Gradient domain blending is a blending technique, which was first proposed in [1]. While blending image $G$ into image $S$, Gradient domain blending uses derivatives of $G$ as reference, rather than directly copy the intensity value form $G$. This makes the blending more natural, and work with images with different color, shade, and illumination. Gradient domain blending is first formulated as a continuous Poisson problem, and then discretized using image grids. You are recommended to read [1] for more theory behind it.

Figure 1 illustrates the notations: let $S$ be the image definition domain, and let $\Omega$ be a closed subset of $S$ with boundary $\partial \Omega$. Let $g$ be the subset of image $G$ to be blended, and $v$ be the gradient field of $g$. $v$ would be used as the reference in later blending. Let $f^*$ be a known scalar function defined over $S$ minus the interior of $\Omega$ and let $f$ be an unknown scalar function defined over the interior of $\Omega$. Note that after discretization, all the variables are defined in finite image grids. For each pixel $p$ in $S$, let $N_p$ be the set of its 4-connected neighbors in $S$, and let $< p, q >$ denote a pixel pair such that $q \in N_p$.

The insight in gradient domain blending is that, after blending, the derivative of $g$ remains while intensity values are consistent with image $S$ on the blending boundary. The problem can be formulated as the optimization:

$$\min_{f|\Omega} \sum_{<p,q> \cap \Omega \neq \emptyset} (f_p - f_q - v_{pq})^2, \text{ with } f_p = f^*_p \text{ for all } p \in \partial \Omega$$

where $v_{pq}$ is the projection of $v\left(\frac{p+q}{2}\right)$ on the oriented edge $[p, q]$.

Its solution satisfies the following simultaneous linear equations:

$$\text{for all } p \in \Omega, |N_p| f_p - \sum_{q \in N_p \cap \Omega} f_q = \sum_{q \in N_p \cap \Omega} f^*_q + \sum_{q \in N_p} v_{pq}$$

The discretized pseudo code version is as below:

$$4f_{(row, col)} - f_{(row-1, col)} - f_{(row, col-1)} - f_{(row+1, col)} - f_{(row, col+1)} = \text{desired gradient}$$

where $f_{(row, col)}$ is the image pixel intensity in image (row, col), and desired gradient is $|(dx, dy)|$. Therefore, you could form a sparse, symmetric, and positive-definite linear system. Solving this linear system yields
final blending results.

Figure 2: Example for gradient domain blending.