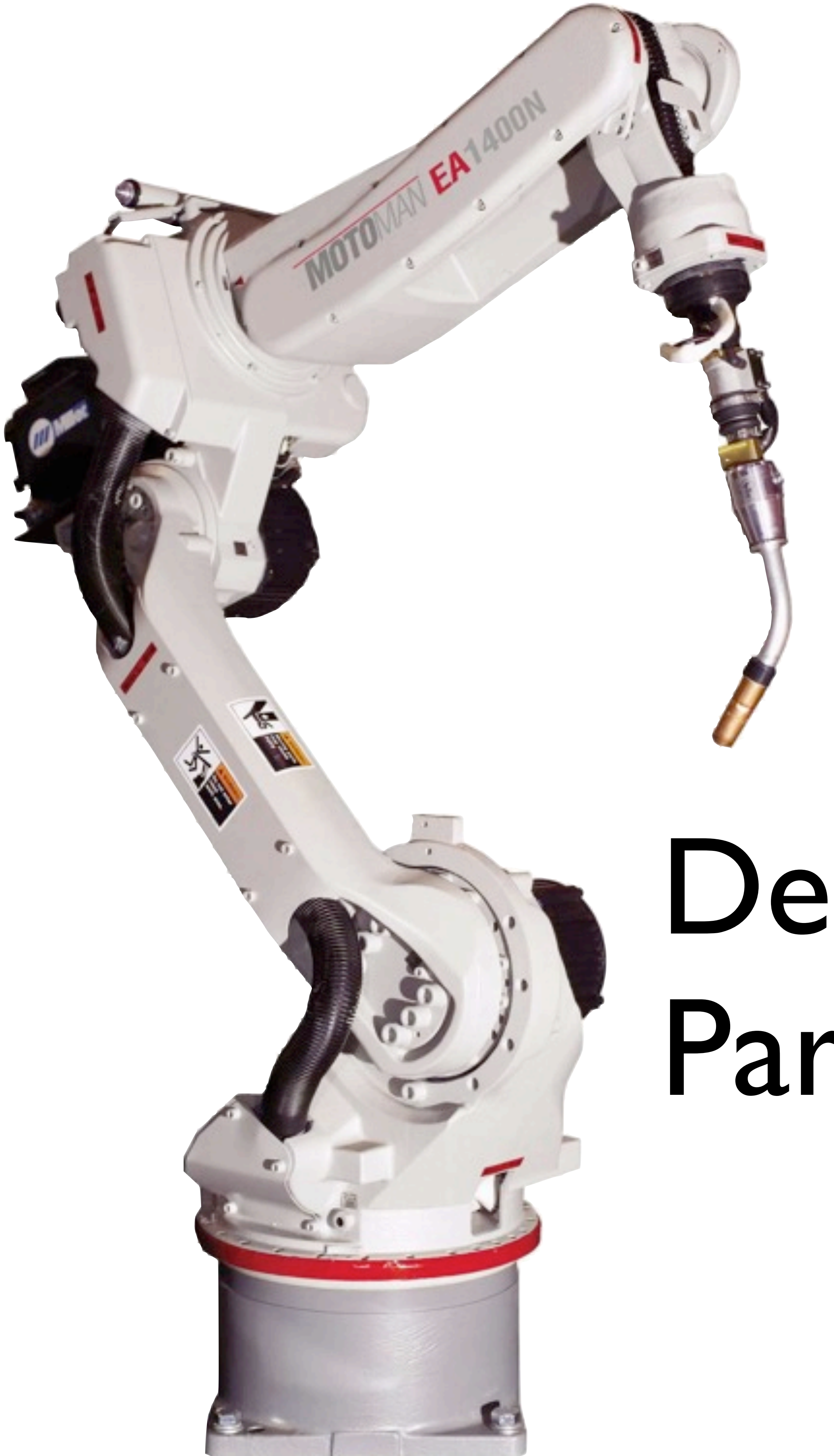
$$\mathbf{T}_3^0 = \begin{bmatrix} c_1^* & 0 & -s_1^* & -d_3^* s_1^* \\ s_1^* & 0 & c_1^* & d_3^* c_1^* \\ 0 & -1 & 0 & d_2^* + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

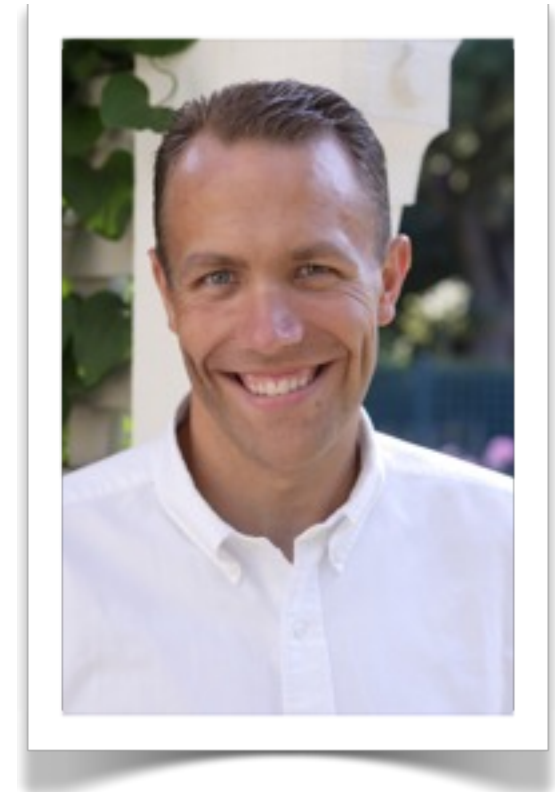
This is the general idea of forward kinematics for manipulators.

Notice that there were many choices we had to make regarding frame placement, which means there are many equally good solutions.

The robotics community has agreed on a set of conventions to ensure uniformity.


Discovery





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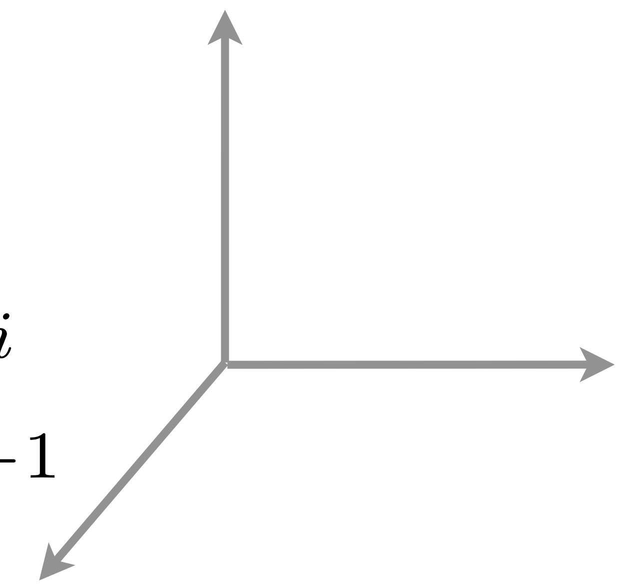
Denavit-Hartenberg Parameters

The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains



start by attaching a frame to each link:
the joint variable for joint i+1 acts along/around z_i
the axis x_i is perpendicular to, and intersects, z_{i-1}

the following conventions make this process easier (p. 82 in SHV):

if z_{i-1} is parallel to z_i	orient x_i toward z_{i-1}
if z_{i-1} intersects z_i	orient x_i normal to the plane formed by z_{i-1} and z_i
if z_{i-1} is not coplanar with z_i	orient x_i along normal with z_{i-1}

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

The **Denavit-Hartenberg convention** defines four parameters and some rules to help characterize arbitrary kinematic chains

a_i **Link Length**	the distance perpendicular to z_i and z_{i-1}, measured along x_i
α_i **Link Twist**	the angle between z_{i-1} and z_i, measured in the plane normal to x_i (right-hand rule around x_i)
d_i **Link Offset**	the distance along z_{i-1} from o_{i-1} to the intersection with x_i
θ_i **Joint Angle**	the angle between x_{i-1} and x_i, measured in the plane normal to z_{i-1} (right-hand rule around z_{i-1})

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

The **Denavit-Hartenberg transform** results from successive rotations and translations via the four DH parameters

The transform from i-1 to i:

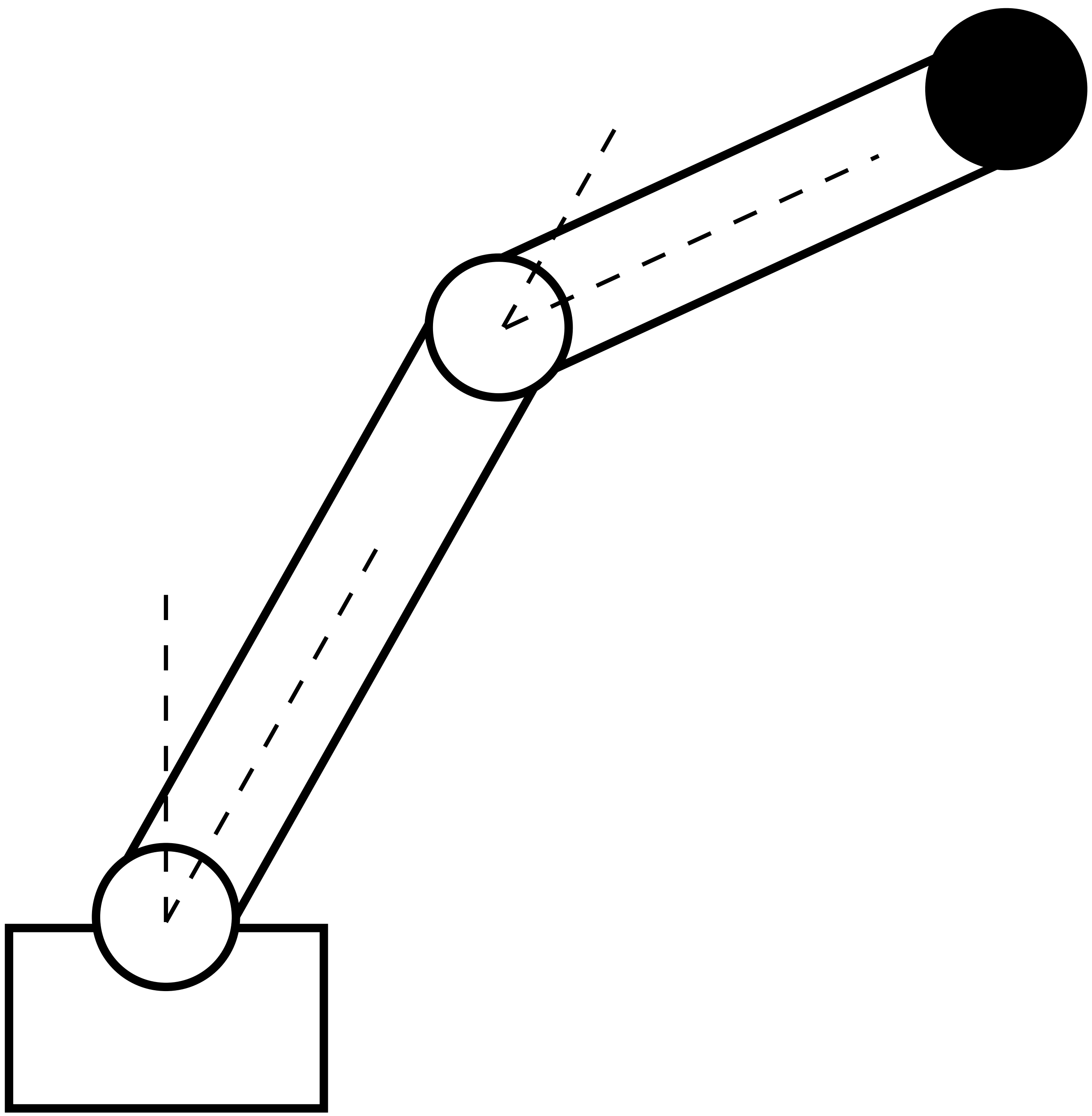
$$A_i = \mathrm{Rot}_{z,\theta_i}\ \mathrm{Trans}_{z,d_i}\ \mathrm{Trans}_{x,a_i}\ \mathrm{Rot}_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\begin{aligned}
A_i &= \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Denavit & Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," ASME Journal of Applied Mechanics, June 1955

Planar RR Robot



Change to due Thursday, September 27

Homework 2:
Manipulator Kinematics and DH Parameters

MEAM 520, University of Pennsylvania
Katherine J. Kuchenbecker, Ph.D.

September 18, 2012

This assignment is due on Tuesday, September 25, by 5:00 p.m. sharp. You should aim to turn the paper part in during class that day. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. The code must be emailed according to the instructions at the end of this document. Late submissions of either or both parts will be accepted until 5:00 p.m. on Wednesday, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

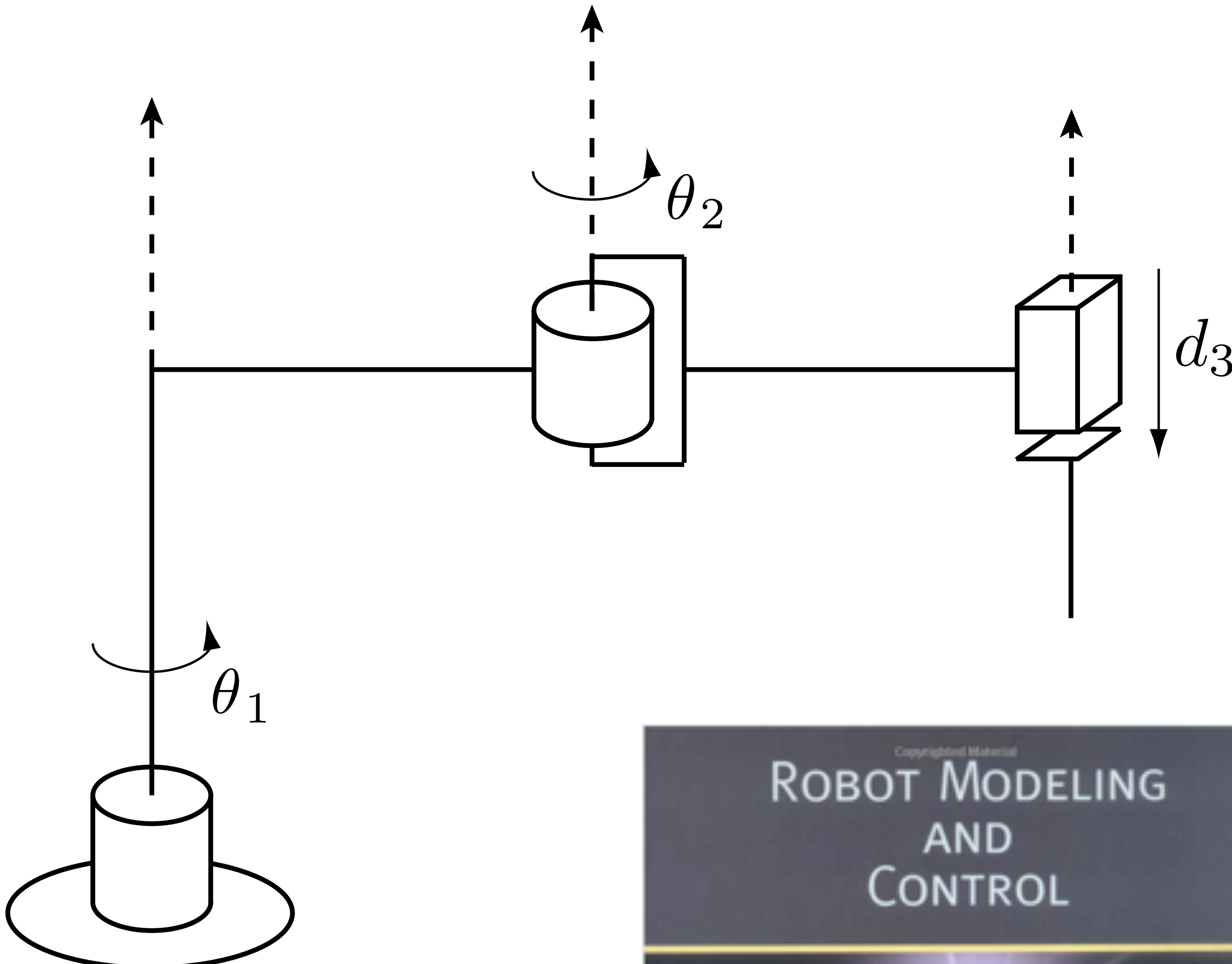
Written Problems *(30 points)*

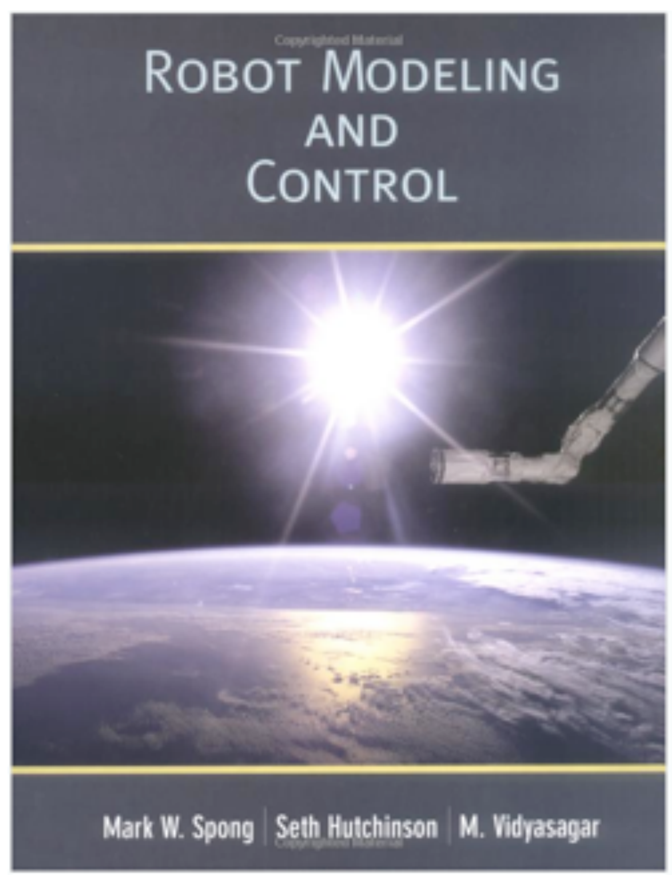
The first set of problems are written, including two from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions when provided. Write in pencil, show your work clearly, box your answers, and staple your pages together.

1. Custom problem – Kinematics of Baxter *(5 points)*
   Rethink Robotics recently released a new robot named Baxter. Watch YouTube videos of Baxter (e.g., http://www.youtube.com/watch?v=rjPFqkFyrOY) to learn about its kinematics. Draw a schematic of the serial kinematic chain of Baxter's left arm (the one the woman is touching in the picture above.) Use the book's conventions for how to draw revolute and prismatic joints in 3D.

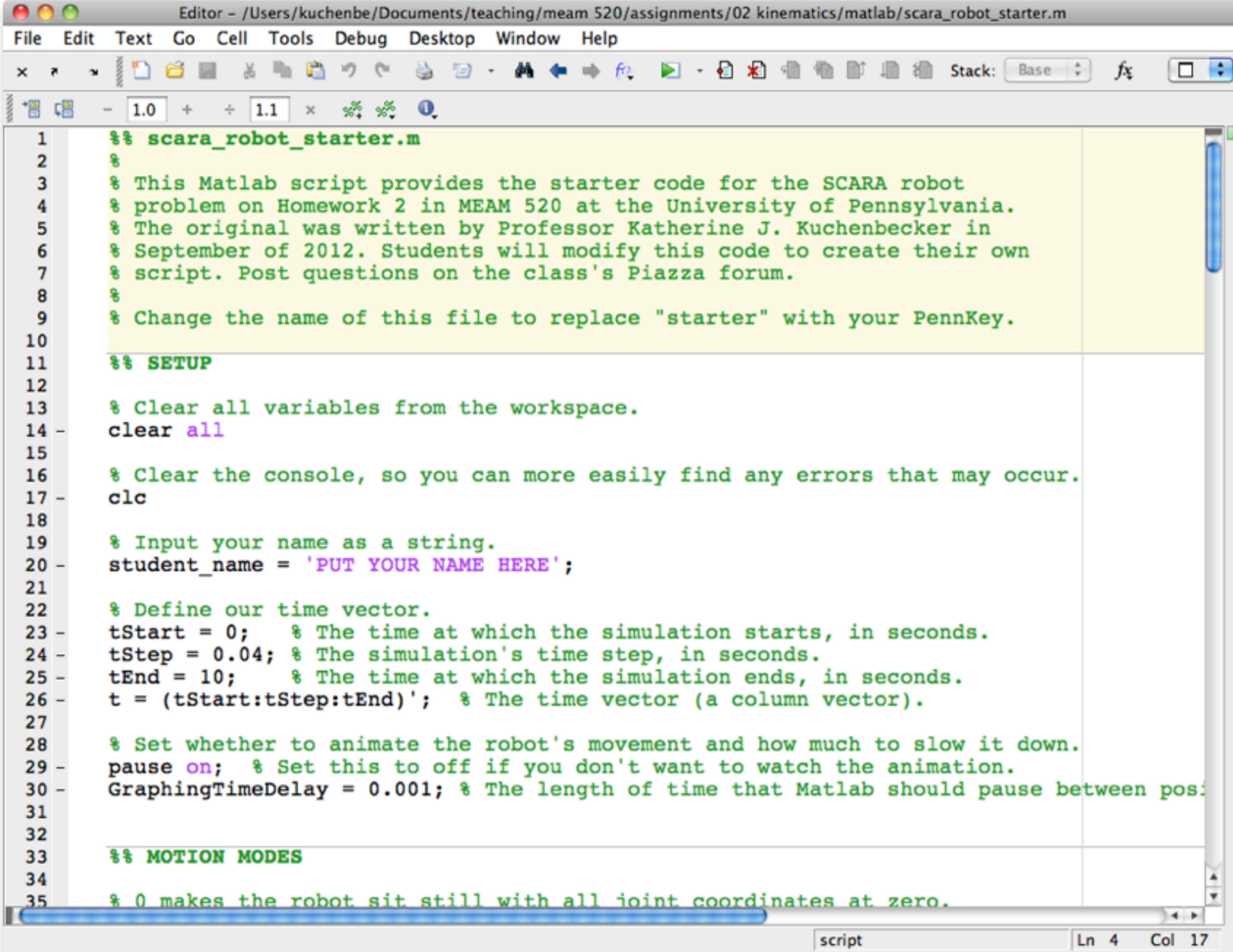
2. SHV 3-7, page 113 – Three-link Cartesian Robot *(10 points)*
   Your solution should include a schematic of the manipulator with appropriately placed coordinate frames, a table of the DH parameters, and the final transformation matrix. Then answer the following question: What are the x, y, and z coordinates of the tip of the robot's end-effector in the base frame (as a function of the robot parameters and the joint coordinates)?

DH Parameters for SCARA Manipulator

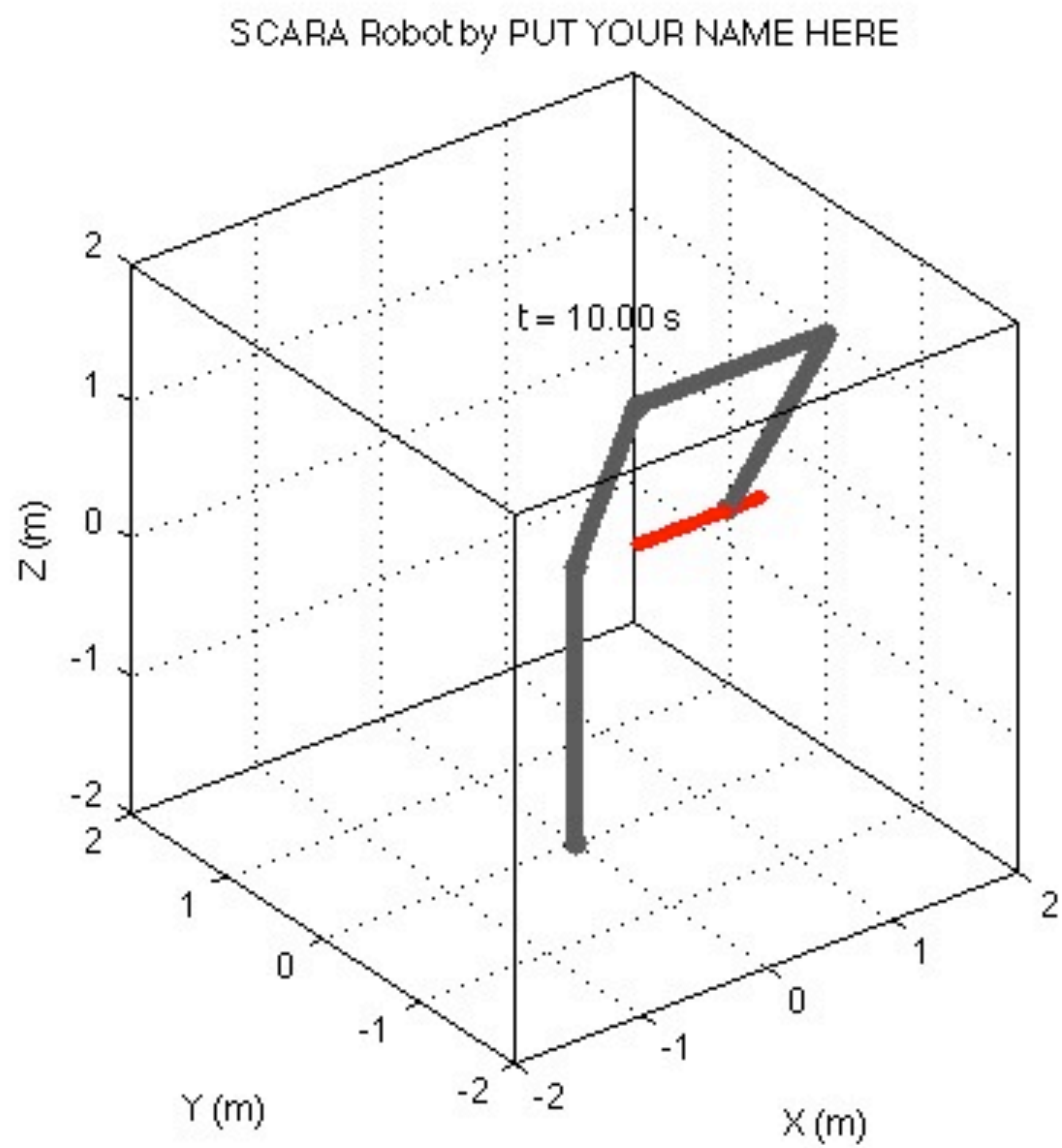




pages 91-93


Editor - /Users/kuchenbe/Documents/teaching/meam 520/assignments/02 kinematics/matlab/scara_robot_starter.m

```
%% scara_robot_starter.m
%
% This Matlab script provides the starter code for the SCARA robot
% problem on Homework 2 in MEAM 520 at the University of Pennsylvania.
% The original was written by Professor Katherine J. Kuchenbecker in
% September of 2012. Students will modify this code to create their own
% script. Post questions on the class's Piazza forum.
%
% Change the name of this file to replace "starter" with your PennKey.

%% SETUP

% Clear all variables from the workspace.
clear all

% Clear the console, so you can more easily find any errors that may occur.
clc

% Input your name as a string.
student_name = 'PUT YOUR NAME HERE';

% Define our time vector.
tStart = 0;   % The time at which the simulation starts, in seconds.
tStep = 0.04; % The simulation's time step, in seconds.
tEnd = 10;    % The time at which the simulation ends, in seconds.
t = (tStart:tStep:tEnd)';  % The time vector (a column vector).

% Set whether to animate the robot's movement and how much to slow it down.
pause on;  % Set this to off if you don't want to watch the animation.
GraphingTimeDelay = 0.001; % The length of time that Matlab should pause between pos


%% MOTION MODES

% 0 makes the robot sit still with all joint coordinates at zero.
```


SCARA Robot by PUT YOUR NAME HERE
t = 10.00 s
Z (m)
2
1
0
-1
-2
Y (m)
2
1
0
-1
-2
X (m)
-2
-1
0
1
2

Questions ❓