3. PRESENTING EXPERIMENTAL DATA

3.1 INTRODUCTION

Some form of analysis must be performed on all experimental data. The analysis may be a simple verbal appraisal of the test results, or it may take the form of a complex theoretical analysis of the errors involved in the experiment and matching of the data with fundamental physical principles. Even new principles may be developed in order to explain some unusual phenomenon. Our discussion in this chapter will consider the analysis of data to determine errors, precision, and general validity of experimental measurements. The correspondence of the measurements with physical principles is another matter, quite beyond the scope of our discussion. Some methods of graphical data presentation will also be discussed. The interested reader should consult the monograph by Wilson [2] for many interesting observations concerning correspondence of physical theory and experiment.

The experimentalist should always know the validity of data. The automobile test engineer must know the accuracy of the speedometer and gas gage in order to express the fuel-economy performance with confidence. A nuclear engineer must know the accuracy and precision of many instruments just to make some simple radioactivity measurements with confidence. In order to specify the performance of an amplifier, an electrical engineer must know the accuracy with which the appropriate measurements of voltage, distortion, etc., have been conducted. Many considerations enter into a final determination of the validity of the results of experimental data, and we wish to present some of these considerations in this chapter.

Errors will creep into all experiments regardless of the care that is exerted. Some of these errors are of a random nature, and some will be due to gross blunders on the part of the experimenter. Bad data due to obvious blunders may be discarded immediately. But what of the data points that just "look" bad? We cannot throw out data because they do not conform with our hopes and expectations unless we see something obviously wrong. If such "bad" points fall outside the range of normally expected random deviations, they may be discarded on this basis of some consistent statistical data analysis. The key here is "consistent". The elimination of data points must be consistent and should not be dependent on human whims and bias based on what "ought to be". In many instances it is very difficult for the individual to be consistent and unbiased. The pressure of a deadline, disgust with previous experimental failures, and normal impatience all can influence rational thinking processes. However, the competent experimentalist will strive to maintain consistency in the primary data analysis. Our objective in this chapter is to show how one may go about maintaining this consistency.

3.2 CAUSES AND TYPES OF EXPERIMENTAL ERRORS

In this section we present a discussion of some of the types of errors that may be present in experimental data and begin to indicate the way these data may be handled. First, let us distinguish between single-sample and multi-sample data.

Single-sample data are those in which some uncertainties may not be discovered by repetition. Multi-sample data are obtained in those instances where enough experiments are performed so that the reliability of the results can be assured by statistics. Frequently, cost will prohibit the collection of multi-sample data, and the experimenter must be content with single-sample data and prepared to extract as much information as possible from experiments. The reader should consult Refs [1-3] for further discussion on this subject, but we state a simple example at this time. If one measures pressure with a pressure gage and a single instrument is the only one used for the entire set of observations, then some of the error that is present in the measurement will be sampled only once no matter how many times the reading is repeated. Consequently, such as experiment is a single-sample experiment. On the other hand, if more than one pressure gage is used for the same total set of observations, we might say that a multi-sample experiment has been performed. The number of observations will then determine the success of this multi-sample experiment in accordance with accepted statistical principles.

The magnitude of an experimental error is ultimately unknown. If the experimenter knows what the error was, he or she would correct it and it would no longer be an error. In other words, the real errors in experimental data are those factors that are always vague to some extent and carry some amount of uncertainly. Our task is to determine just how uncertain a particular observation may be and to devise a consistent way of specifying the uncertainty in analytical form. A reasonable definition may be taken as the possible range that error may have. This uncertainty may vary a great deal depending upon the circumstances of the experiment. Perhaps it is better to speak of experimental uncertainty instead of experimental error because of magnitude of an error is always uncertain. Both terms are used in practice, however, so the reader should be familiar with the meaning attached to the terms and the ways that they relate to each other.

At this point, we may mention some of the types of errors that may cause uncertainty in an experimental measurement. First, there can always be those gross blunders in apparatus or instrument construction that may invalidate the data. Hopefully, the careful experimenter will be able to eliminate most of these errors. Second, there may be certain fixed errors that will cause repeated readings to be in error by roughly the same amount but for some unknown reason. These fixed errors are sometimes called systematic errors. Third, there are the random errors, which may be caused by personal fluctuations, random mechanical and electronic fluctuations in the apparatus or instruments, various influences of friction, etc. These random errors usually follow a certain statistical distribution, but not always. In many instances it is very difficult to distinguish between fixed errors and random errors.

The experimentalist may sometimes use theoretical methods to estimate the magnitude of a fixed error. For example, consider the measurement of the temperature of a hot gas stream flowing in a duct with a mercury-in-glass thermometer. It is well known that heat may be conducted from the stem of the thermometer into the surrounds. In other words, the fact that part of the thermometer is exposed to the surroundings at a temperature different from the gas temperature to be measured may influence the temperature of the stem of the thermometer. Therefore, the temperature we read on the thermometer is not the true temperature of the gas, and it will not make any difference how many readings are taken we shall always have an error resulting from the heat-transfer condition of the stem of the thermometer. This is fixed error, and its magnitude may be estimated with theoretical calculations based upon known heat transfer processes and thermal properties of the gas and the glass thermometer.

3.3 ERROR ANALYSIS ON A COMMONSENSE BASIS

We have already noted that it is somewhat more explicit to speak of experimental uncertainty rather than experimental error. Suppose that we have satisfied ourselves with the uncertainty in some basic experimental measurements, taking into consideration such factors as instrument accuracy, competence of the using the instruments, etc. Often, the primary measurements must be combined to calculate a particular result that is desired. We shall be interested in knowing the uncertainty in the final result due to the uncertainties in the primary measurements. This may be done by a commonsense analysis of the data, which may take many forms. To find the worst-case error, all the errors in the primary measurements are combined in the most detrimental way. Consider the calculation of electric power from

$$P = EI$$

where E (voltage) and I (current) are measured as

$$E = 100\text{ V} \pm 2\text{ V}$$
$$I = 10\text{ A} \pm 0.2\text{ A}$$

The nominal value of the power is 100 x 10 =1000 W. By taking the worst possible variations in voltage and current, we could calculate.

$$P_{max} = (100 + 2)(10 + 0.2) = 1040.4\text{ W}$$
$$P_{min} = (100 - 2)(10 - 0.2) = 960.4\text{ W}$$

Thus, using this method of calculation, the uncertainty in the power is +4.04 percent, -3.96 percent. It is quite unlikely that the power would be in error by these amounts because the voltmeter variations would probably not correspond with the ammeter variations. When the voltmeter reads an extreme "high," there is no reason why the ammeter must also read an extreme "high" at that particular instant; indeed, this combination is most unlikely.

The simple calculation applied to the electric-power equation above is a useful way of inspecting experimental data to determine what errors *could* result in a final calculation; however, the test is too severe and should be used only for rough inspections of data. It is significant to note, however, that if the results of the experiments appear to be in error by *more* than the amounts indicated by the above calculation, then the experimenter had better examine the data more closely. In particular, the experimenter should look for certain fixed errors in the instrumentation (such as the reading on the bathroom scale when no weight is on it), which may be eliminated by applying either theoretical or empirical corrections.

As another example we might conduct an experiment where heat is *added* to a container of water. If our temperature instrumentation should indicate a *drop* in temperature of the water, our good sense (i.e., knowledge of the laws of nature) would tell us that something is wrong and the data point(s) should be thrown out. No sophisticated analysis procedures are necessary to discover this kind of error.

3.4 UNCERTAINTY ANALYSIS

A method of estimating uncertainty in experimental results has been presented by Kline and McClintock [3]. The method is based on a careful specification of the uncertainties in the various primary experimental measurements. For example, a certain pressure reading might be expressed as

$$p = 100\ \text{kN/m}^2 \pm\ 1\ \text{kN/m}^2$$

When the plus or minus notation is used to designate the uncertainty, the person making this designation is stating the degree of accuracy with which he or she *believes* the measurement has been made. We may note that this specification is in itself uncertain because the experimenter is naturally uncertain about the accuracy of these measurements.

If a very careful calibration of an instrument has been performed recently, with standards of very high precision, then the experimentalist will be justified in assigning a much lower uncertainty to measurements than if they were performed with a gage or instrument of unknown calibration history.

To add a further specification of the uncertainty of a particular measurement, Kline and McClintock propose that the experimenter specify certain odds for the uncertainty. The above equation for pressure might thus be written

$$p = 100\ \text{kN/m}^2 \pm\ 1\ \text{kN/m}^2\ (20\ \text{to}\ 1)$$

In other words, the experimenter is willing to bet with 20 to 1 odds that the pressure measurement is within $\pm\ 1\ \text{kN/m}^2$. It is important to note that the specification of such odds can *only* be made by the experimenter based on the total laboratory experience.

Suppose a set of measurements is made and the uncertainty in each measurement may be expressed with the same odds. These measurements are then used to calculate some desired result of the experiments. We wish to estimate the uncertainty in the calculated result on the basis of the uncertainties in the primary measurements. The result R is a given function of the independent variables $x_1, x_2, x_3, ..., x_n$. Thus,

$$R = R(x_1, x_2, x_3, ..., x_n) \tag{3.1}$$

Let w_r be the uncertainty in this result and $w_1, w_2, ..., w_n$ be the uncertainties in the independent variables. If the uncertainties in the independent variables are all given with same odds, then the uncertainty in the result having these odds is given in Ref. [1] as

$$\mathbf{W}_R = \left[\left(\frac{\partial R}{\partial x_1} w_1\right)^2 + \left(\frac{\partial R}{\partial x_2} w_2\right)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} w_n\right)^2\right]^{1/2} \tag{3.2}$$

If this relation is applied to the electric power relation of the previous section, the expected uncertainty (also called the root-mean-square error) is 2.83 percent instead of 4.04 percent.

Example 3.1 The resistance of a certain size of copper wire is given as

$$R = R_o [1 + \alpha (T-20)]$$

where $R_o = 6\Omega \pm 0.3$ percent is the resistance at 20˚C, $\alpha = 0.004˚C^{-1} \pm 1$ percent is the temperature coefficient of resistance, and the temperature of the wire is $T = 30 \pm 1˚C$. Calculate the resistance of the wire and its uncertainty.

Solution. The nominal resistance is

$$R = (6)[1 + (0.004)(30 - 20)] = 6.24\Omega$$

The uncertainty in this value is calculated by applying Eq. (3.2). The various terms are:

$$\frac{\partial R}{\partial R_o} = 1 + \alpha(T - 20) = 1 + (0.004)(30 - 20) = 1.04$$

$$\frac{\partial R}{\partial \alpha} = R_o(T - 20) = (6)(30 - 20) = 60$$

$$\frac{\partial R}{\partial T} = R_o \alpha = (6)(0.004) = 0.024$$

$$w_{r_o} = (6)(0.003) = 0.018\Omega$$

$$w_\alpha = (0.004)(0.01) = 4 \times 10^{-5} \,˚C^{-1}$$

$$w_T = 1˚C$$

Thus, the uncertainty in the resistance is

$$w_R = [(1.04)^2(0.018)^2 + (60)^2(4x10^{-5})^2 + (0.024)^2(1)^2]^{1/2}$$

$$= 0.0305\Omega \text{ or } 0.49\%$$

Particular notice should be given to the fact that the uncertainty propagation in the result w_R predicted by Eq. 3.2 depends on the squares of the uncertainties in the independent variables w_n. This means that if the uncertainty in one variable is significantly larger than the uncertainties in the other variables, say, by a factor of 5 or 10, then it is the largest uncertainty that predominates and the others may probably be neglected.

To illustrate, suppose there are three variables with a product of sensitivity and uncertainty $[(\partial R/\partial x)w_x]$ of magnitude 1, and one variable with a magnitude of 5. The uncertainty in the result would be

$$\left(5^2 + 1^2 + 1^2 + 1^2\right)^{1/2} = \sqrt{28} = 5.29$$

The importance of this brief remark concerning the relative magnitude of uncertainties is evident when one considers the design of an experiment procurement of instrumentation, etc. Very little is gained by trying to reduce the "small" uncertainties. Because of the square propagation it is the large ones that predominate, and any improvement in the overall experimental result must be achieved by improving the instrumentation or technique connected with these relatively large uncertainties. In the examples and problems that follow, both in this chapter and throughout the book, the reader should always note the relative effect of uncertainties in primary measurements on the final result.

The reader is cautioned to examine possible experimental errors *before* the experiment is designed and conducted. Equation (3.2) may be used very effectively for such analysis, as we shall see in the sections and chapters that follow. A further word of caution may be added here. It is equally as unfortunate to overestimate uncertainty as to underestimate it. An underestimate gives false security, while an overestimate may make one discard important results, miss a real effect, or buy much too expensive instruments. The purpose of this chapter is to indicate some of the methods for obtaining reasonable estimates of experimental uncertainty.

In the previous discussion of experimental planning we noted that an uncertainty analysis may aid the investigator in selecting alternative methods to measure a particular experimental variable. It may also indicate how one may improve the overall accuracy of a measurement by attacking certain critical variables in the measurement process. The next three examples illustrate these points.

Example 3.2 Selection of measurement method. A resistor has a nominal stated value of 10Ω ± 1 percent. A voltage is impressed on the resistor, and the power dissipation is to be calculated in two different ways: (1) from $P = E^2/R$ and (2) from $P = EI$. In (1) only a voltage measurement will be made, while both current and voltage will be measured in (2). Calculate the uncertainty in the power determination in each case when the measured values of E and I are:

$$E = 100 \text{ V} \pm 1\%$$
$$I = 10 \text{ A} \pm 1\%$$

FIGURE EXAMPLE 3.2

Power measurement across a resistor.

Solution. The schematic is shown in the accompanying figure. For the first case we have

$$\frac{\partial P}{\partial E} = \frac{2E}{R} \qquad \frac{\partial P}{\partial R} = -\frac{E^2}{R^2}$$

and we apply Eq. (3.2) to give

$$w_P = \left[\left(\frac{2E}{R}\right)^2 w_E^2 + \left(-\frac{E^2}{R^2}\right)^2 w_R^2\right]^{1/2} \qquad \text{(a)}$$

Dividing by $P = E^2/R$ gives

$$\frac{w_P}{P} = \left[4\left(\frac{w_E}{E}\right)^2 + \left(\frac{w_R}{R}\right)^2\right]^{1/2} \qquad \text{(b)}$$

Inserting the numerical values for uncertainty,

$$\frac{w_P}{P} = \left[4(0.01)^2 + (0.01)^2\right]^{1/2} = 2.236\%$$

For the second case we have

$$\frac{\partial P}{\partial E} = I \qquad \frac{\partial P}{\partial I} = E$$

and after similar algebraic manipulation, we obtain

$$\frac{w_P}{P} = \left[\left(\frac{w_E}{E}\right)^2 + \left(\frac{w_I}{I}\right)^2\right]^{1/2} \qquad \text{(c)}$$

Inserting the numerical values of uncertainty,

$$\frac{w_P}{P} = \left[(0.01)^2 + (0.01)^2\right]^{1/2} = 1.414\%$$

Thus, the second method of power determination provides considerably less uncertainty than the first method, even though the primary uncertainties in each quantity are the same. In this example the utility of the uncertainty analysis is that it affords the individual a basis for *selection of a measurement method* to produce a result with less uncertainty.

Example 3.3 Instrument selection. The power measurement in Example 3.2 is to be conducted by measuring voltage and current across the resistor with the circuit shown in the accompanying figure. The voltmeter has an internal resistance R_m, and the value of R is known only approximately. Calculate the nominal value of the power dissipated in R and the uncertainty for the following conditions:

$R = 100\Omega$ (not known exactly)
$R_m = 1000\Omega \pm 5\%$
$I = 5$ A $\pm 1\%$
$E = 500$ V $\pm 1\%$

FIGURE EXAMPLE 3.3
Effect of meter impedance on measurement.

Solution. A current balance on the circuit yields

$$I_1 + I_2 = I$$

$$\frac{E}{R} + \frac{E}{R_m} = I$$

and

$$I_1 = I - \frac{E}{R_m} \qquad \text{(a)}$$

The power dissipated in the resistor is

$$P = EI_1 = EI - \frac{E^2}{R_m} \qquad \text{(b)}$$

The nominal value of the power is thus calculated as

$$\mathrm{P} = (500)(5) - \frac{500^2}{1000} = 2250 \text{ W}$$

In terms of known quantities the power has the functional form P = $f(E,I,R_m)$, and the derivatives

$$\frac{\partial P}{\partial E} = I - \frac{2E}{R_m} \qquad \frac{\partial P}{\partial I} = E$$

$$\frac{\partial P}{\partial R_m} = \frac{E^2}{R_m^2}$$

The uncertainty for the power is now written as

$$w_P = \left[\left(I - \frac{2E}{R_m}\right)^2 w_E^2 + E^2 w_I^2 + \left(\frac{E^2}{R_m^2}\right)^2 w_{R_m}^2\right]^{1/2} \qquad \text{(c)}$$

Inserting the appropriate numerical values gives

$$w_P = \left[\left(5 - \frac{1000}{1000}\right)^2 5^2 + (25 \times 10^4)(25 \times 10^{-4}) + \left(25 \times \frac{10^4}{10^6}\right)^2 (2500)\right]^{1/2}$$

$$= [16 + 25 + 6.25]^{1/2}(5)$$

$$= 34.4 \text{ W}$$

or

$$\frac{w_P}{P} = \frac{34.4}{2250} = 1.53\%$$

In order of influence on the final uncertainty in the power we have

1. Uncertainty of current determination
2. Uncertainty of voltage measurement
3. Uncertainty of knowledge of internal resistance of voltmeter

Comment. There are other conclusions we can draw from this example. The relative influence of the experimental quantities on the overall power determination is noted above. But this listing may be a bit misleading in that it implies that the uncertainty of the meter impedance does not have a large effect on the final uncertainty in the power determination. This results from the fact that R_m>>R($R_m = 10R$). If the meter impedance were lower, say, 200Ω, we would find that it was a dominant factor in the overall uncertainty. For a very *high* meter impedance there would be little influence, even with a very inaccurate knowledge of the exact value of R_m. Thus, we are led to the simple conclusion that we need not worry too much about the precise value of the internal impedance of the meter as long as it is very large compared with the resistance we are measuring the voltage across. This fact should influence *instrument selection* for a particular application.

3.5 EVALUATION OF UNCERTAINTIES FOR COMPLICATED DATA REDUCTION

We have seen in the preceding discussion and examples how uncertainty analysis can be a useful tool to examine experimental data. In many cases data reduction is a rather complicated affair and is often performed with a computer routine written specifically for the task. A small adaptation of the routine can provide for direct calculation of uncertainties when analytical determination of the partial derivatives in Eq. (3.2) is difficult. We still assume that this equation applies, although it could involve several computational steps. We also assume that we are able to obtain estimates by some means of the uncertainties in the primary measurements, i.e., w_1, w_2, etc.

Suppose a set of data is collected in the variables $x_1, x_2, ..., x_n$ and a result calculated. At the same time one may perturb the variables by $\Delta x_1, \Delta x_2, \ldots$, and calculate new results. Then

$$R(x_1) = R(x_1, x_2, ..., x_n)$$

$$R(x_1 + \Delta x_1) = R(x_1 + \Delta x_1, x_2, ..., x_n)$$

$$R(x_2) = R(x_1, x_2,...,x_n)$$

$$R(x_2 + \Delta x_2) = R(x_1, x_2 + \Delta x_2,...,x_n)$$

For small enough values of Δx the partial derivatives can be well approximated by the finite difference expressions

$$\frac{\partial R}{\partial x_1} \approx \frac{R(x_1 + \Delta x_1) - R(x_1)}{\Delta x_1}$$

$$\frac{\partial R}{\partial x_2} \approx \frac{R(x_2 + \Delta x_2) - R(x_2)}{\Delta x_2}$$

and these values could be inserted in Eq. (3.2) to calculate the uncertainty in the result.

At this point we must again alert the reader to the ways uncertainties or errors of instruments are normally specified. Suppose a pressure gage is available and the manufacturer states that it is accurate within ± 1.0 percent. This statement normally refers to *percent of full scale*. So a gage with a range of 0 to 100 kPa would have an uncertainty of ± 10 percent when reading a pressure of only 10 kPa. Of course, this means that the uncertainty in the calculated result, either as an absolute value or percentage, can vary depending on the range of operation used to make the primary measurements. The above procedure can be used to advantage in complicated data-reduction schemes.

A very full description of this technique and many other considerations of uncertainty analysis are given by Moffat [4]. An example of an industry standard on uncertainty analysis is given in Ref. [5].

Example 3.5. Calculate the uncertainty of the wire resistance in Example 3.1 using the technique described in this section.

Solution. In Example 3.1 we have already calculated the nominal resistance at 6.24Ω. We now perturb the three variables R_o, α, and *T* by small amounts to evaluate the partial derivatives. We shall take

$$\Delta R_o = 0.01 \qquad \Delta\alpha = 1 \times 10^{-5} \qquad \Delta T = 0.1$$

Then

$$R(R_o + \Delta R_o) = (6.01)[1 + (0.004)(30\text{-}20)] = 6.2504$$

and the derivative is approximated as

$$\frac{\partial R}{\partial R_o} \approx \frac{R(R_o + \Delta R_o) - R}{\Delta R_o} = \frac{6.2504 - 6.24}{0.01} = 1.04$$

or the same result as in Example 3.1. Similarly,

$$R(\alpha + \Delta\alpha) = (6.01)[1 + (0.00401)(30\text{-}20)] = 6.2406$$

$$\frac{\partial R}{\partial \alpha} \approx \frac{R(\alpha + \Delta\alpha) - R}{\Delta\alpha} = \frac{6.2406 - 6.24}{1 \text{ x } 10^{-5}} = 60$$

$$R(T + \Delta T) = (6)[1 + (0.004)(30.1\text{-}20)] = 6.2424$$

$$\frac{\partial R}{\partial T} \approx \frac{R(T + \Delta T) - R}{\Delta T} = \frac{6.2424 - 6.24}{0.1} = 0.024$$

All derivatives are the same as in Example 3.1. Hence the uncertainty in R is the same, i.e. 0.0305Ω.

3.6 GRAPHICAL ANALYSIS AND CURVE FITTING

Successful analysis of experimental data requires good understanding of the physical processes behind the data. Unless thought through carefully, curve-plotting and cross-plotting usually generate an excess of displays, which are confusing not only to the management or supervisory personnel who must pass on the experiments, but sometimes even to the experimenter.

Assuming that the engineer knows what is to be examined with graphical presentations, the plots may be carefully prepared and checked against appropriate theories. Frequently, a *correlation* of the experimental data is desired in terms of analytical expression between variables that were measured in the experiment; the easiest to plot and understand is a linear relationship. It is most convenient, then, to try to plot the data in such a linear form, which could sometimes be accomplished by a coordinate transformation.

Table 3.1 summarizes several different types of functions and transformations that may be used to produce straight lines on graph paper. The graphical measurements, which may be made to determine the various constants, are also shown. It may be remarked that the method of least squares may be applied to all these relations to obtain the best straight line to fit the experimental data. A number of computer software packages are available to accomplish the functional plots illustrated in Table 3.1. See, for example, Refs. [6], [7], and [8].

Note that when using logarithmic or semilog graph paper is unnecessary to make log calculations; the scaling of the paper automatically accomplishes this.

3.7 GENERAL CONSIDERATIONS IN DATA ANALYSIS

This chapter has considered a variety of topics: statistical analysis, uncertainty analysis, curve plotting, least squares, etc. that arise in a variety of experimental investigations. As a summary to this chapter let us now give an approximate outline of the manner in which one would go about analyzing a set of experimental data.

1. *Examine the data for consistency*. No matter how hard one tries, there will always be some data points that appear to be grossly in error. If we add heat to a container of water, the temperature must rise, and so if a particular data point indicates a *drop* in temperature for a heat *input*, that point might be eliminated. In other words, the data should follow consistency with laws of nature, and points that do not appear proper in that way should be eliminated. If very many data points fall in the category of "inconsistent," perhaps the entire experimental procedure should be investigated for gross mistakes or miscalculation.

2. *Perform a statistical analysis of data where appropriate*. A statistical analysis is only appropriate when measurements are repeated several times. If this is the case, make estimates of such parameters as standard deviation, etc.

3. *Estimate the uncertainties in the results*. We have discussed uncertainties at length. Hopefully, these calculations will have been performed in advance and the investigator will already know the influence of different variables by the time the final results are obtained.

4. *Anticipate the results from theory*. Before trying to obtain correlations of the experimental data, the investigator should carefully review the theory appropriate to the subject and try to glean some information that will indicate the trends the results may take. Important dimensionless groups, pertinent functional relations, and other information may lead to a fruitful interpretation of the data.

5. *Validate the data*. The experimental investigator should make sense of the data in terms of physical theories or on the basis of previous experimental work in the field. Certainly, the results of the experiments should be analyzed to show how they conform to or differ from previous investigations or standards that may be employed for such measurements.

6. *Correlate the data*. Develop the mathematical relationship between the parameter of interest and the independently measured variables that define it. For example, the equation $Nu = cR_e^{0.8}Pr^{0.4}$ is the mathematical relationship between the Nusselt number and the Reynolds and Prandtl numbers which are the independent variables sufficient to determine it.

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