1. Convolution

Assume we have the following 1D image $I$ and symmetric filter $f$:

$$I = [0.3 \ 0.4 \ 0.9 \ 0 \ 0.2 \ 0.1 \ 0.4 \ 0.9 \ 0] \quad f = [0.2 \ 0.2 \ 0.2 \ 0.2]$$

- Convolve $I$ with $f$ assuming around-boundary reflection padding. Show your computations.

- The above filter $f$ is called a box blur. Traditionally, we have used a Gaussian blurring filter in this class. Give an example of an application in computer vision in which the Gaussian blur may have an advantage over the box blur.

- Separate the following two-dimensional filter into a one-dimensional horizontal filter and a one-dimensional vertical filter.

$$I = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

2. Deconvolution

Deconvolution is the process of recovering the input kernel or image from the convolved result. Let $I$ be a $5 \times 5$ image:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

After convolving image $I$ by a $3 \times 3$ kernel $k$ you obtain the following image $J$:

$$J = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \\ 2 & 6 & 8 & 6 & 2 \\ 1 & 4 & 6 & 4 & 1 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix}$$

Given $J = I \otimes k$, and knowing $I$, what is the kernel $k$?

3. Affine Transformation
As discussed in class, both lines and points can be represented as triplets of numbers in homogeneous coordinates. Homogeneous coordinates for lines $l_1, l_2, l'_1, l'_2$ and points $A, B, A', B'$ have been given below. This information is sufficient to determine a transformation between $I$ and $I'$.

$$ A = (-1, 2) \quad B = (3, 5) $$

$$ A' = (-2, 2\sqrt{3}) \quad B' = (4\sqrt{3} - 5, 4 + 5\sqrt{3}) $$

$$ l_1 : 3x - y - 10 = 0 \quad l_2 : x + y - 6 = 0 $$

$$ l'_1 : \frac{3\sqrt{3} + 1}{4}x + \frac{3 - \sqrt{3}}{4}y - 13 = 0 \quad l'_2 : \frac{\sqrt{3} - 1}{4}x + \frac{\sqrt{3} + 1}{4}y - 7 = 0 $$

- Calculate the intersection of $l_1, l_2$ and also the intersection of $l'_1$ and $l'_2$ with homogeneous coordinates.
- You are told that the transformation mapping $I$ to $I'$ is an Affine transformation involving a rotation, scale and translation (in that order).
There exists a point $P$ in $I$ such that

$$P = (2, 3)$$

Utilizing the concepts studied in class and applied in project 2, compute the position of the warp of $P$, $P'$ in $I'$.

4. Exploring Estimate TPS

A TPS model is built by solving a set of linear equations such that the model maps the control points to the target points. This model consists of 3 affine parameters and a weighted combination of functions centered on each control point. This problem attempts to illustrate what the estimate TPS function is doing on a simpler problem in one dimension.

Consider the plot above. We want to find a function that maps the control points to the target points (2 to 0, 4 to 8, 6 to 20). Our model will be:

$$f(x) = \sum_{i=1}^{3} w_i U_i(x)$$

Where the $U_i$ are basis functions given by the plots below and the $w_i$ are the parameters in the model. (Note that the affine terms that would be present in a TPS model have been removed to reduce the number of variables)

Solve for $w_1$, $w_2$, and $w_3$. 
5. Image Stitching

The task is to stitch images A, B and C together. After computing feature matching and RANSAC, we determine the homography transformation for warping points from image A to
image B as $T_B^A$ and from image C to image B as $T_C^B$.

\[
T_B^A = \begin{bmatrix}
1 & -1/5 & -300 \\
1/3 & 1 & -150 \\
0 & 0 & 1
\end{bmatrix}
\quad \text{and} \quad
T_C^B = \begin{bmatrix}
3/4 & 1/5 & 30 \\
-1/3 & 1 & 30 \\
0 & 0 & 1
\end{bmatrix}.
\]

• The size of A, B, and C is 601 rows (height) and 501 columns (width), i.e., (500, 600). The origin is (0, 0).

Let $A_{\text{morph}}$ ($C_{\text{morph}}$) be the image resulting from transformation $T_B^A$ ($T_C^B$), which aligns with image B.

What are the spatial extent of the morphed images $A_{\text{morph}}$ and $C_{\text{morph}}$? What is the size (height and width) of stitched image $S$?

6. RANSAC

A transformation from triangle A in image 1 to triangle B in image 2 can be modeled as an affine transformation. Because of noise in the data, we decide to use RANSAC for affine fitting.
• Provide a description of how RANSAC works here for this task. It could be a paragraph or pseudo code.
You may assume that you have points correspondences between images, a function to calculate the affine transformation, and a function to compute the distance of a point to the triangle.
• Assume that there are 20% outliers, what is the minimum number of RANSAC iterations needed to get, with probability 90%, at least one random sample that is free from outliers?

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