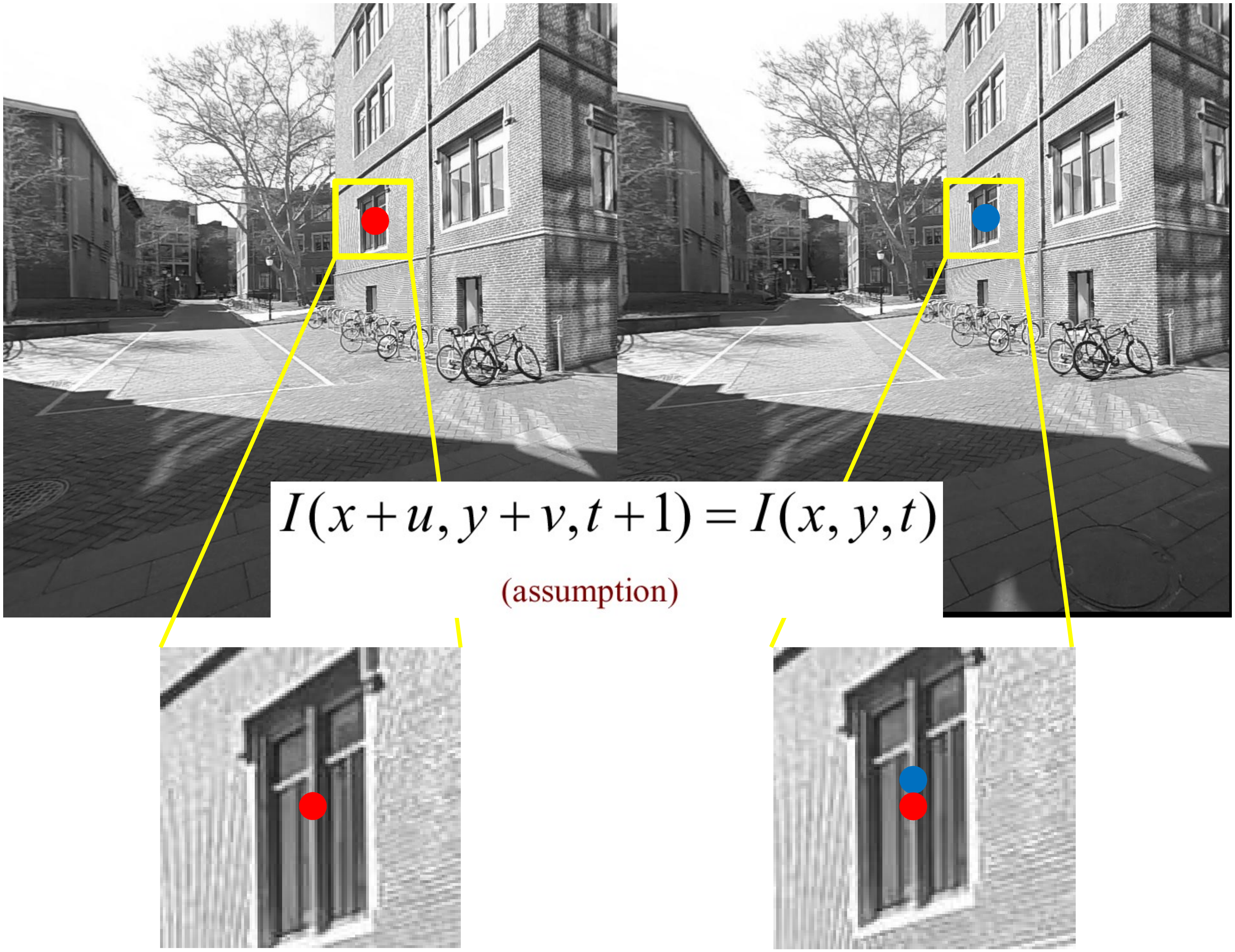
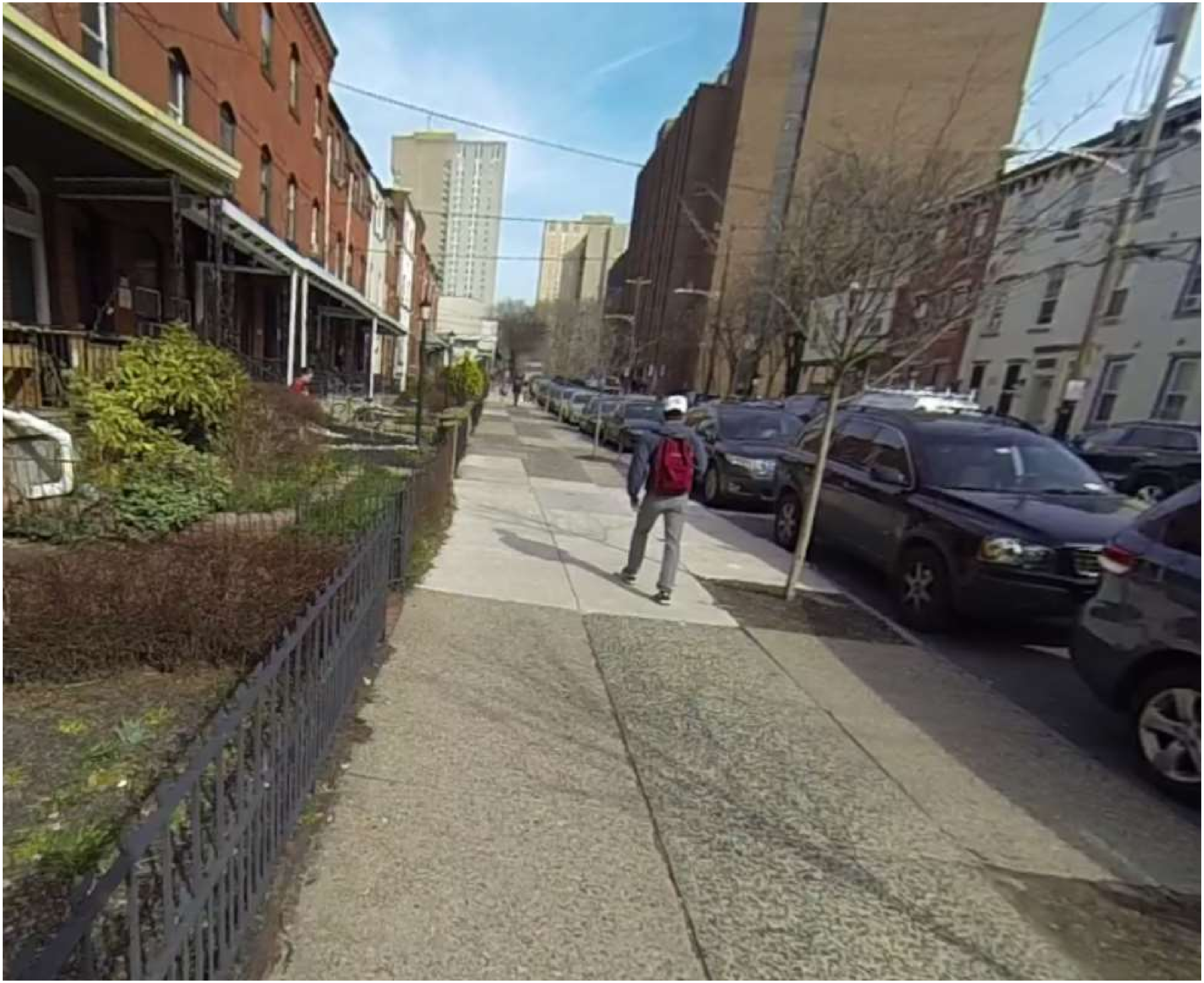
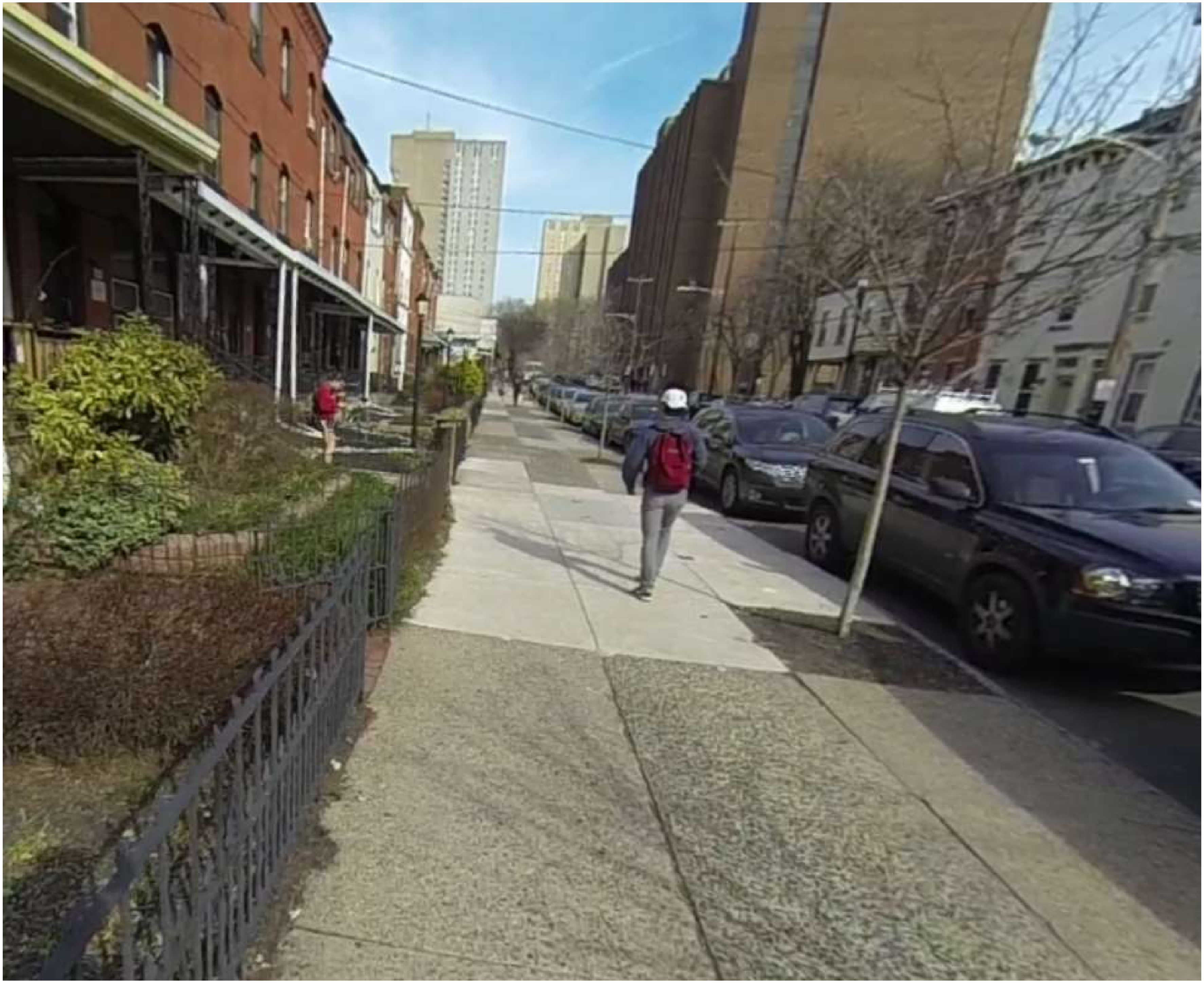
Optical Flow: 2D point correspondences



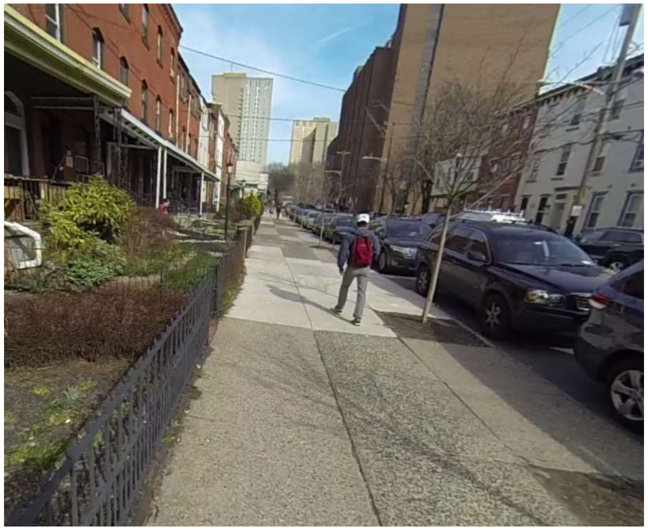
$$I(x+u, y+v, t+1) = I(x, y, t)$$

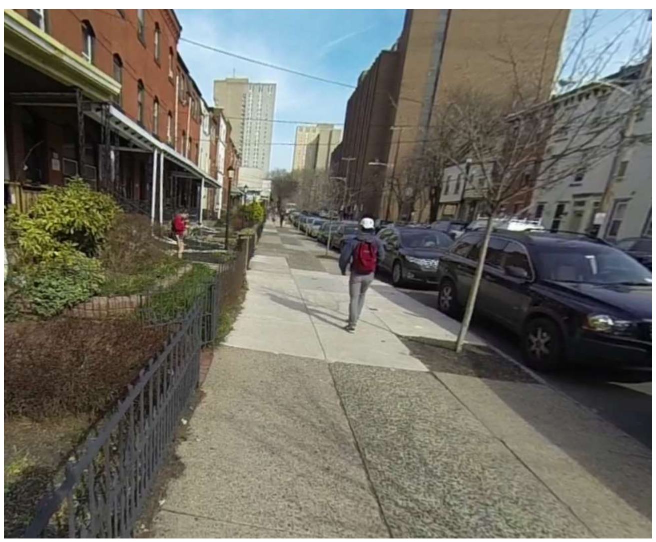
(assumption)

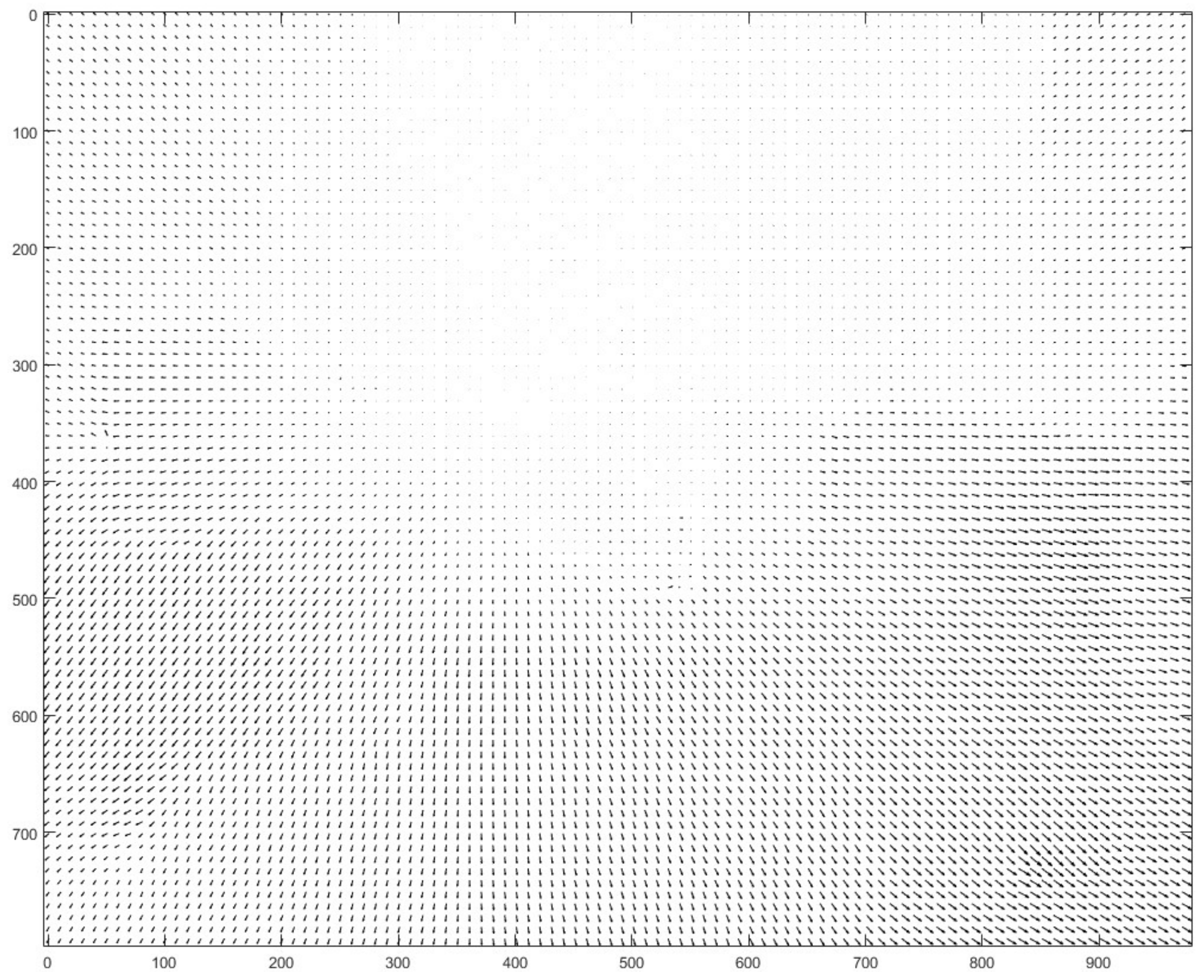




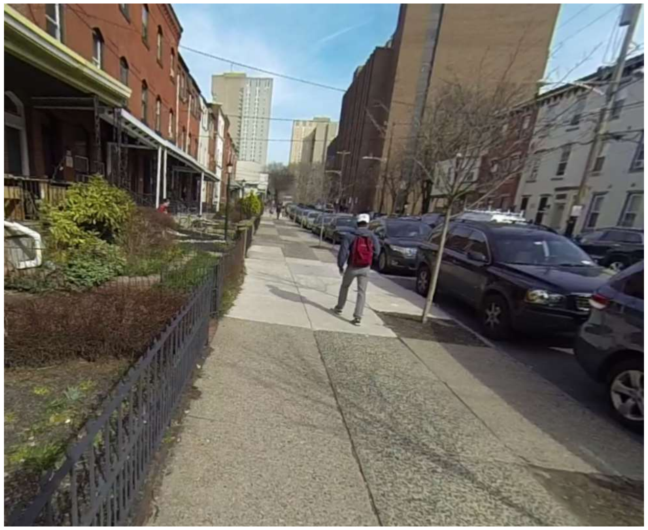
Dense optical flow encodes object motion

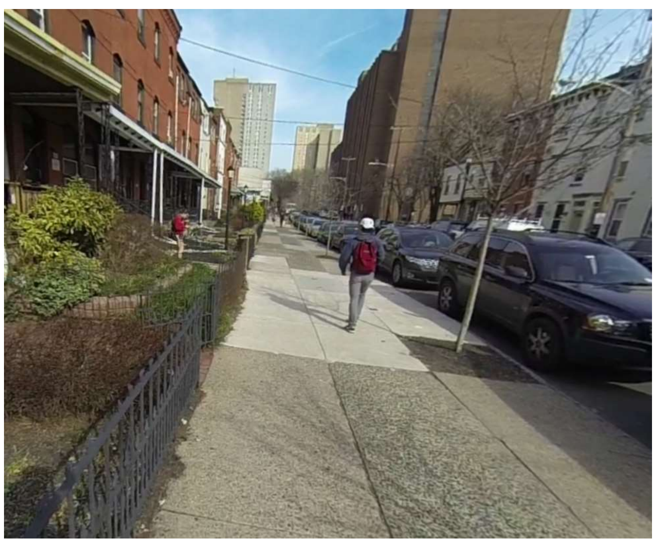


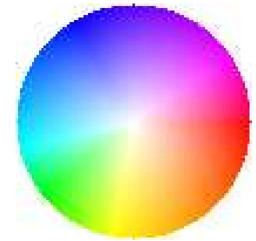


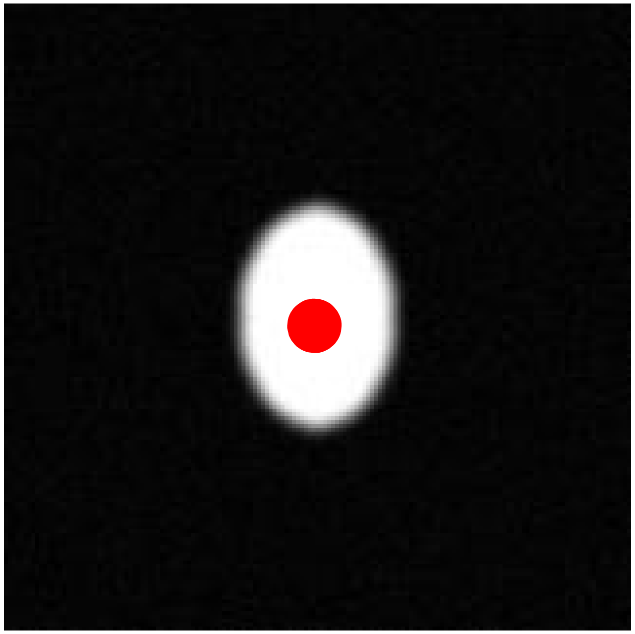


Dense optical flow encodes object motion



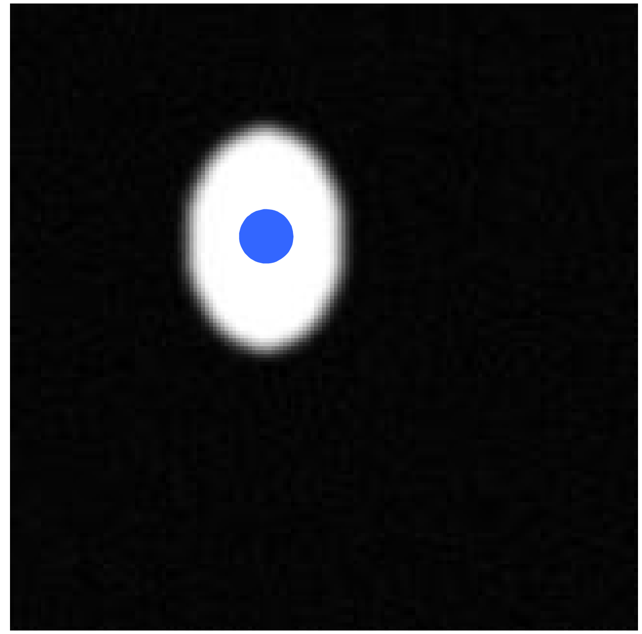






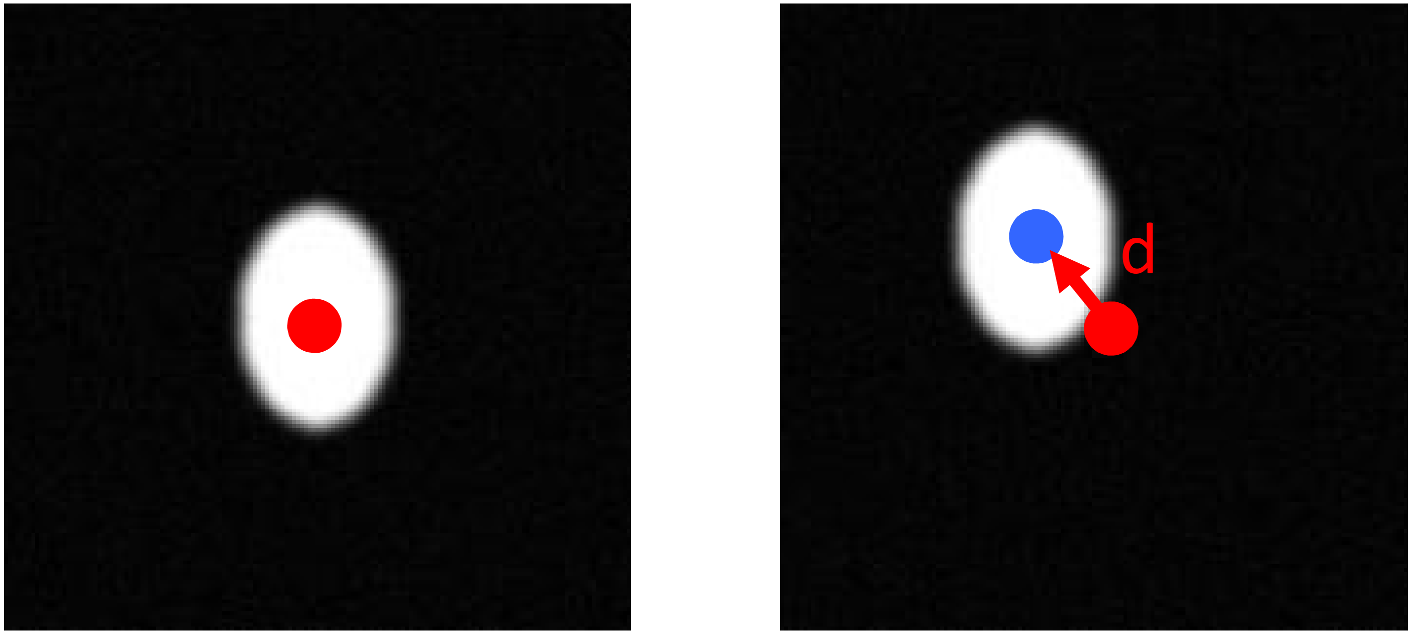
$\mathbf{I}(\mathbf{x})$

$t = 0$



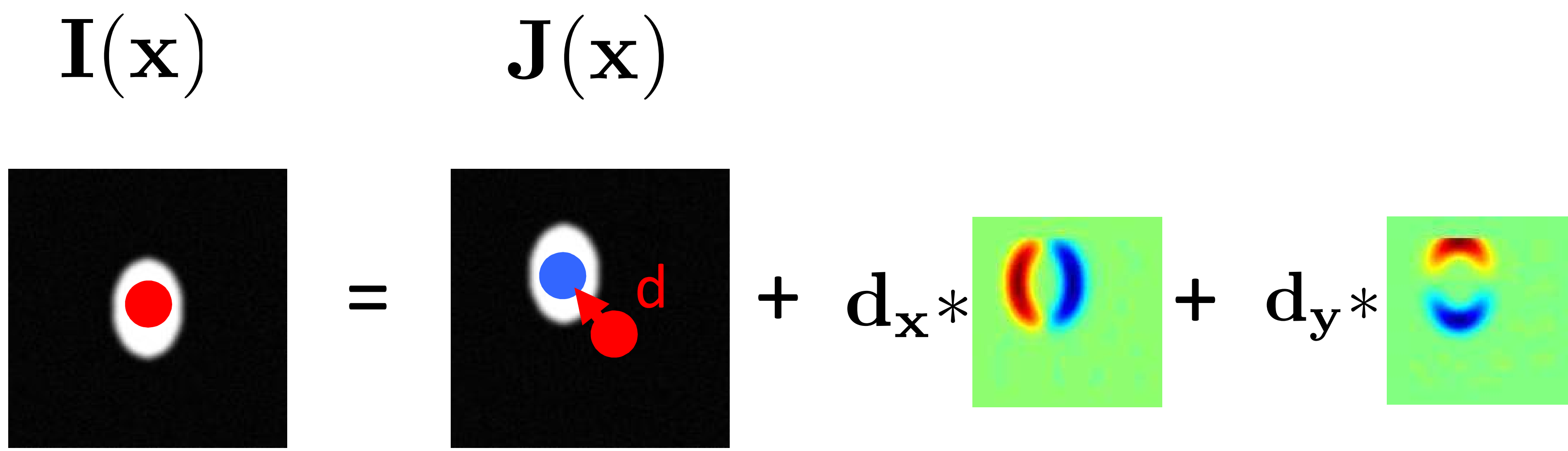
$\mathbf{J}(\mathbf{x})$

$t = 1$



$\mathbf{I}(\mathbf{x})$ $\qquad\qquad$ $\mathbf{J}(\mathbf{x})$

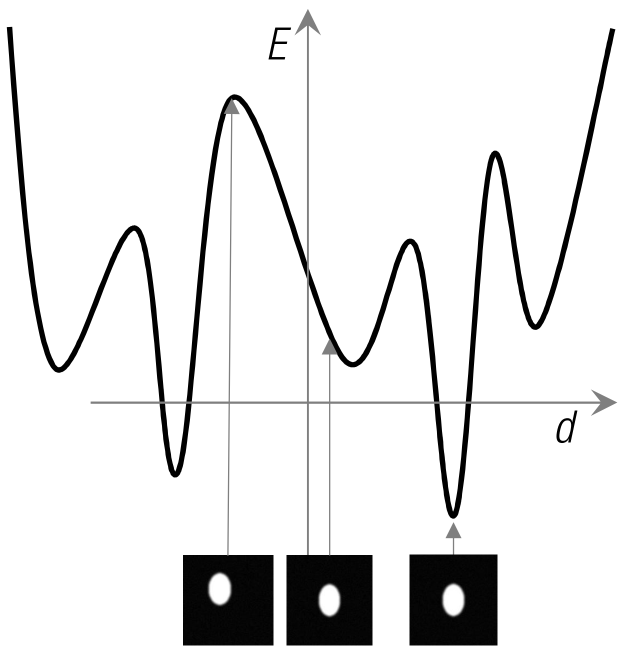
$$I(\mathbf{x}) = J(\mathbf{x} + \mathbf{d})$$

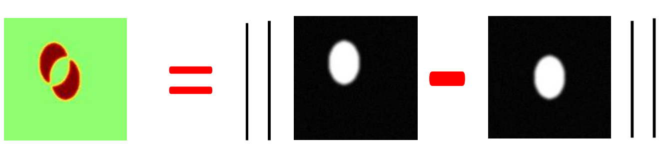


$$I(\mathbf{x}) = J(\mathbf{x} + \mathbf{d})$$

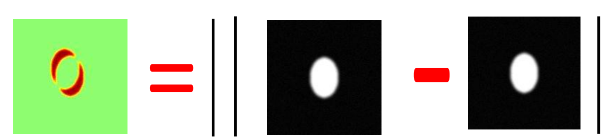
Correspondence cost

$$\min_{\mathbf{d}} E = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$





E(d=(0,0))



E(d=(-7,-9))

Three steps for solving this problem

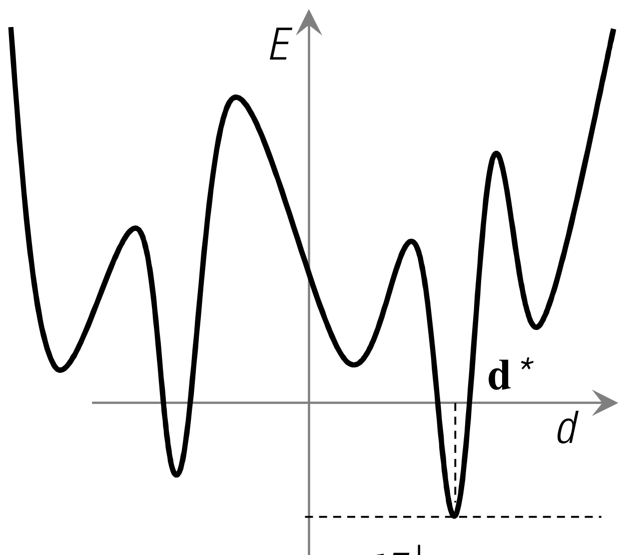
1. Solve for nonlinear least square solution: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

2. Taylor expansion on image $J(\mathbf{x}+\mathbf{d})$

3. Solve for displacement, warp image, and iterate

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathbf{d}) = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$

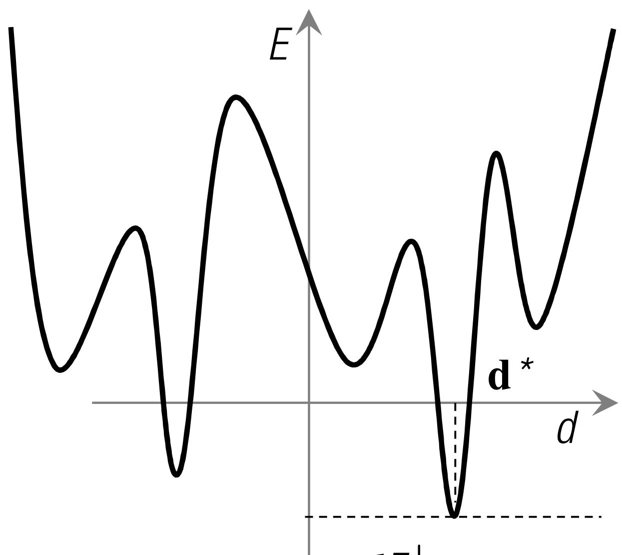


$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

$$E(\mathbf{d}) = \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right)^{\mathrm{T}} \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right)$$

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathbf{d}) = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$



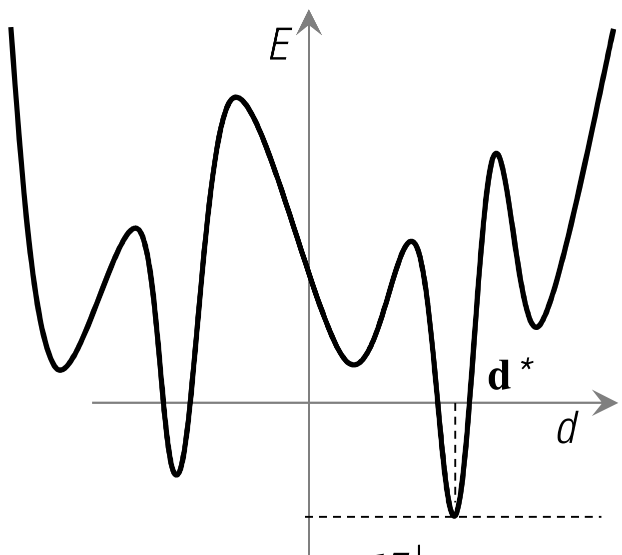
$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

$$E(\mathbf{d}) = \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)^{\mathrm{T}} \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)$$

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x}+\mathbf{d})^{\mathrm{T}}}{\partial \mathbf{d}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)$$

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathbf{d}) = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$



$$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$$

$$E(\mathbf{d}) = \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)^{\mathrm{T}} \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)$$

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x}+\mathbf{d})^{\mathrm{T}}}{\partial \mathbf{d}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)$$

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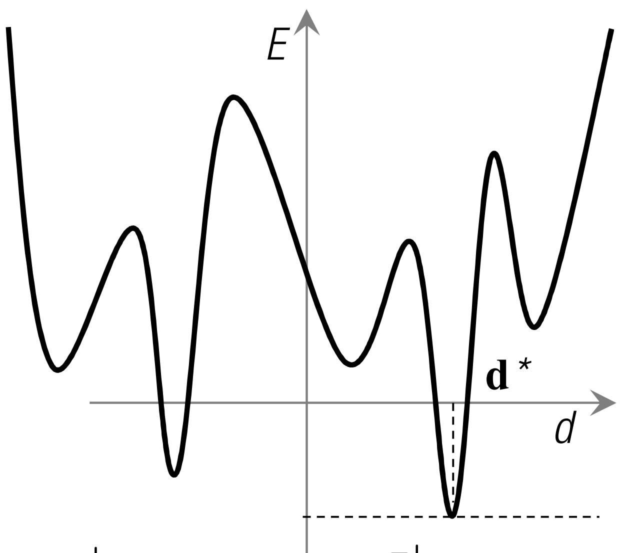
Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathbf{d}) = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$



$\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^\#} = 0$ $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

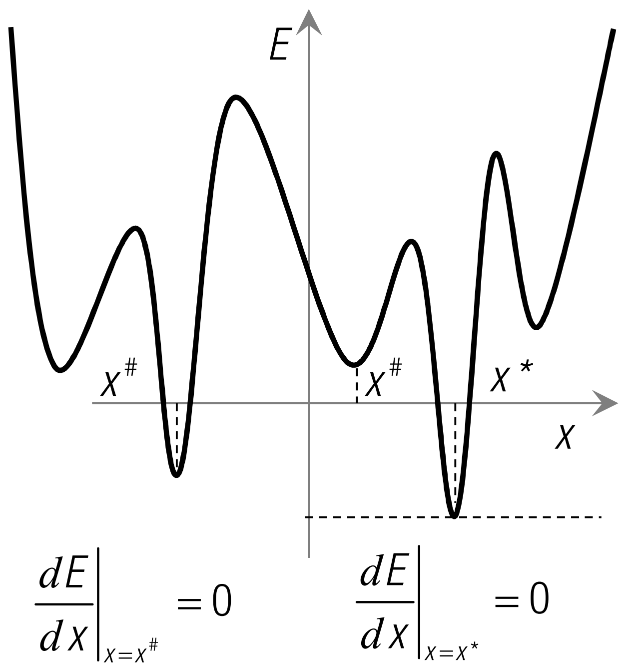
$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right)$$

where $\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} = \left[\frac{\partial J(x,y)}{\partial x}, \frac{\partial J(x,y)}{\partial y} \right]$: Image Gradient

Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$E(\mathbf{d}) = \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right)$$



$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right) = 0$$

Find d such that the above equation is satisfied

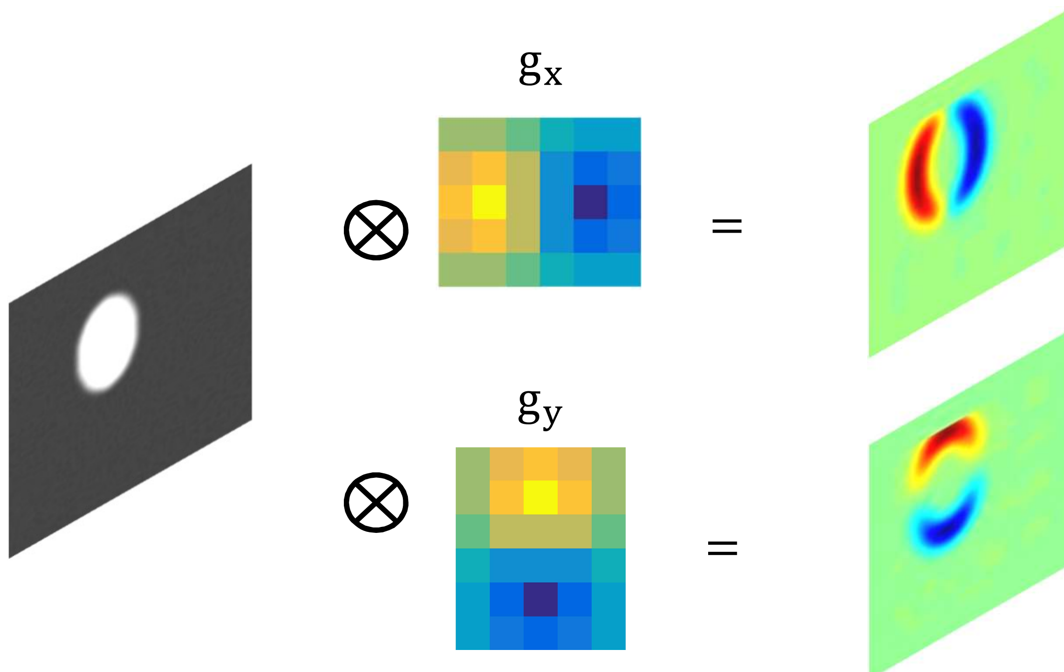
Step 1: $\left.\frac{dE}{d\mathbf{d}}\right|_{\mathbf{d}=\mathbf{d}^*} = 0$

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{T}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right) = 0$$

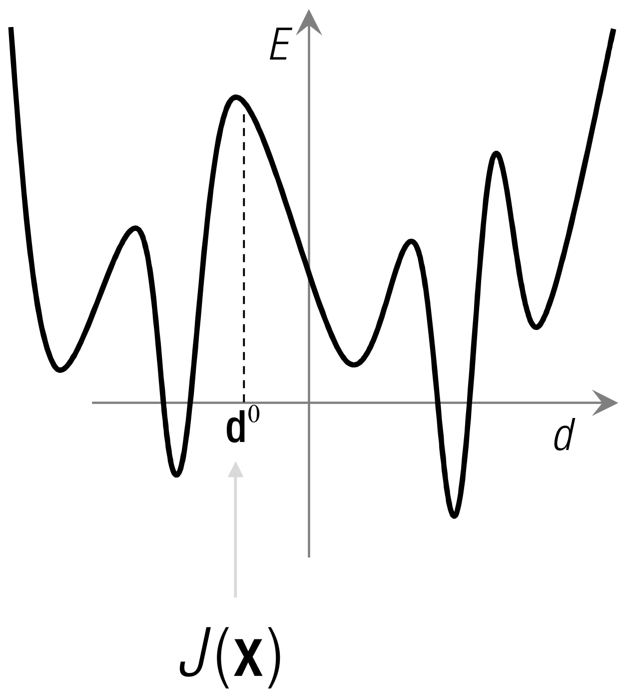
Find d such that the above equation is satisfied

$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta x} =$$

$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta y} =$$



Nonlinear System



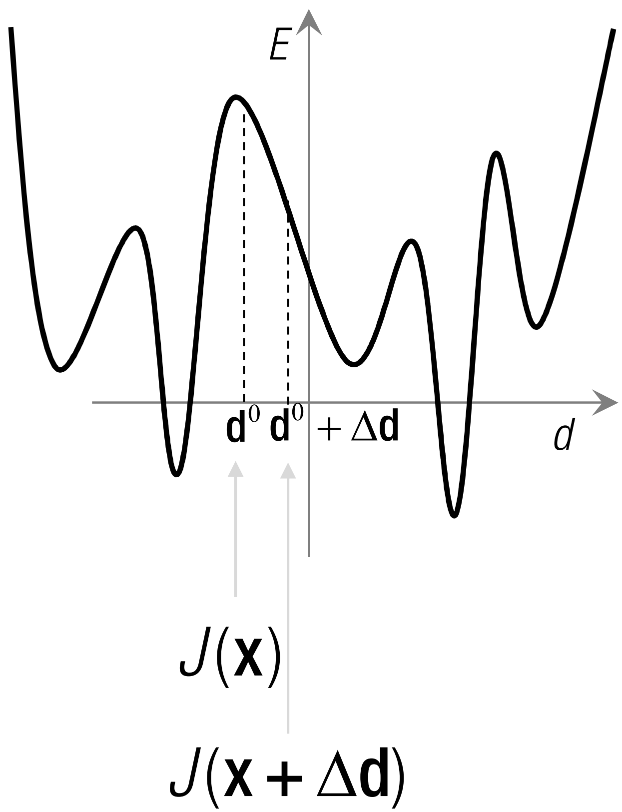
Find d such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right) = 0$$

Idea: how to predict an image when it is shifted by $\Delta\mathbf{d}$

This is a nonlinear process, easy to carry out by image warping, but not easy to write down as an equation.

Nonlinear System



Find d such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{T}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right) = 0$$

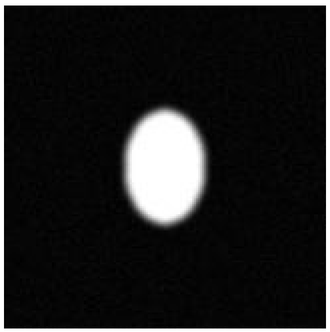
Idea: how to predict an image when it is shifted by $\Delta\mathbf{d}$

Taylor expansion:

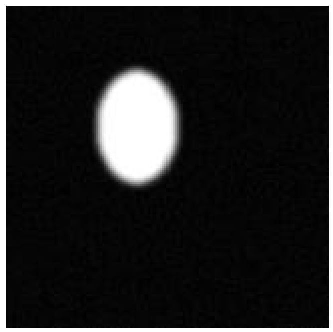
$$J(\mathbf{x}+\Delta\mathbf{d}) = J(\mathbf{x}) + \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d} + \text{H.O.T.}$$

Step 2: Taylor expansion

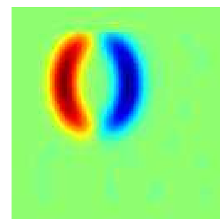
$$J(\mathbf{x}+\Delta\mathbf{d}) = J(\mathbf{x}) + \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d} + \text{H.O.T.}$$



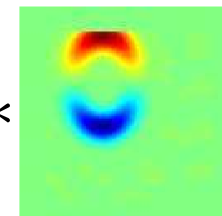
$=$



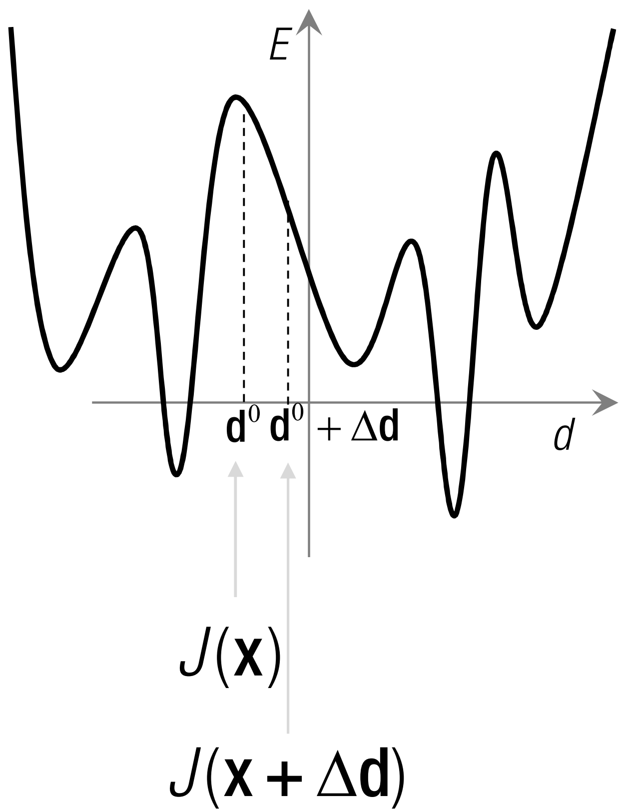
$+ \ \mathbf{d_x} *$



$+ \ \mathbf{d_y} *$



Step 2: Taylor expansion



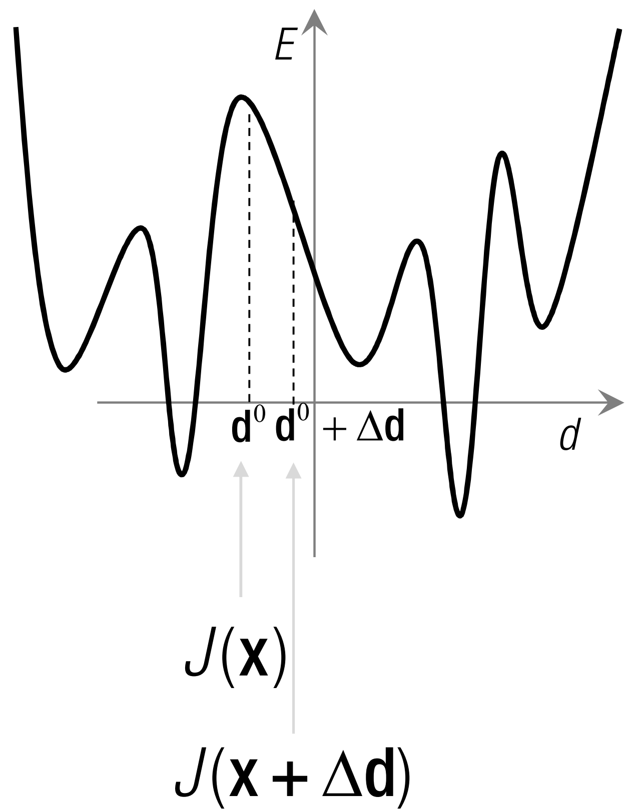
Find d such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\right) = 0$$

$$J(\mathbf{x}+\Delta\mathbf{d}) = J(\mathbf{x}) + \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d} + \text{H.O.T.}$$

$$\rightarrow \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d}\right) = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(I(\mathbf{x}) - J(\mathbf{x})\right)$$

Step 2: Taylor expansion



Find d such that the above equation is satisfied

$$\frac{\partial E}{\partial \mathbf{d}} = 2\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\big(J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x})\big) = 0$$

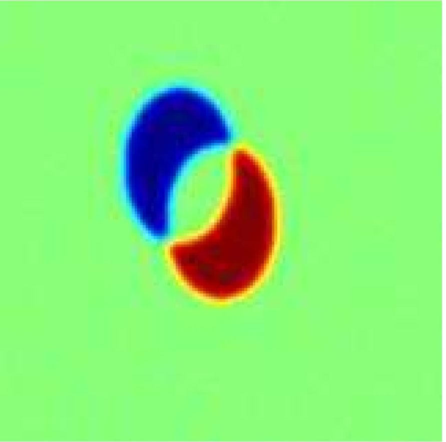
$$J(\mathbf{x}+\Delta\mathbf{d}) = J(\mathbf{x}) + \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d} + \text{H.O.T.}$$

$$\rightarrow \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{d}\right) = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

$$\rightarrow \left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta\mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

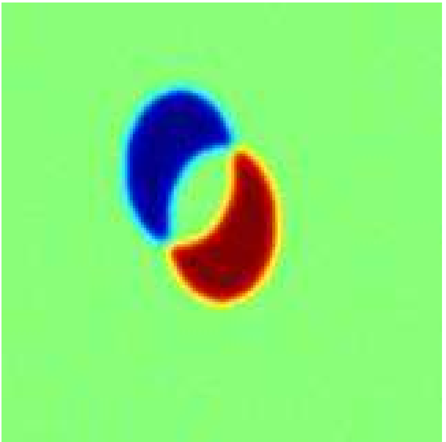
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} \right) \Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}} \left(I(\mathbf{x}) - J(\mathbf{x}) \right)$$

2D unknowns flow vector
per pixel, 2 equations



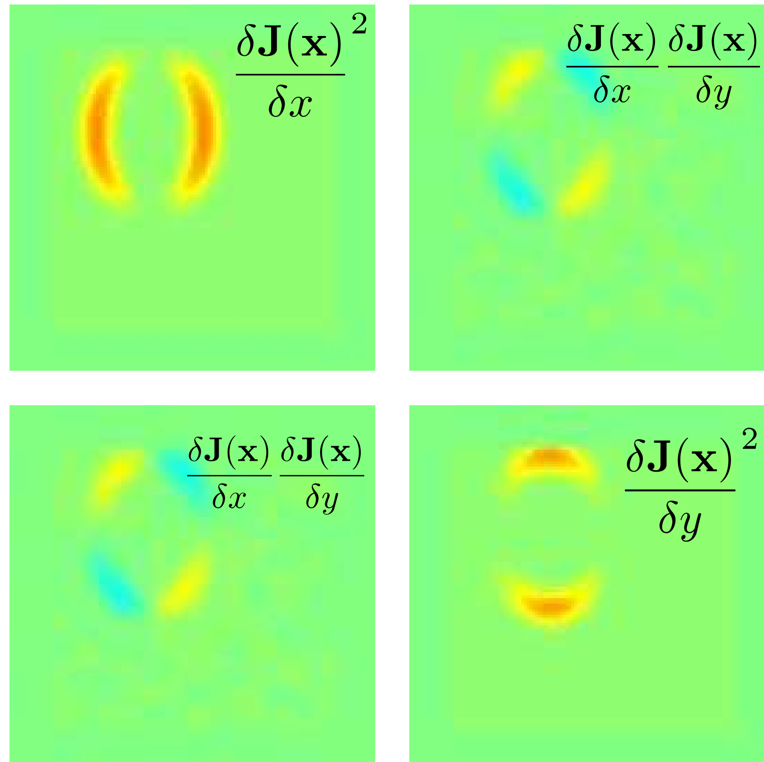
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right) \Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \left(I(\mathbf{x}) - J(\mathbf{x}) \right)$$

2D unknowns flow vector per pixel, 2 equations

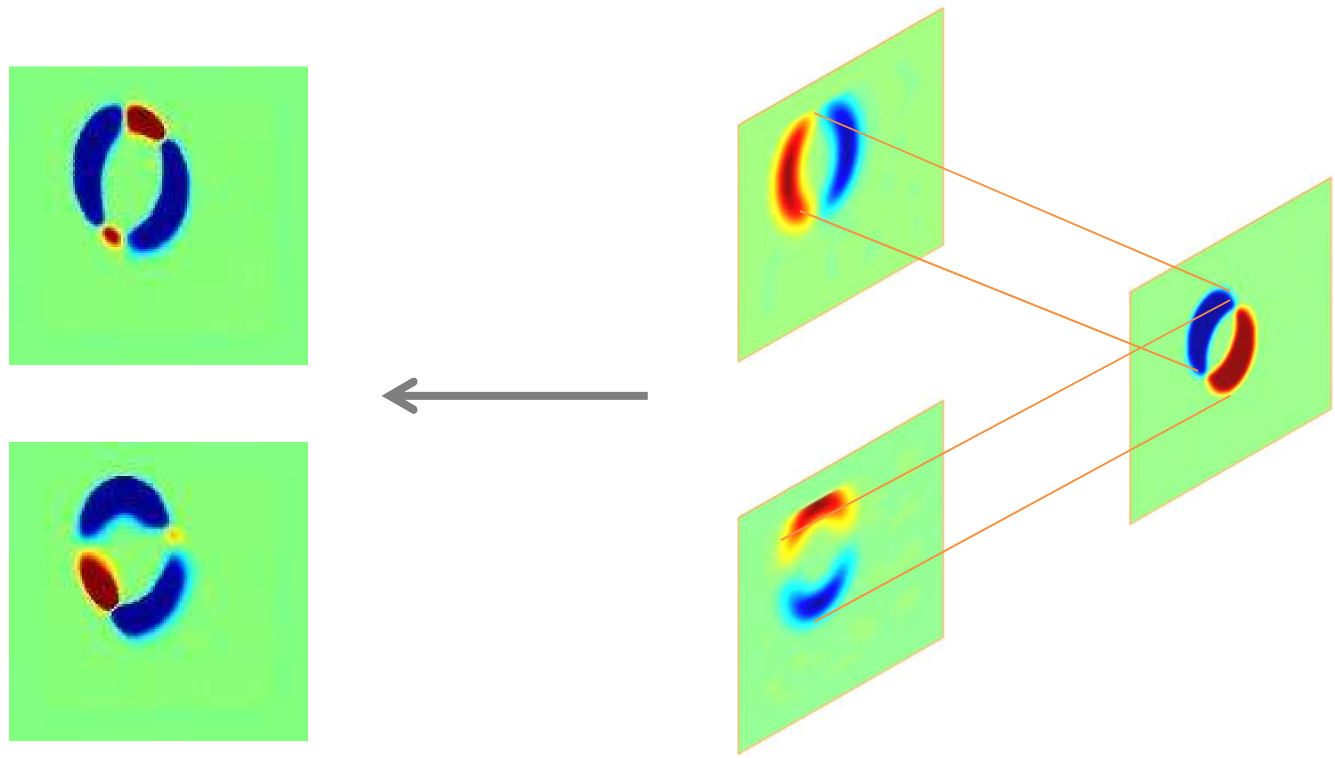


Also known as second moment matrix

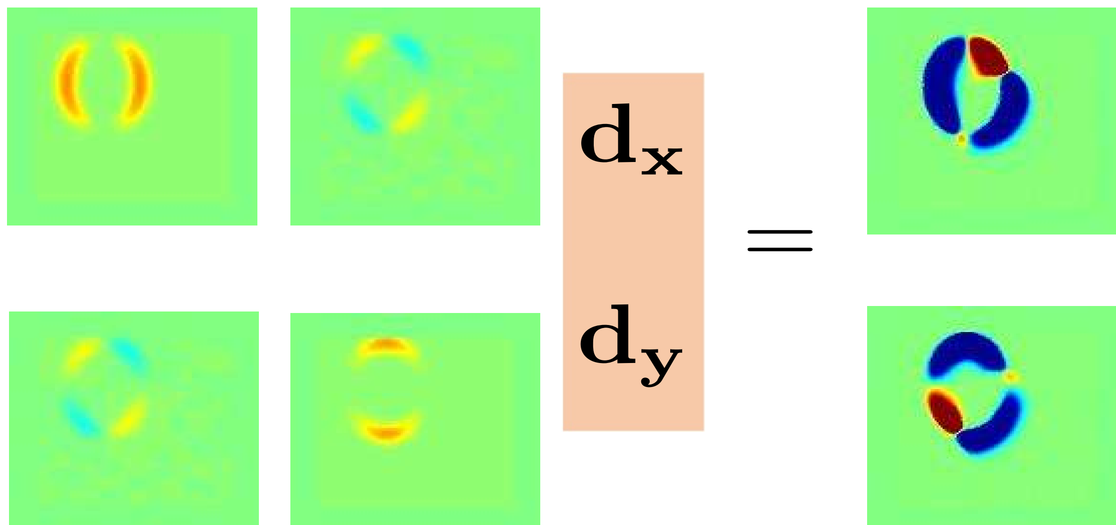
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)$$



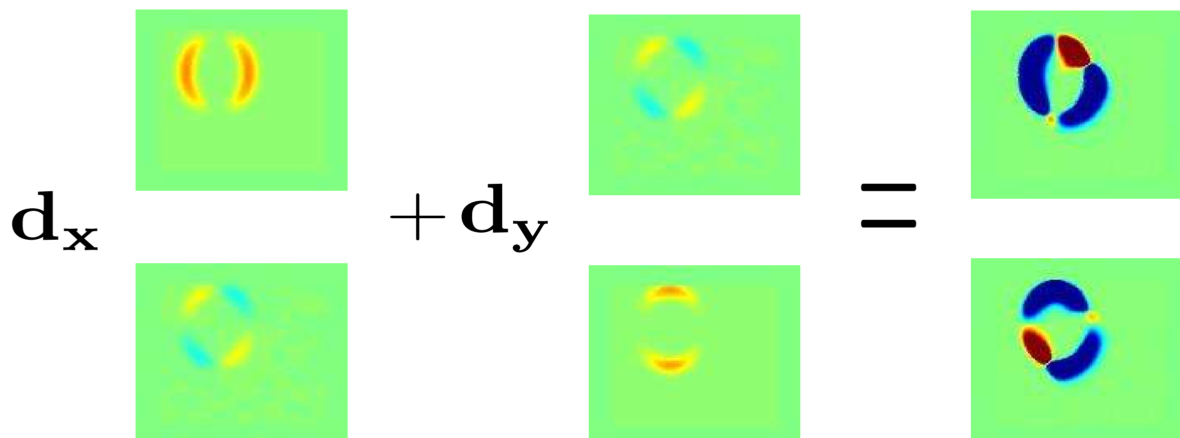
$$\frac{\partial J(\mathbf{x})^{\mathrm{T}}}{\partial \mathbf{x}}\left(I(\mathbf{x}) - J(\mathbf{x})\right)$$

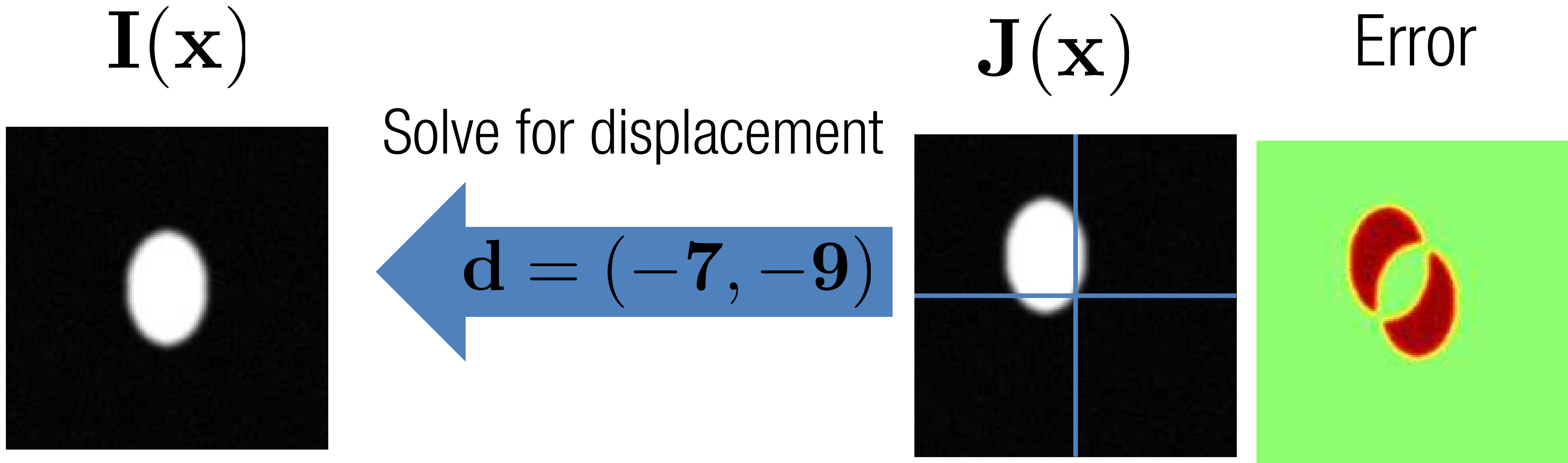


$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} \right) \Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top} \big(I(\mathbf{x}) - J(\mathbf{x}) \big)$$

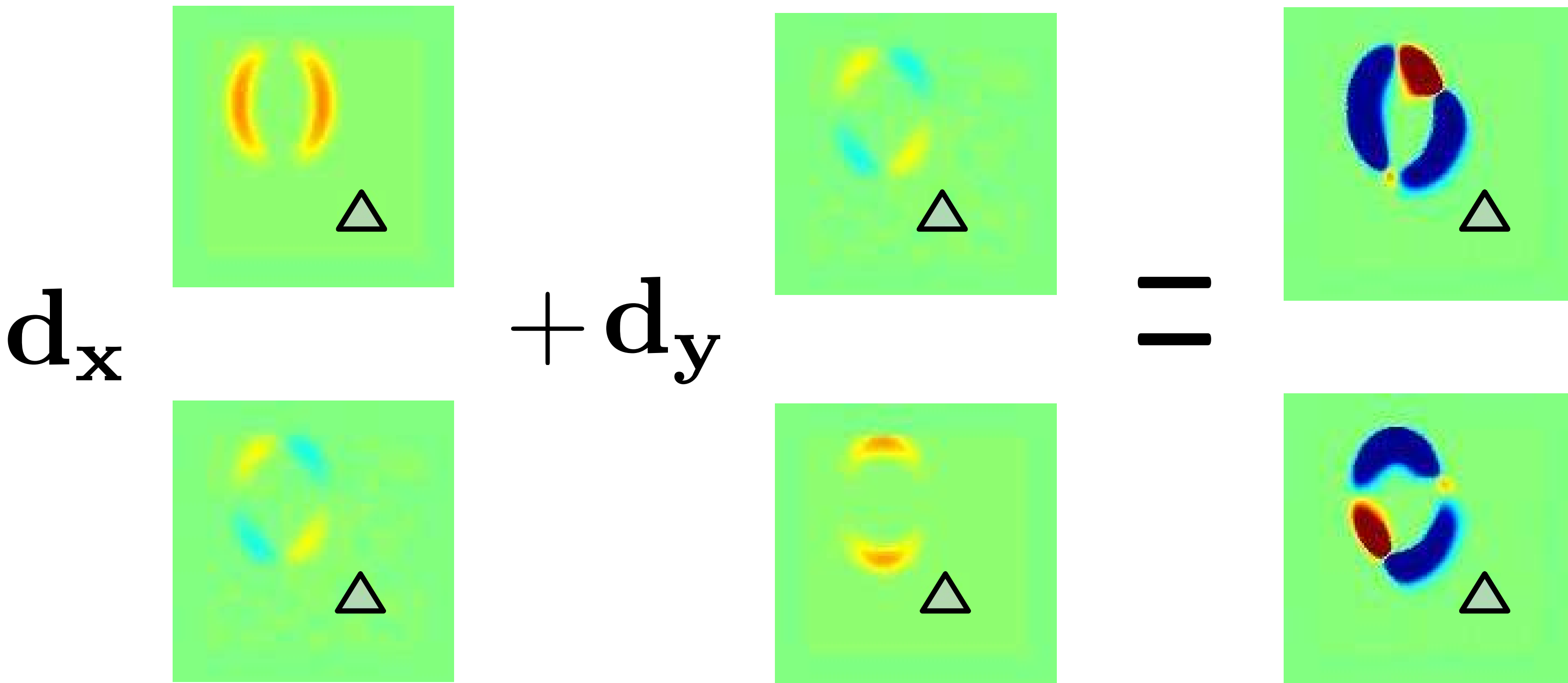


$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} \right) \Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top} \big(I(\mathbf{x}) - J(\mathbf{x}) \big)$$

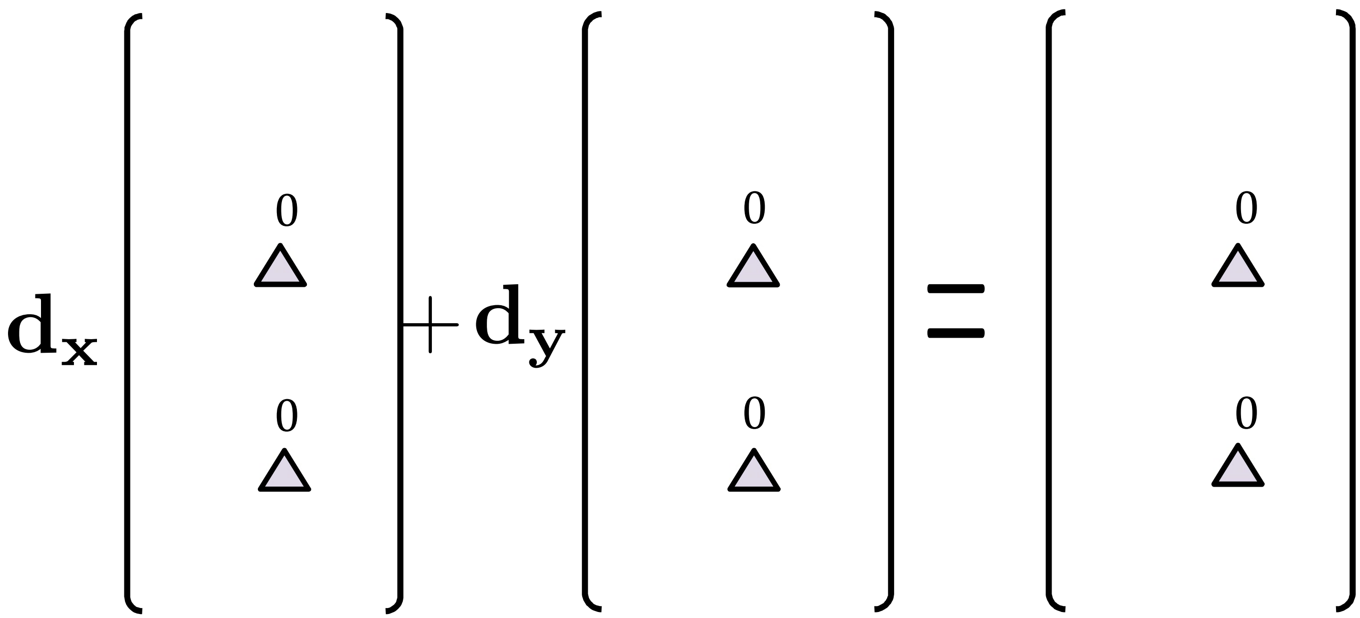




$\mathbf{I}(\mathbf{x})$
Solve for displacement
$\mathbf{d} = (-\mathbf{7}, -\mathbf{9})$
$\mathbf{J}(\mathbf{x})$
Error


$\mathbf{d_x}$
$+\mathbf{d_y}$
=

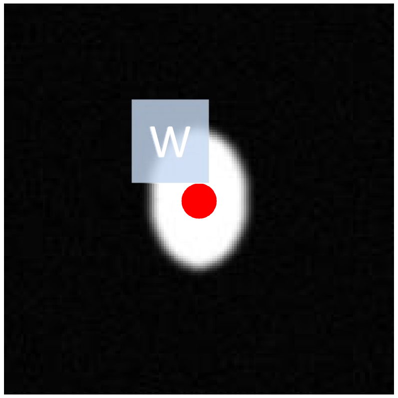
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

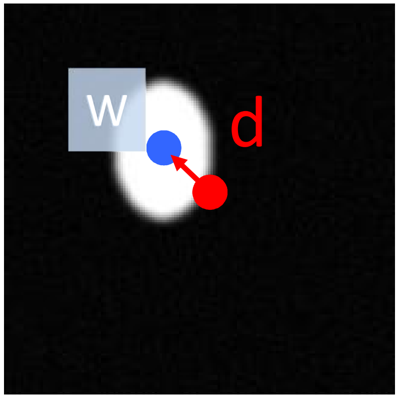


Cannot solve for the displacement

$$\min_{\mathbf{d}} E = \sum_{\mathbf{x} \in \mathbf{W}} \left\| J(\mathbf{x}+\mathbf{d}) - I(\mathbf{x}) \right\|^2$$

Pooling over a window

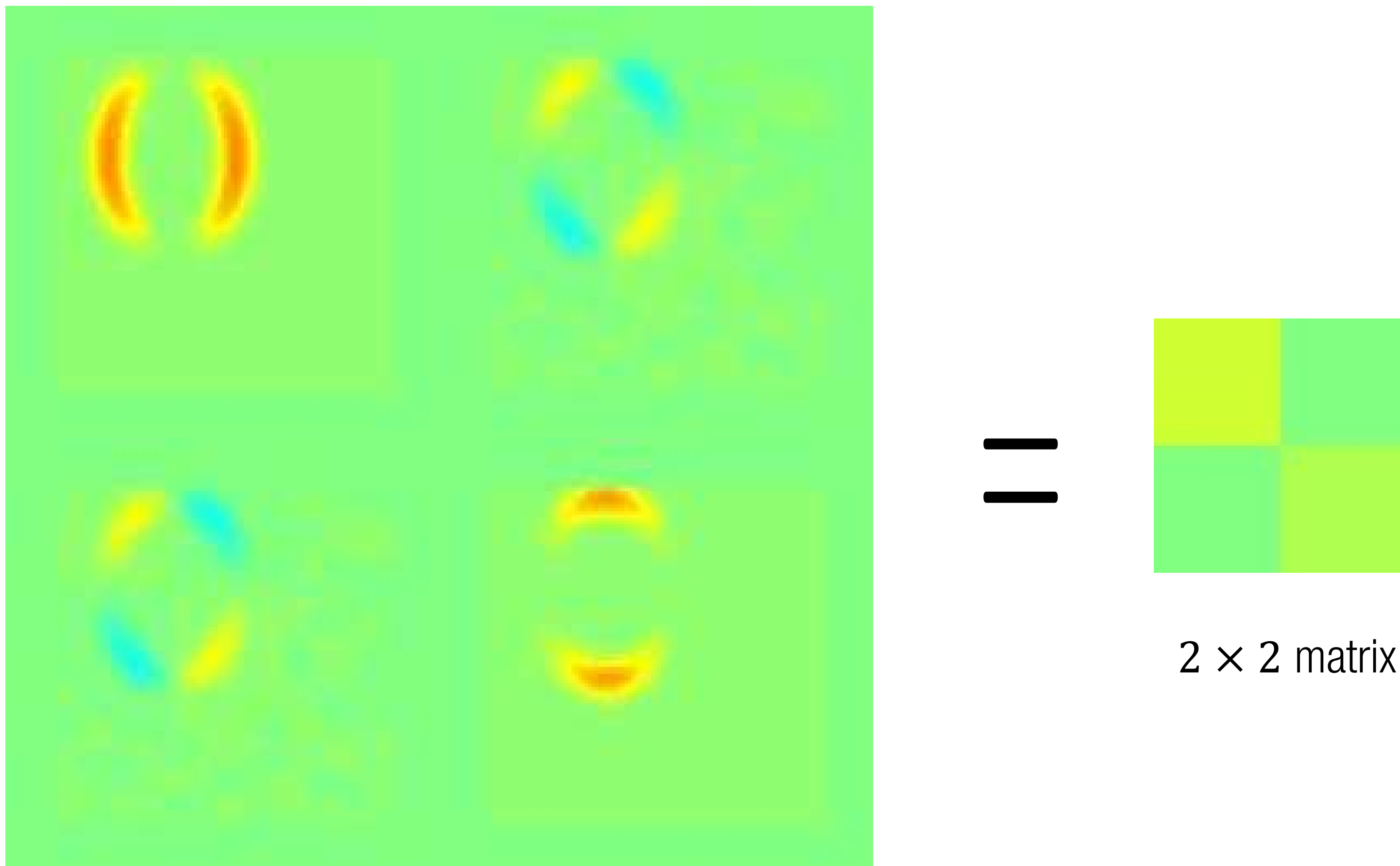




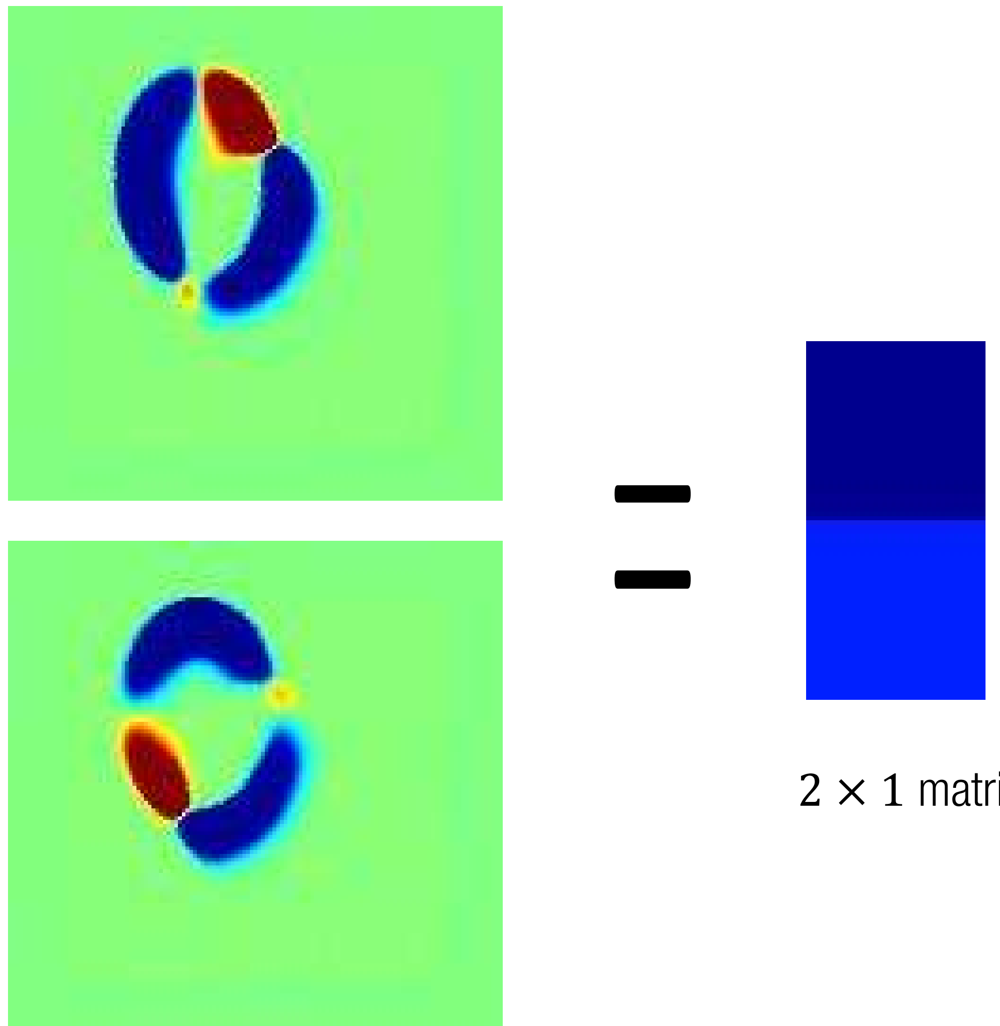
$$\min_{\mathbf{d}} E = \sum_{\mathbf{x} \in \mathbf{W}} \left\| J(\mathbf{x} + \mathbf{d}) - I(\mathbf{x}) \right\|^2$$

$$\sum_{\mathbf{x} \in \mathbf{W}} \left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} \right) \Delta \mathbf{d} = \sum_{\mathbf{x} \in \mathbf{W}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \left(I(\mathbf{x}) - J(\mathbf{x}) \right)$$

Summing over pixels

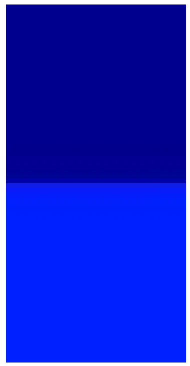


Summing over pixels

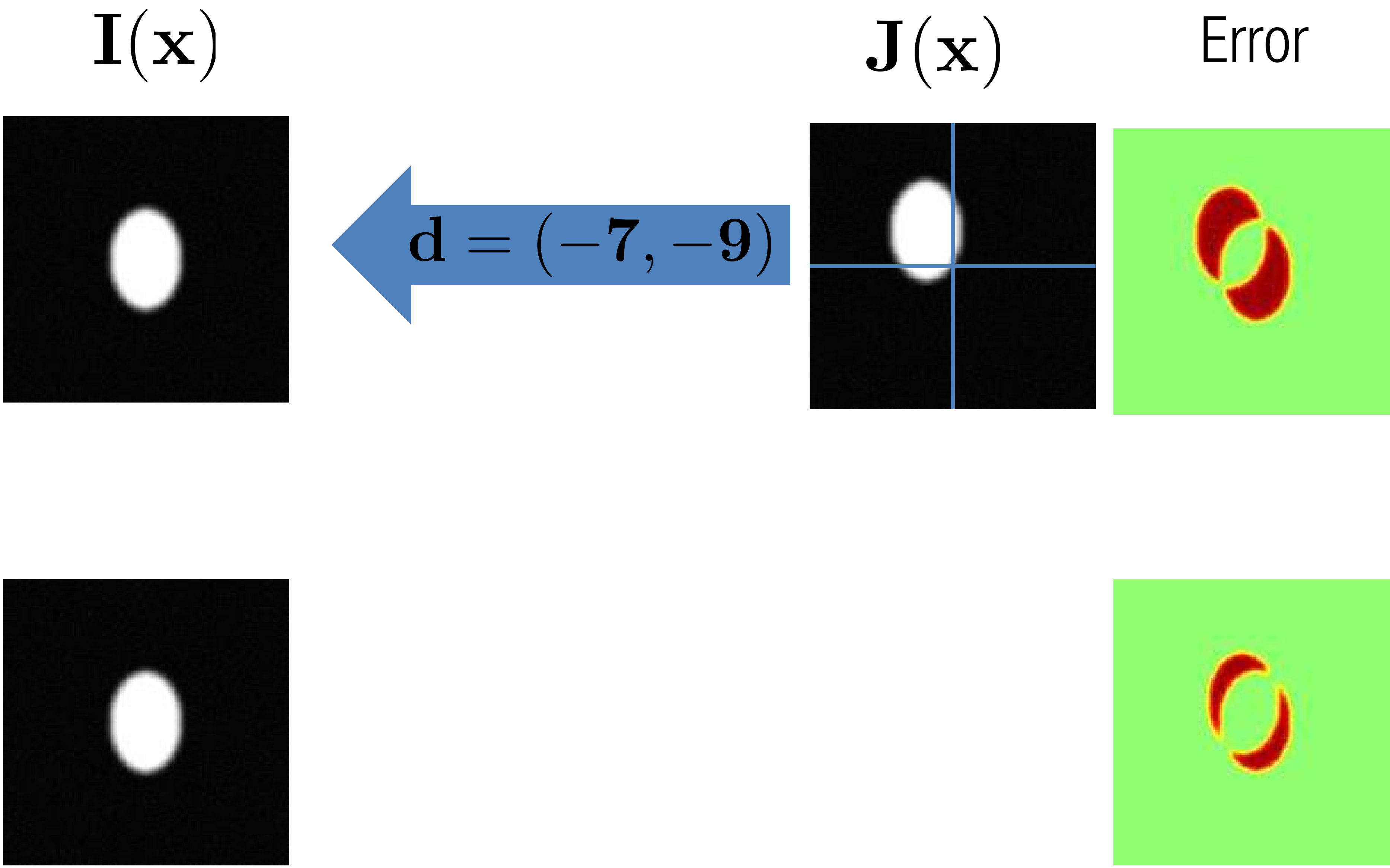


2×1 matrix

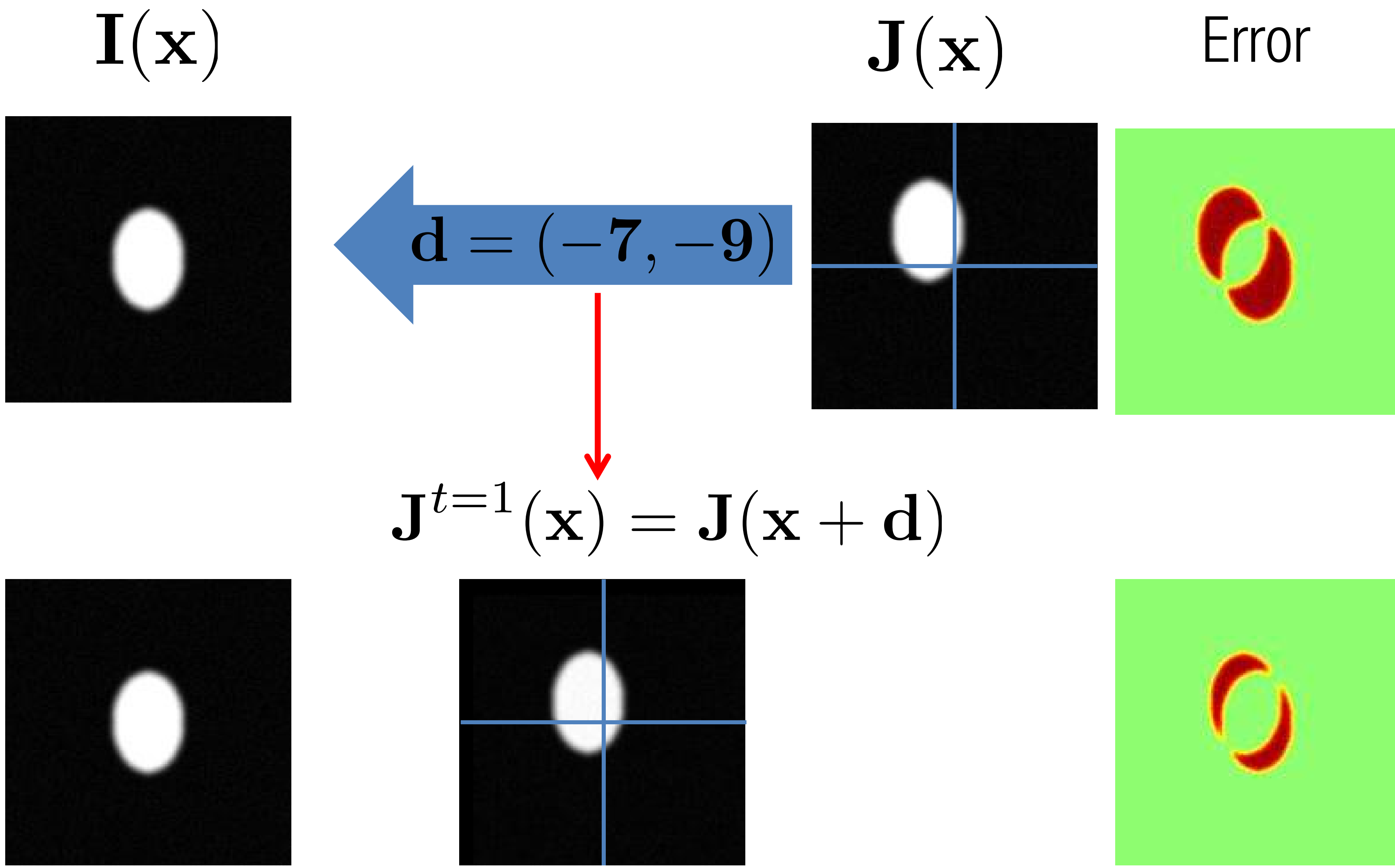
$$\sum_{\mathbf{x}\in W}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta\mathbf{d} = \sum_{\mathbf{x}\in W}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

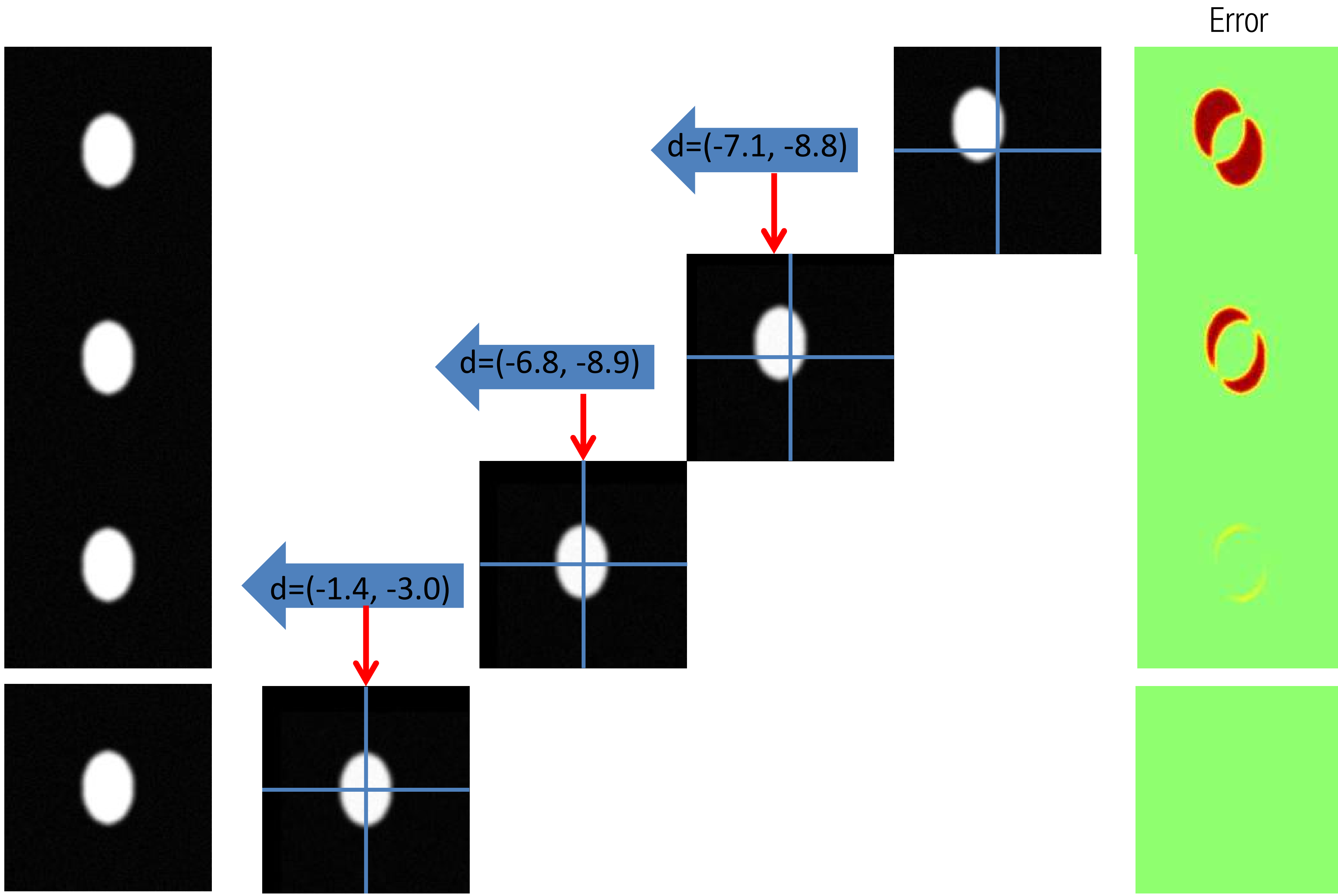
$\mathbf{d_x}$ $+\mathbf{d_y}$ $=$ 

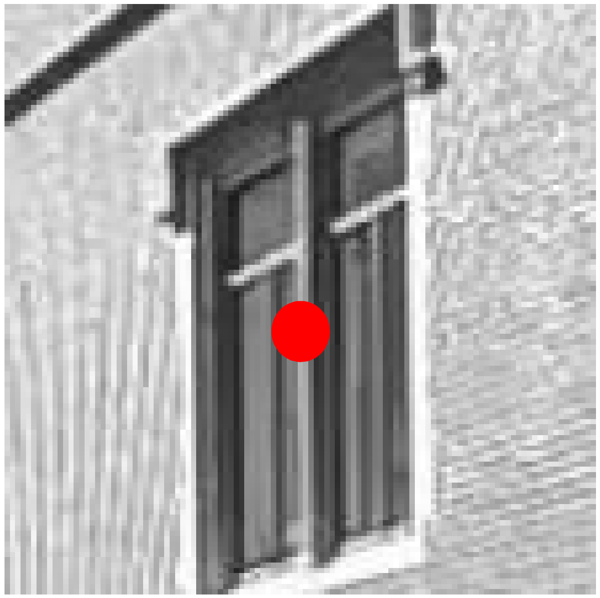
Step 3: Solve for displacement, warp image, and iterate



Step 3: Solve for displacement, warp image, and iterate

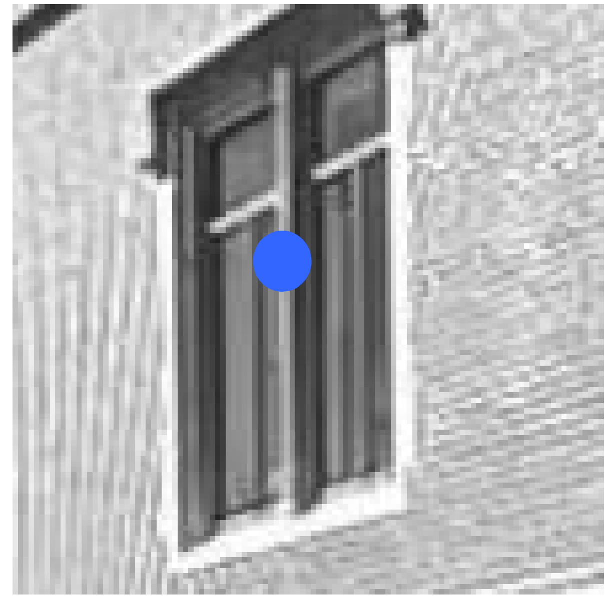



Error
d=(-7.1, -8.8)
d=(-6.8, -8.9)
d=(-1.4, -3.0)



$\mathbf{I}(\mathbf{x})$

$t = 0$



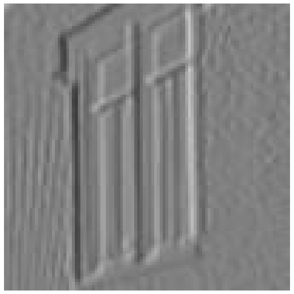
$\mathbf{J}(\mathbf{x})$

$t = 1$

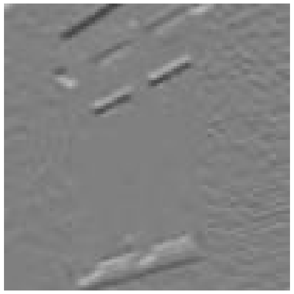
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

$$\left(\frac{\partial J(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

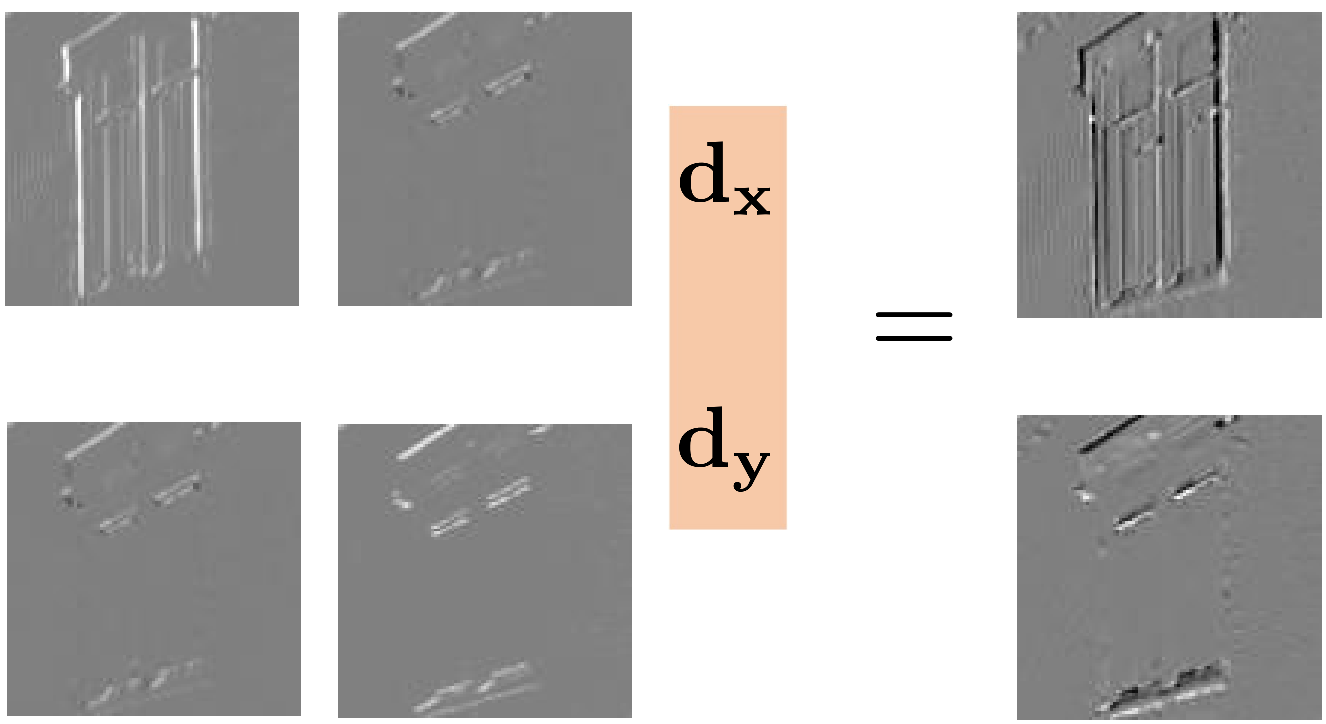
$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta x} =$$



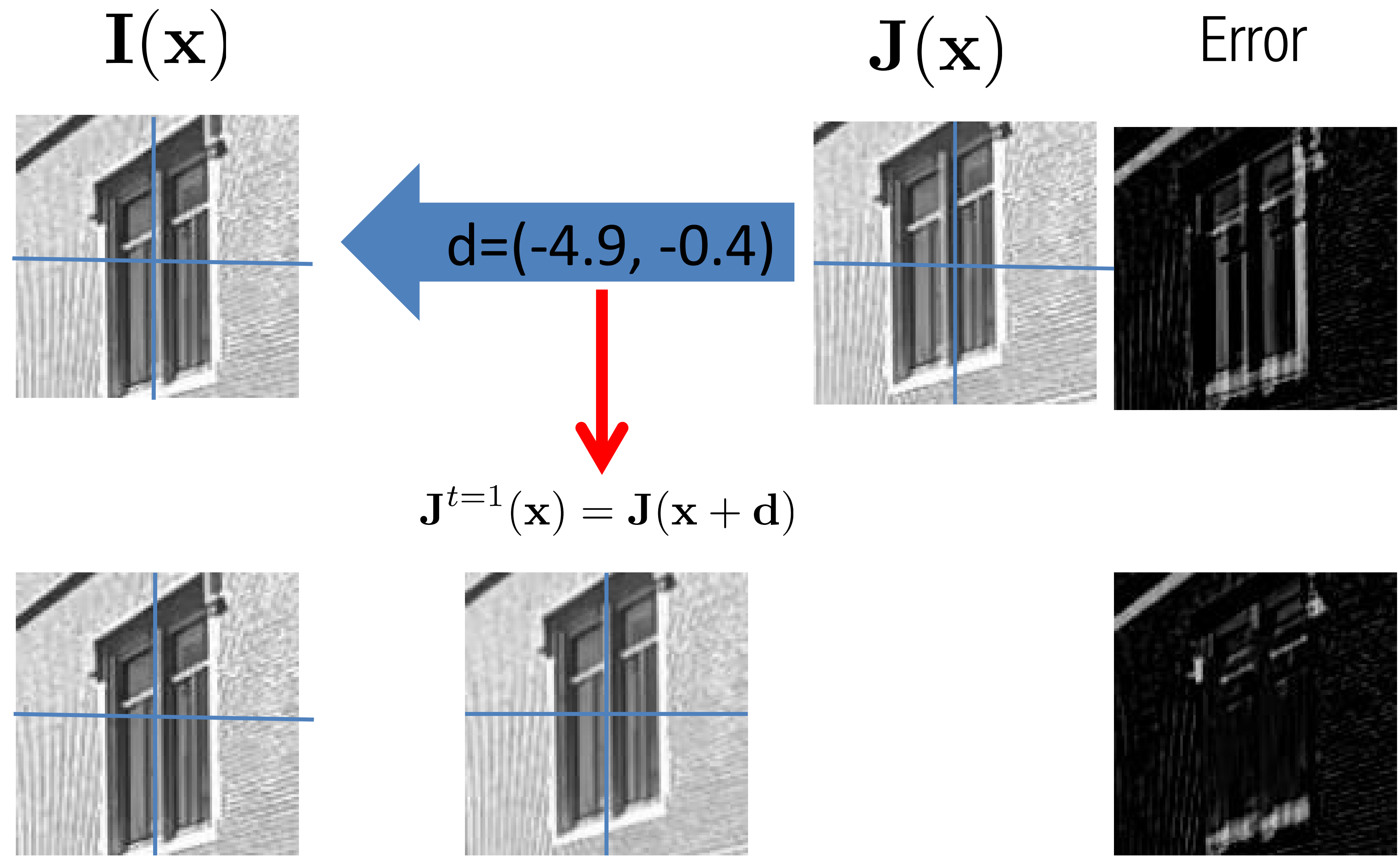
$$\frac{\delta \mathbf{J}(\mathbf{x})}{\delta y} =$$

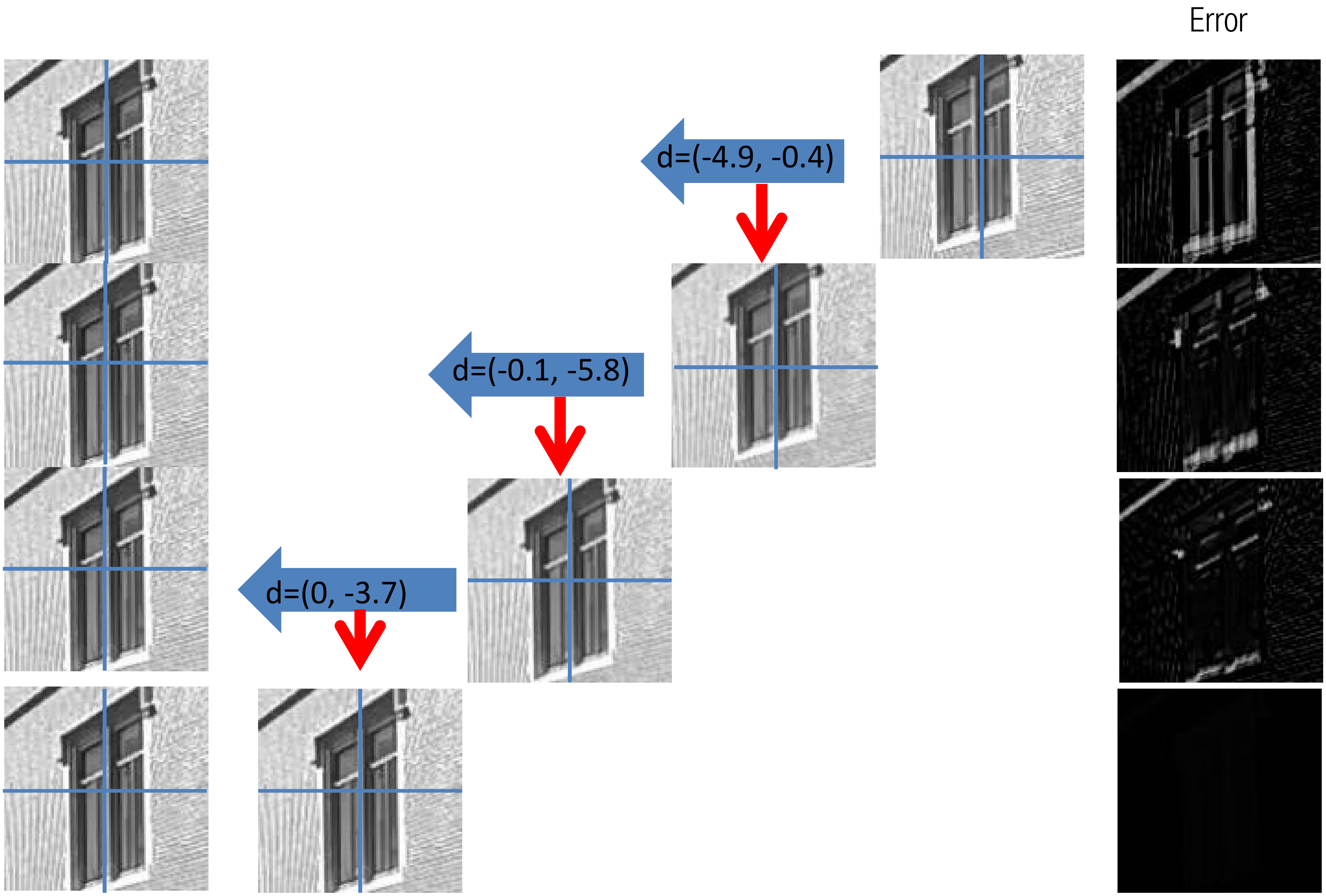


$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$



Step 3: Solve for displacement, warp image, and iterate



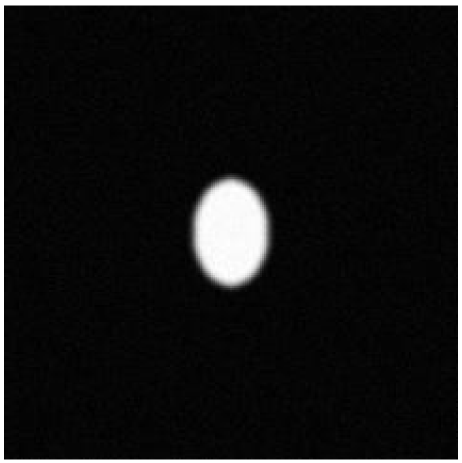


Error
d=(-4.9, -0.4)
d=(-0.1, -5.8)
d=(0, -3.7)

A Failed Case: fast movement

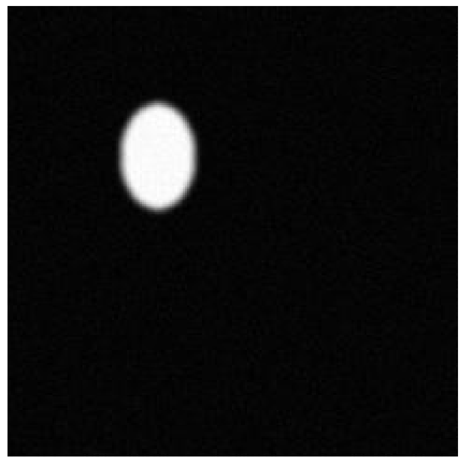
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \left(I(\mathbf{x}) - J(\mathbf{x})\right)$$

A Failed Case: fast movement



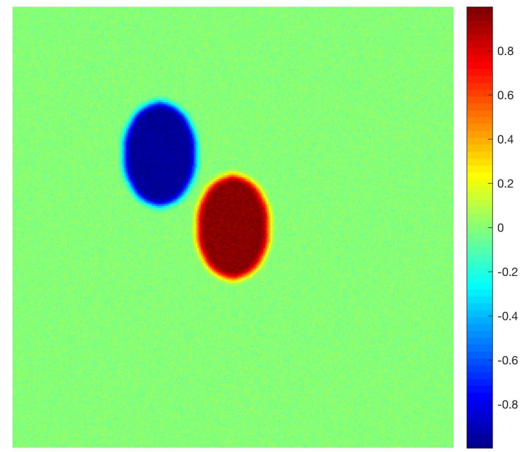
$I(\mathbf{x}) = J(\mathbf{x} + \mathbf{d})$

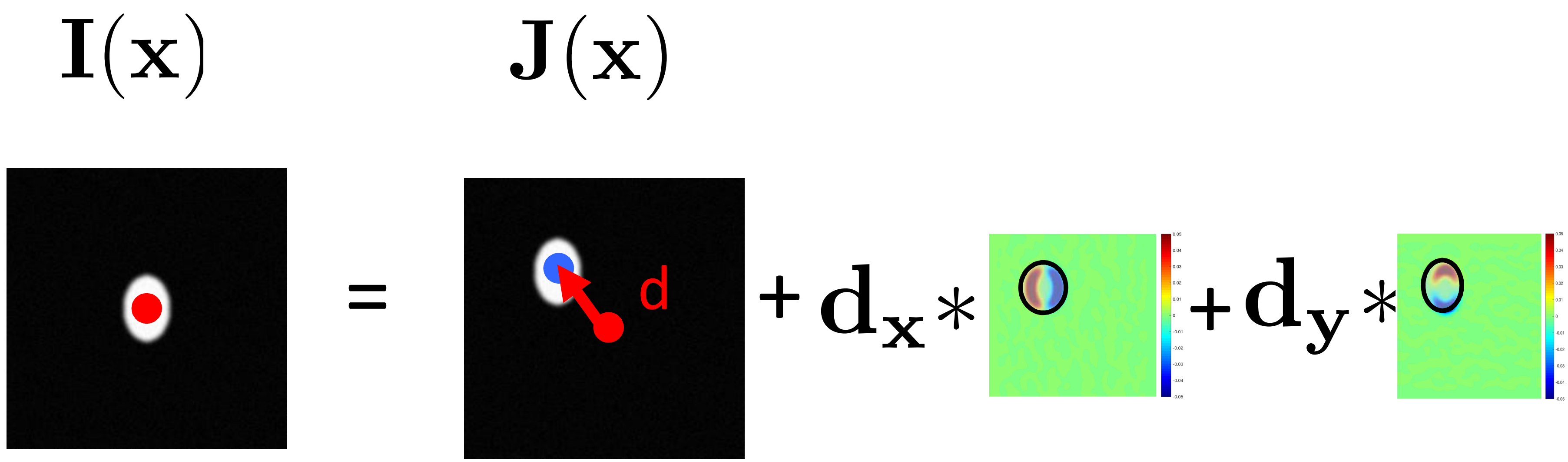
-



$J(\mathbf{x})$

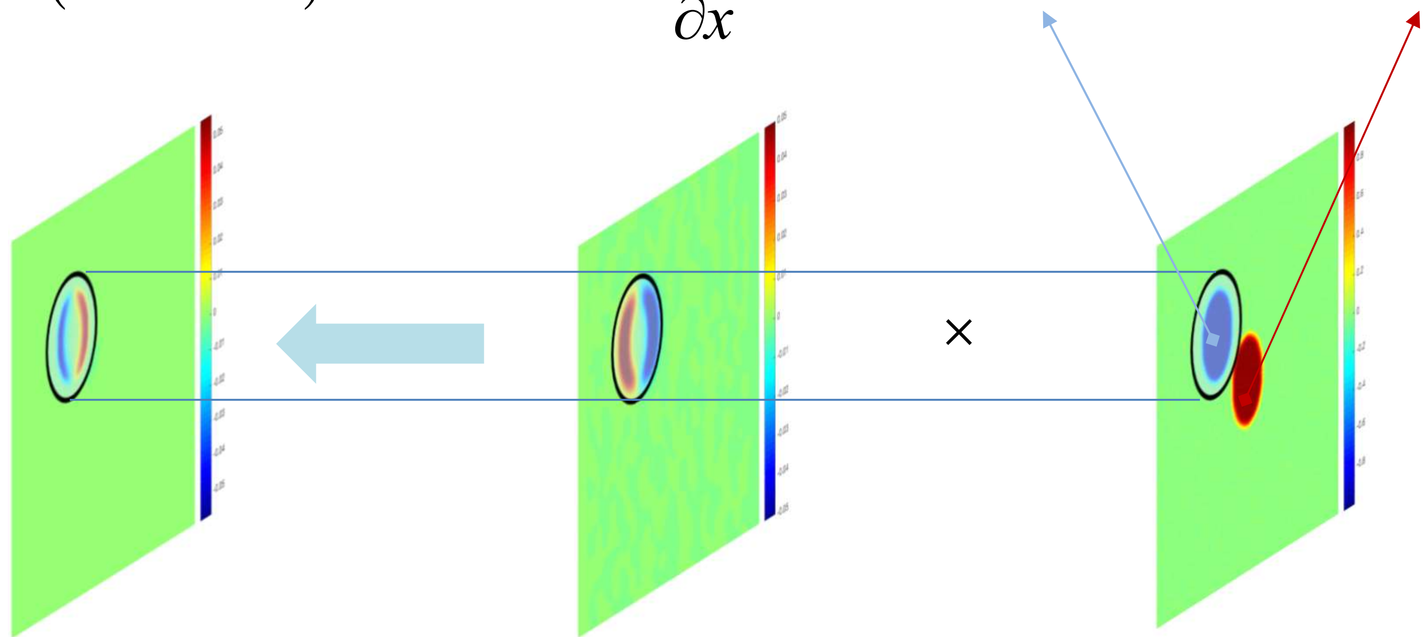
=



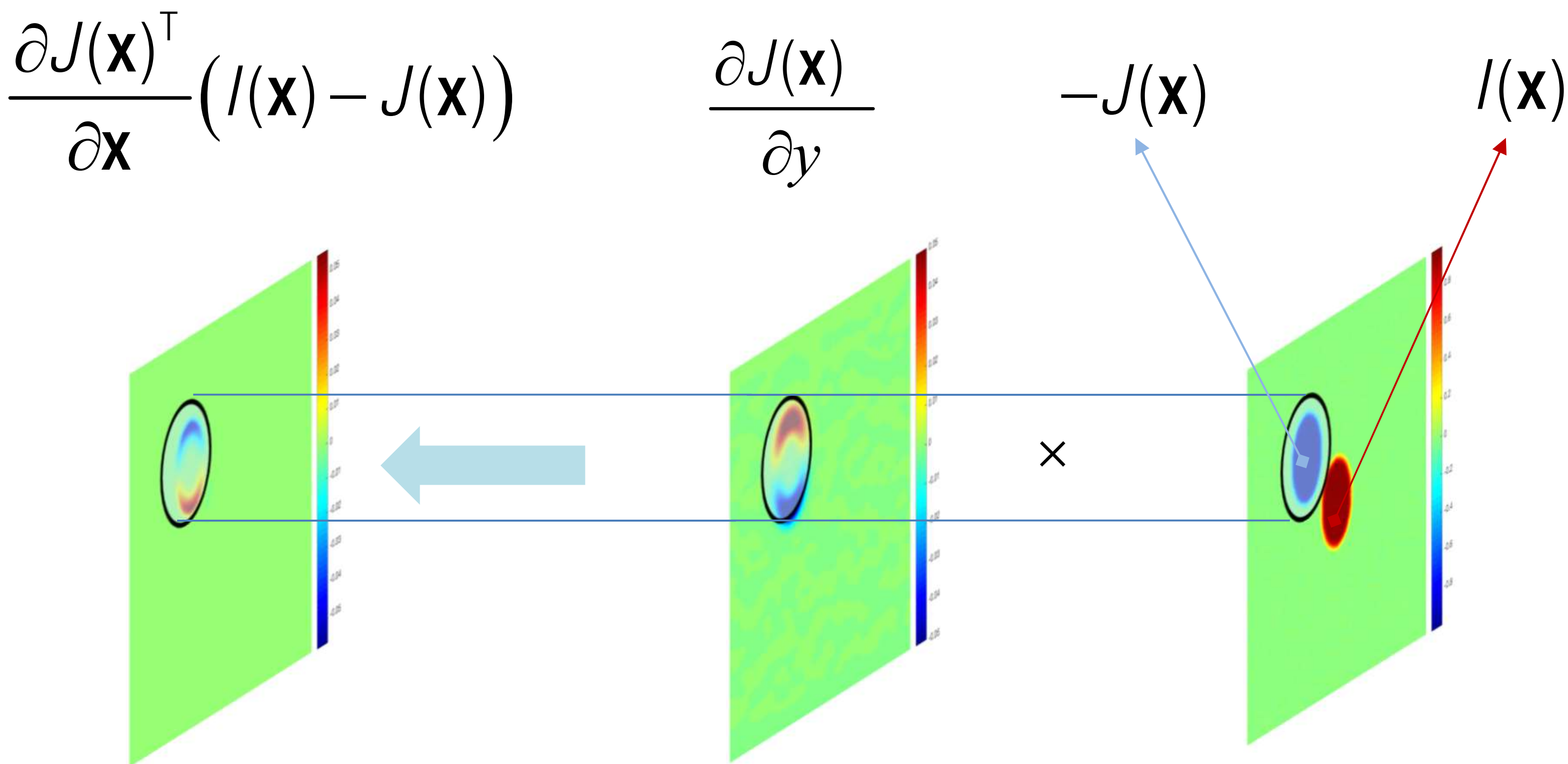


$$I(\mathbf{x}) = J(\mathbf{x} + \mathbf{d})$$

$$\frac{\partial J(\mathbf{x})^{\top}}{\partial \mathbf{x}}\big(I(\mathbf{x}) - J(\mathbf{x})\big) \qquad \frac{\partial J(\mathbf{x})}{\partial x} \qquad -J(\mathbf{x}) \qquad I(\mathbf{x})$$



The influence of I(x) is not incorporated!



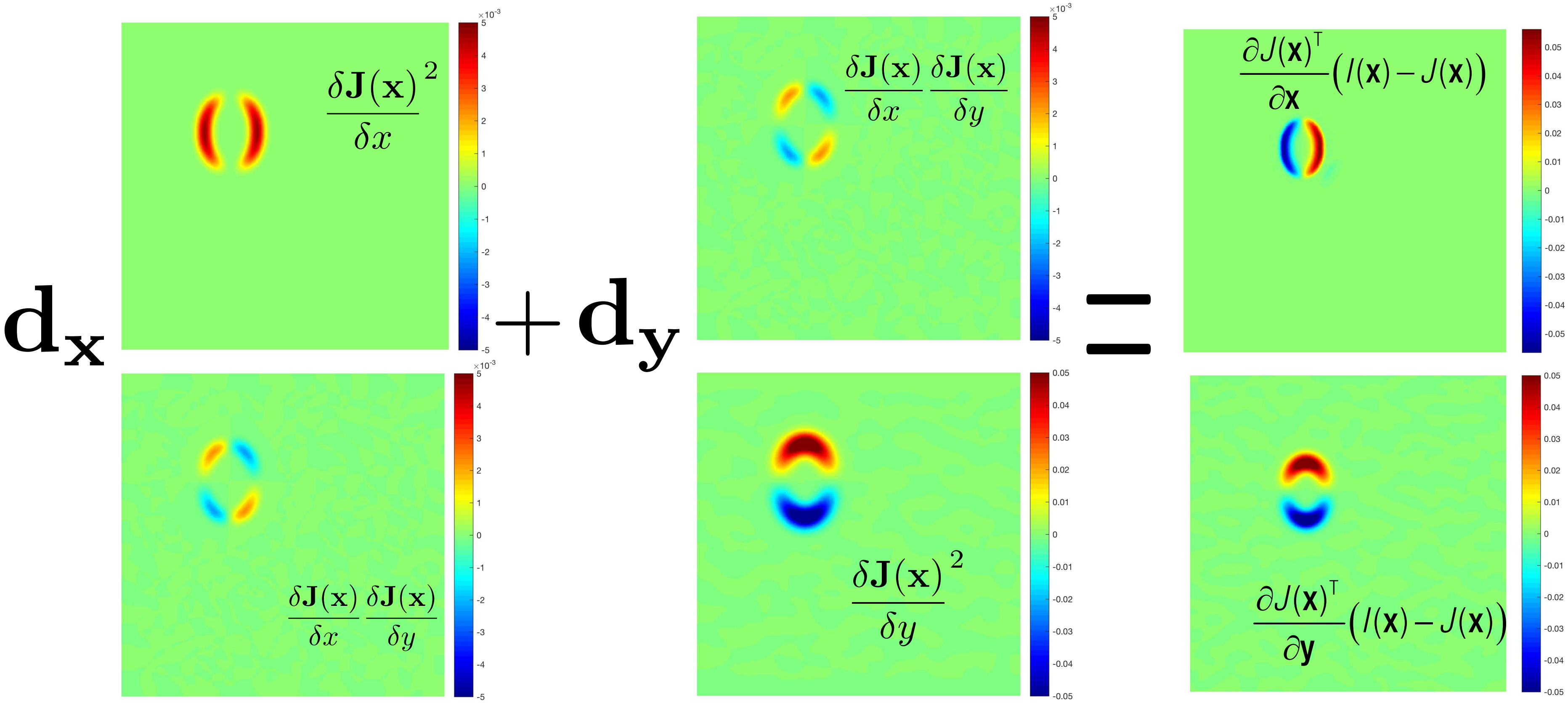
The influence of I(x) is not incorporated!

$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta\mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\top}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

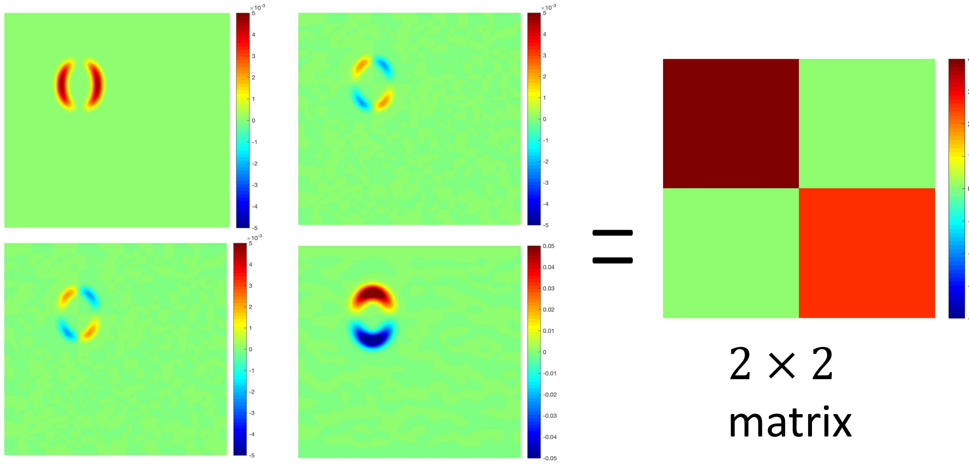
The influence of I(x) is Not included!

Guess what's the corresponding displacement?

$$\left(\frac{\partial J(\mathbf{x})^{T}}{\partial \mathbf{x}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})^{T}}{\partial \mathbf{x}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$



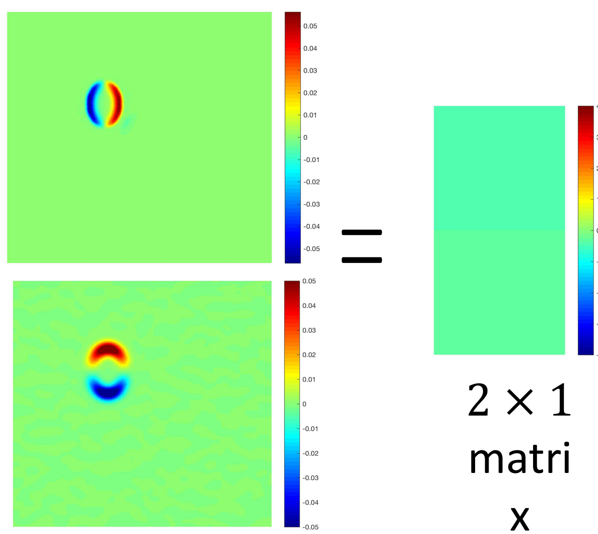
Summing over pixels



2×2
matrix

$$\frac{\partial J(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}$$

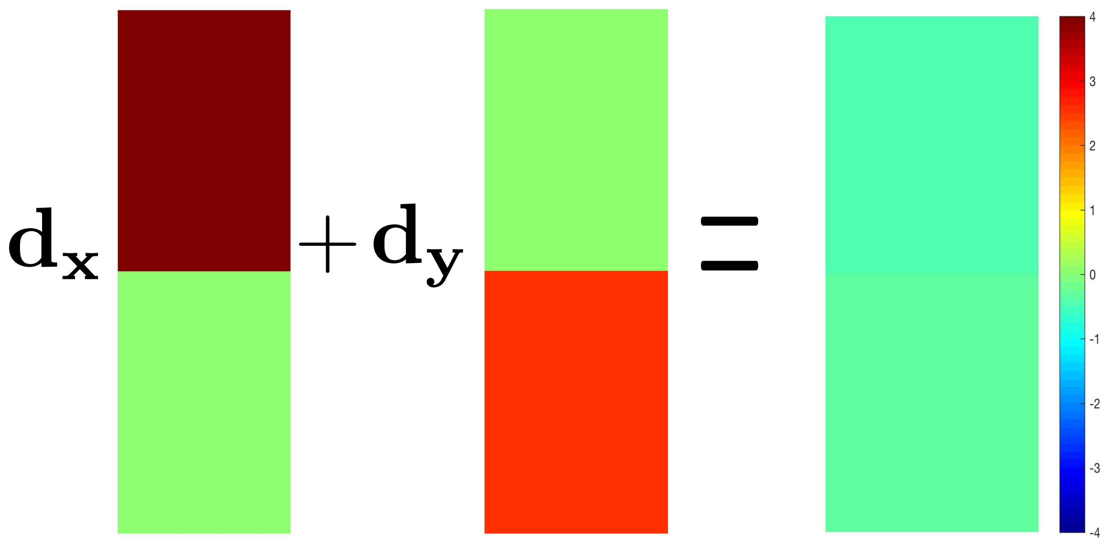
Summing over pixels



2×1
matri
x

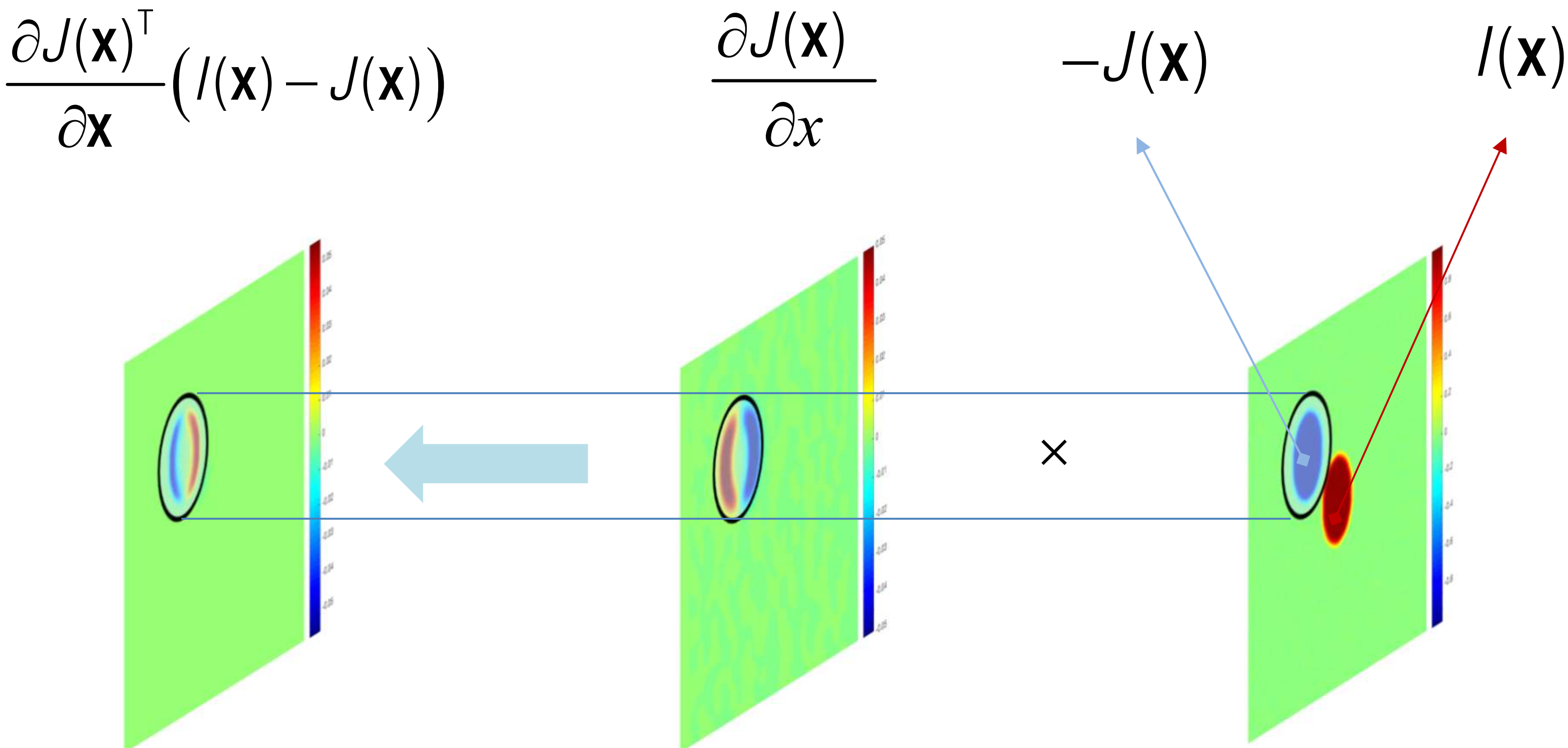
$$\frac{\partial J(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}} \left(I(\mathbf{x}) - J(\mathbf{x}) \right)$$

$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$



[−0.011, 0.09]

Almost zero motion, why?

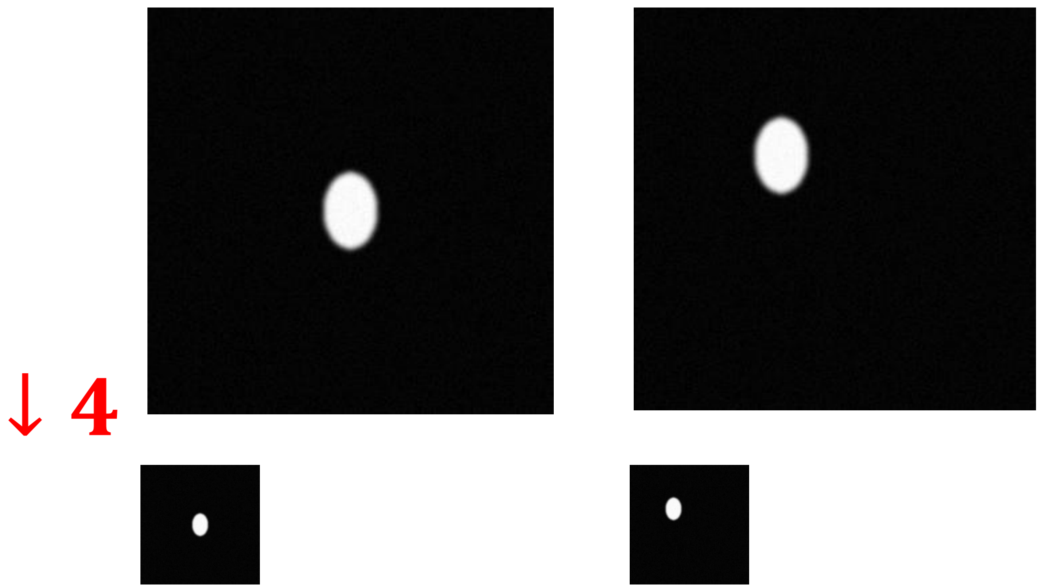


The influence of I(x) is not incorporated!

Solution 1: multi scale optical flow

Solution 2: increase the kernel size of gradient operator

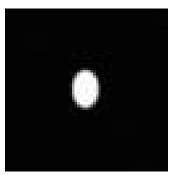
$I(\mathbf{x}) = J(\mathbf{x}+\mathbf{d})$ $J(\mathbf{x})$



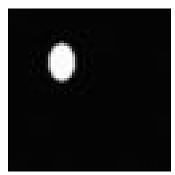
$I_{\downarrow 4}(\mathbf{x})$ $J_{\downarrow 4}(\mathbf{x})$

Solution 1:
multi scale optical flow

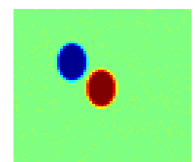
$$\left(\frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\left(I_{\downarrow 4}(\mathbf{x}) - J_{\downarrow 4}(\mathbf{x})\right)$$



$-$

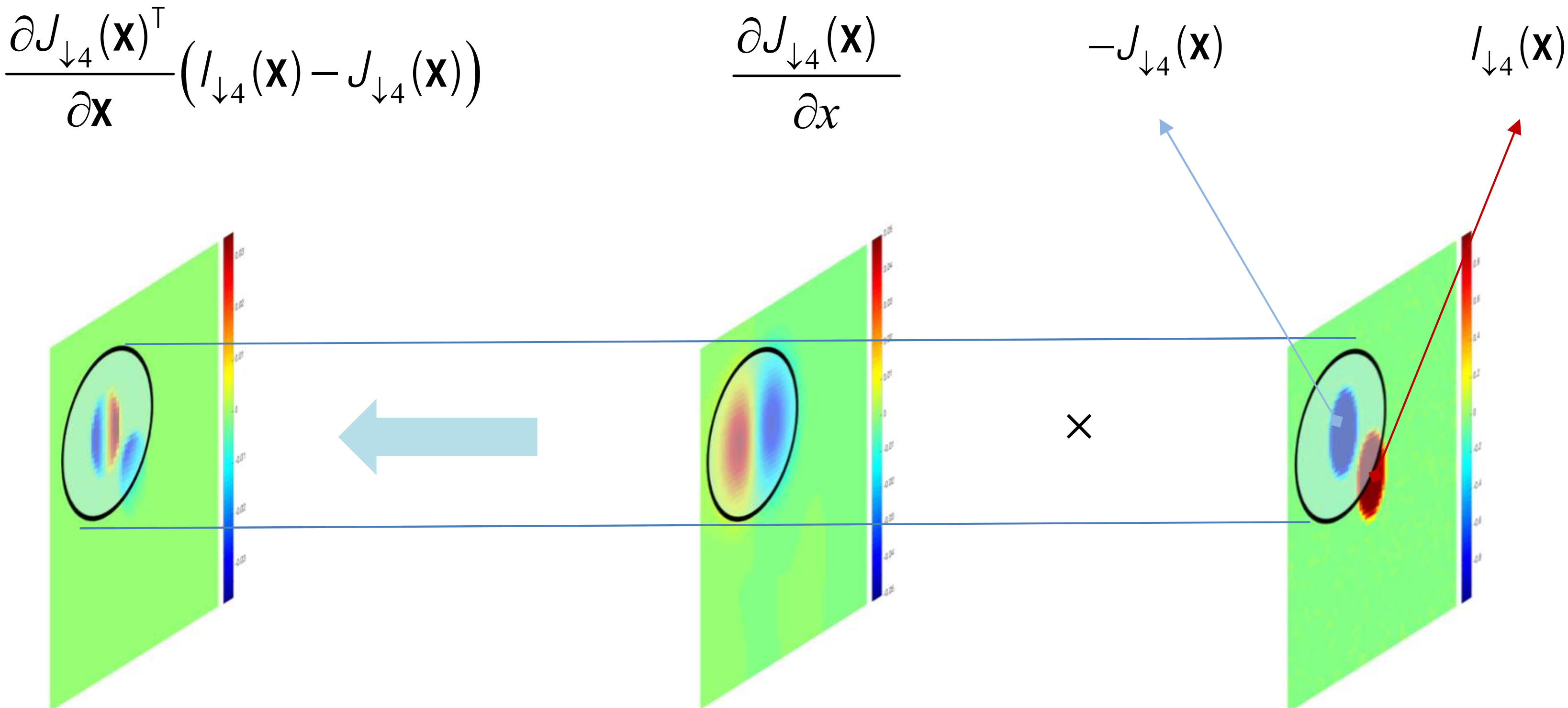


$=$

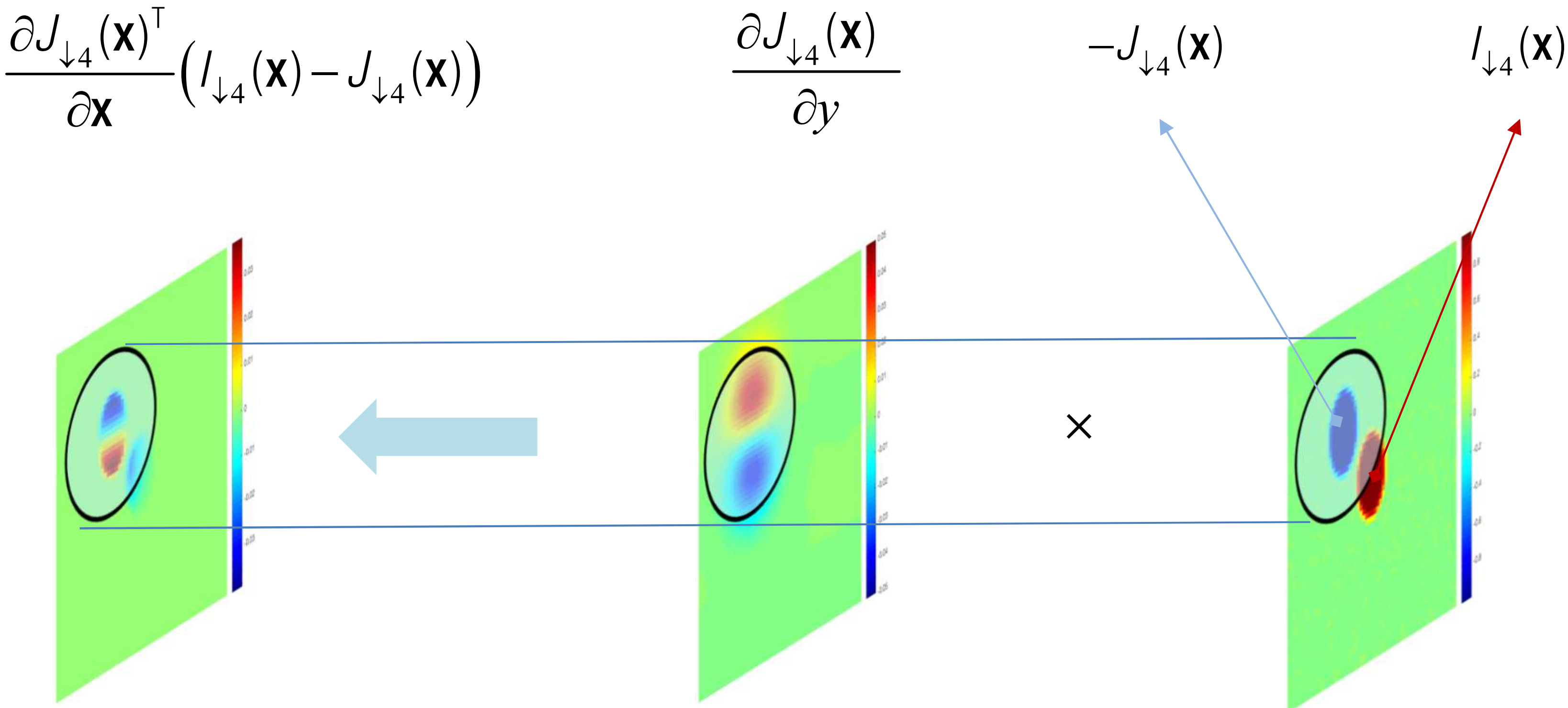


$I_{\downarrow 4}(\mathbf{x})$ $J_{\downarrow 4}(\mathbf{x})$

Start from a coarser resolution image

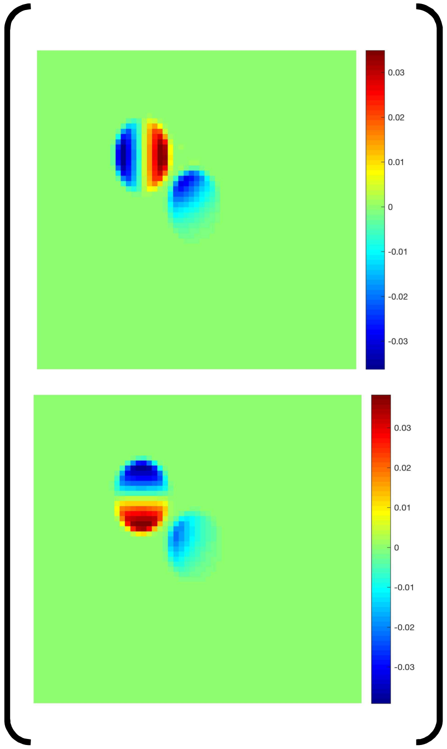


The influence of I(x) is incorporated!

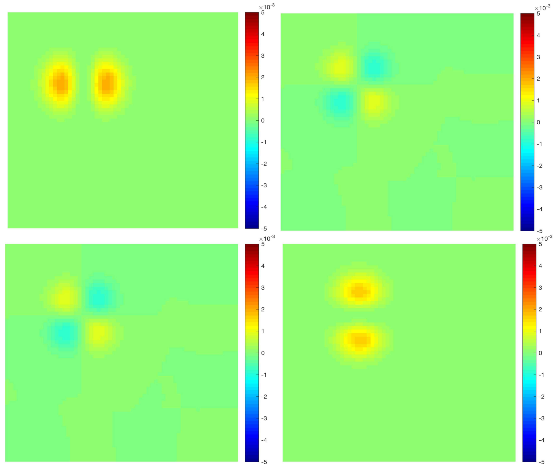


The influence of I(x) is incorporated!

$$\left(\frac{\partial J_{\downarrow 4}(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}} \frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}} \right) \Delta \mathbf{d} = \frac{\partial J_{\downarrow 4}(\mathbf{x})^{\mathsf{T}}}{\partial \mathbf{x}} \left(I_{\downarrow 4}(\mathbf{x}) - J_{\downarrow 4}(\mathbf{x}) \right)$$



Summing over pixels

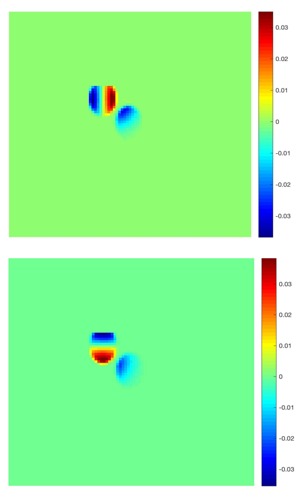


=

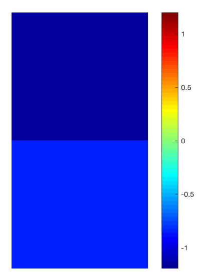
2 × 2
matrix

$$\frac{\partial J_{\downarrow 4}(\mathbf{x})^{T}}{\partial \mathbf{x}} \frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}$$

Summing over pixels



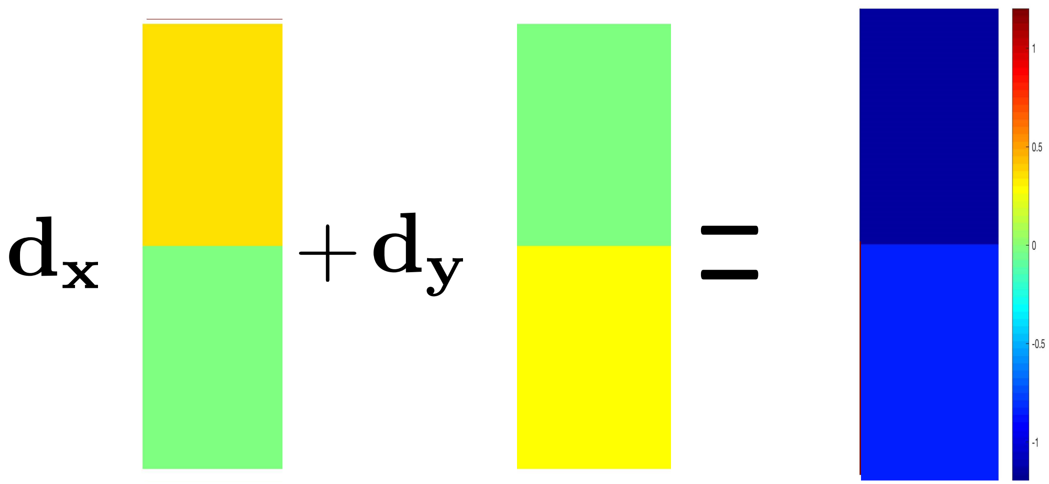
=



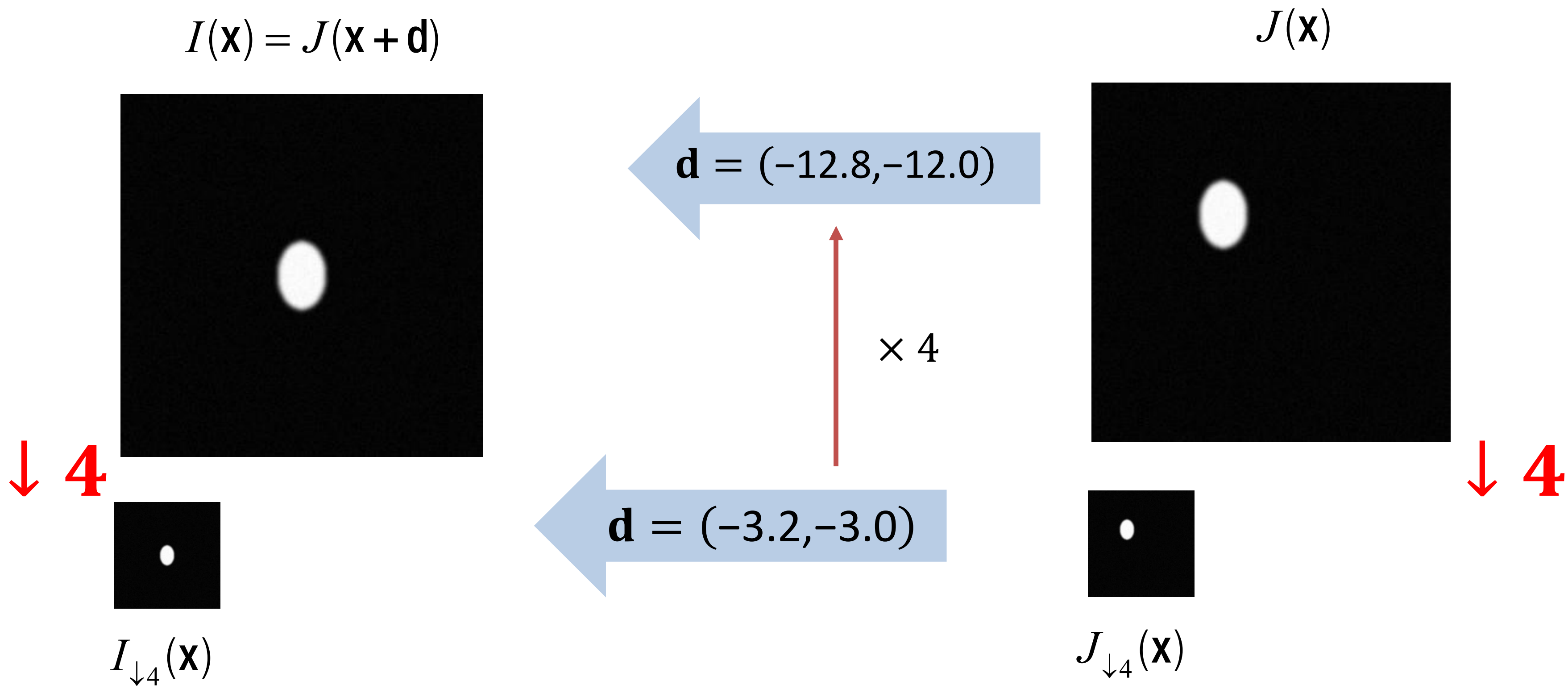
2 × 1
matrix

$$\frac{\partial J_{\downarrow 4}(\mathbf{x})^{T}}{\partial \mathbf{x}} \left(I_{\downarrow 4}(\mathbf{x}) - J_{\downarrow 4}(\mathbf{x}) \right)$$

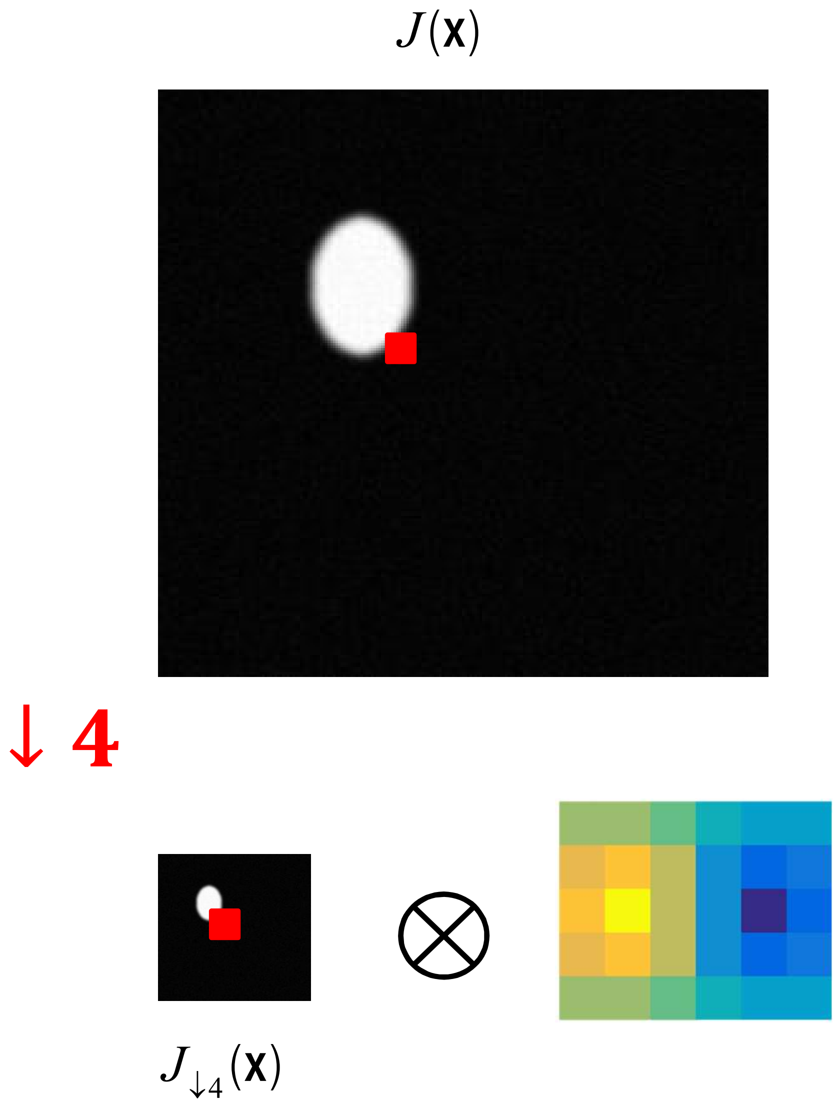
$$\left(\frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}^{\top}\frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J_{\downarrow 4}(\mathbf{x})}{\partial \mathbf{x}}^{\top}\left(I_{\downarrow 4}(\mathbf{x}) - J_{\downarrow 4}(\mathbf{x})\right)$$

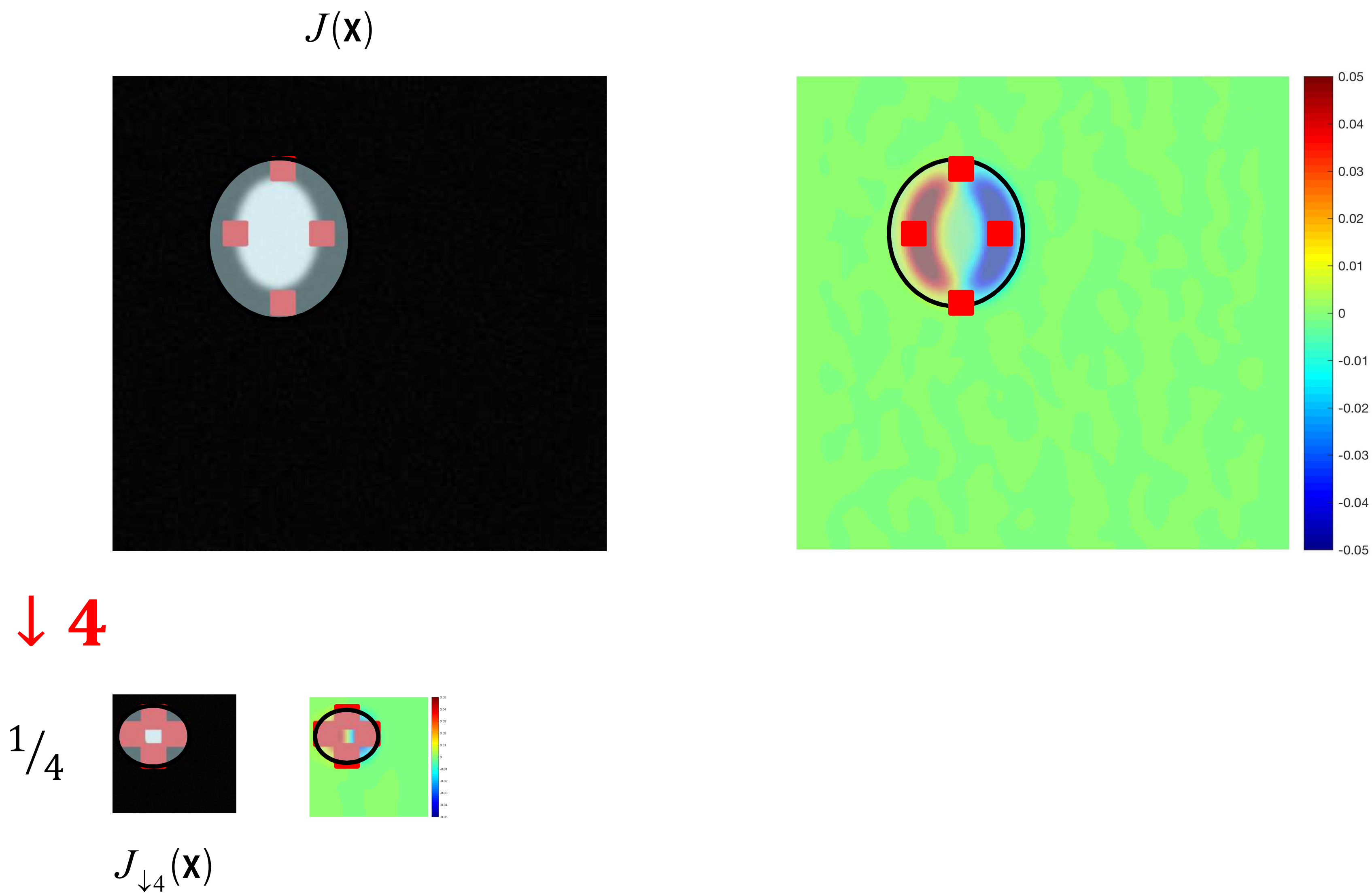


[−3.3 −3.0]

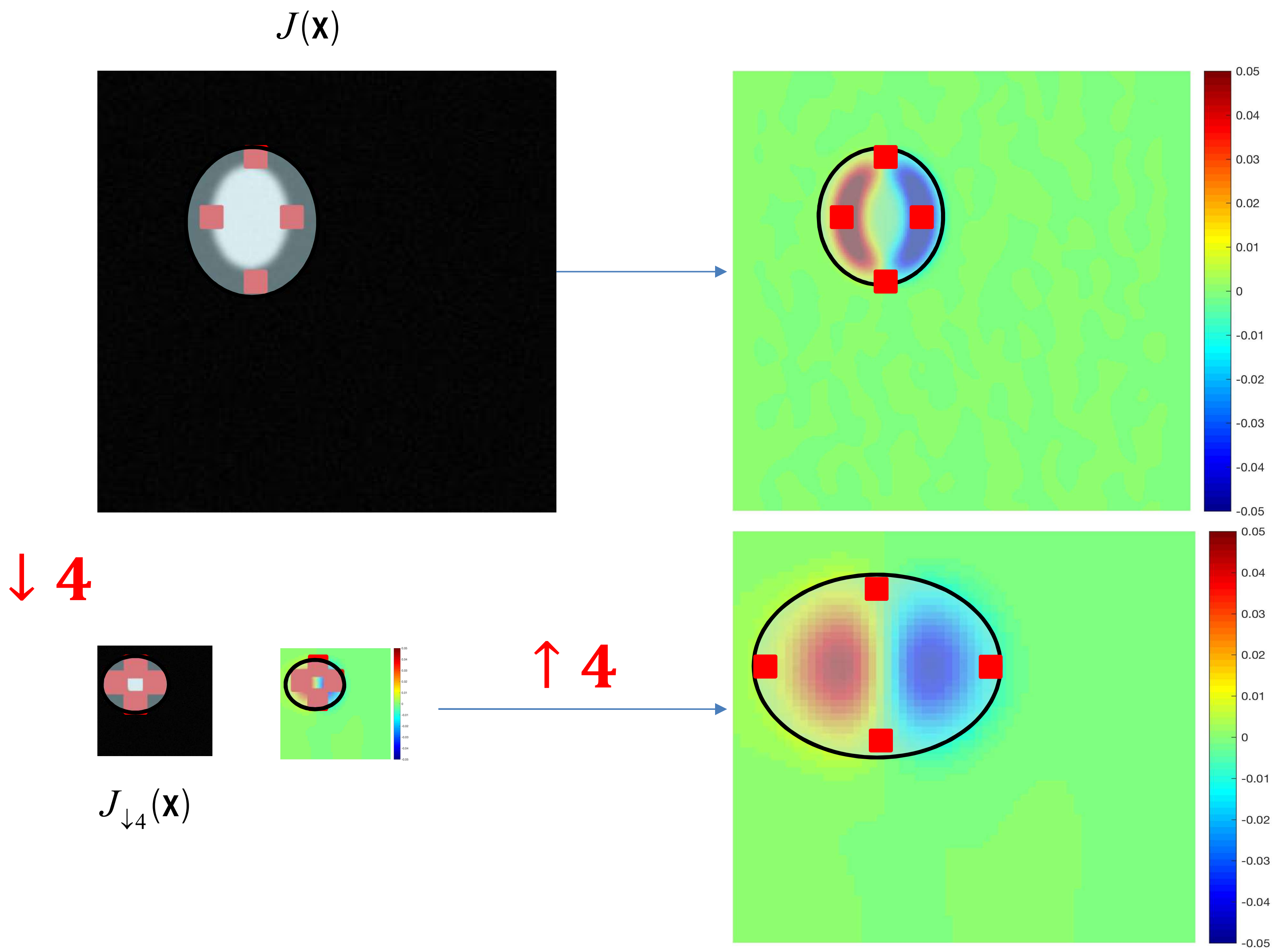


$I(\mathbf{x}) = J(\mathbf{x}+\mathbf{d})$
$J(\mathbf{x})$
$\mathbf{d} = (-12.8, -12.0)$
$\times 4$
$\downarrow 4$
$\downarrow 4$
$\mathbf{d} = (-3.2, -3.0)$
$I_{\downarrow 4}(\mathbf{x})$
$J_{\downarrow 4}(\mathbf{x})$

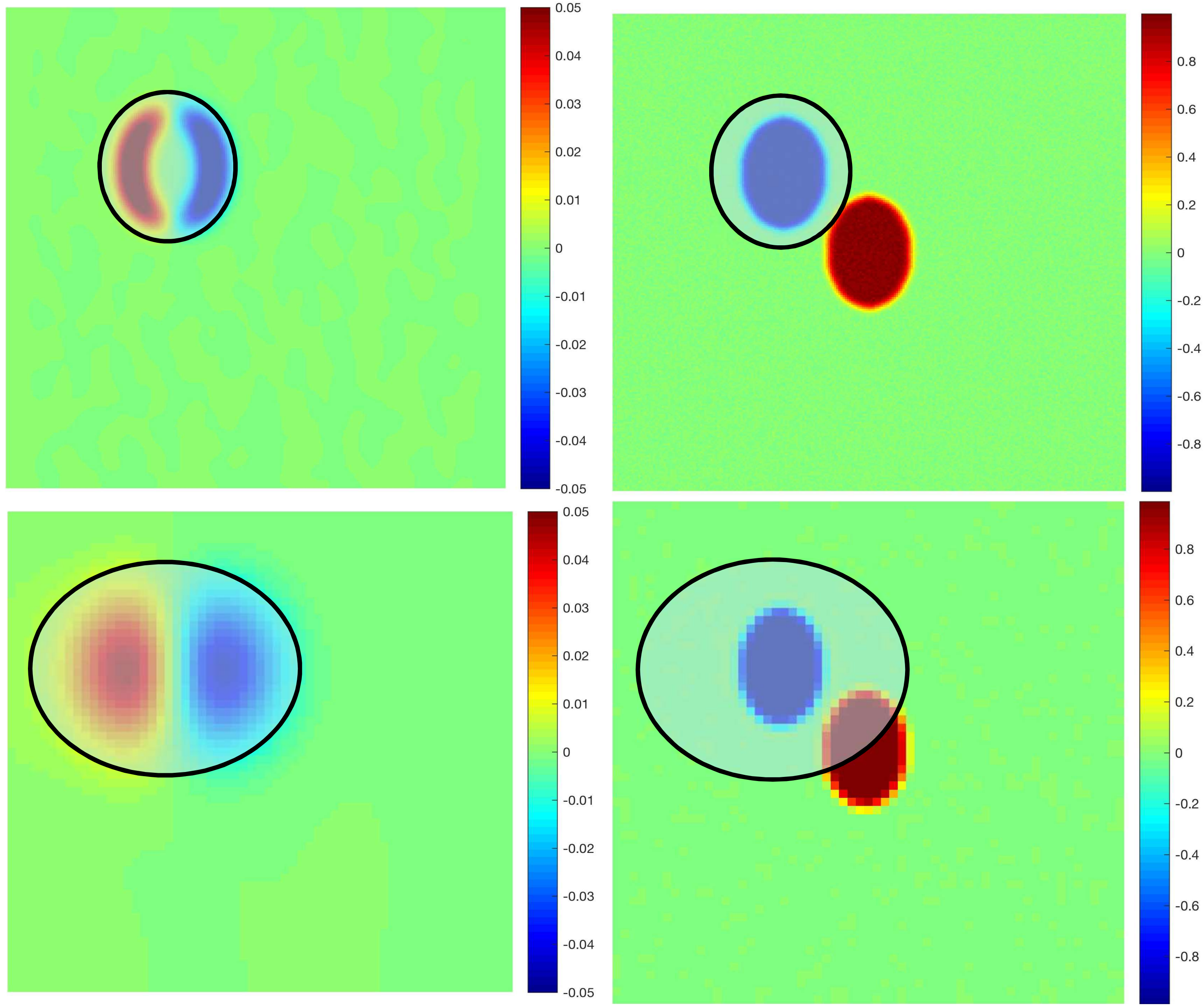

J(x)
↓ 4
J↓4(x)



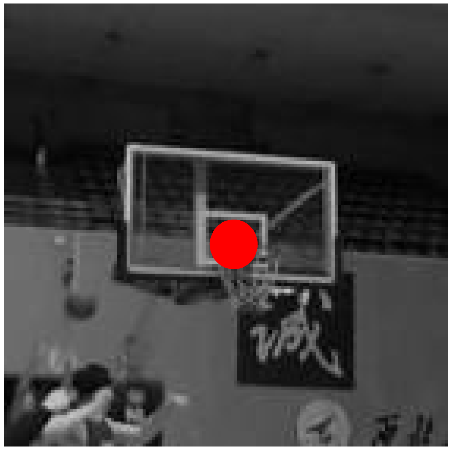
J(x)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
↓ 4
1/4
J↓4(x)



$J(\mathbf{x})$
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
↓ 4
↑ 4
$J_{\downarrow 4}(\mathbf{x})$


0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8

Down sampling image is equivalent to increase the kernel size.



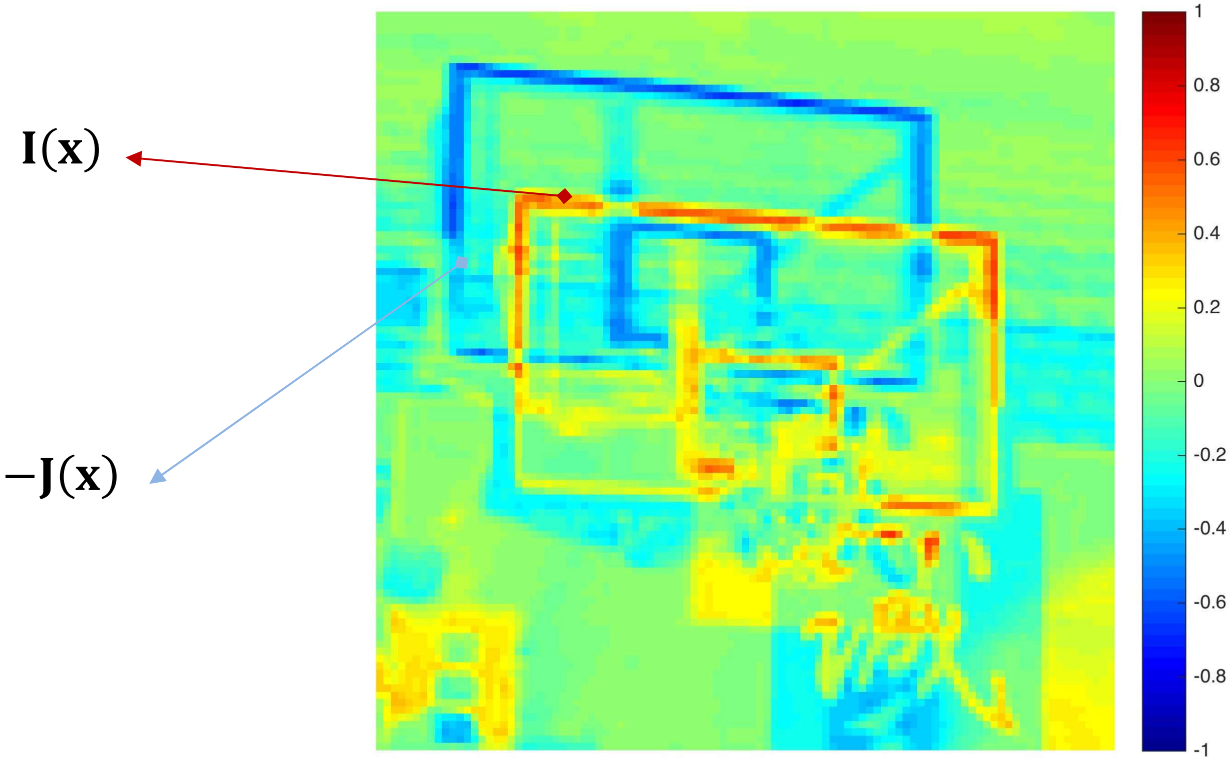
$\mathbf{I}(\mathbf{x})$

$t = 0$

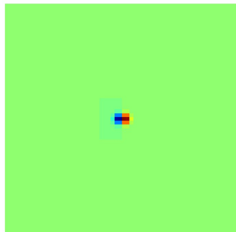
$\mathbf{J}(\mathbf{x})$

$t = 1$

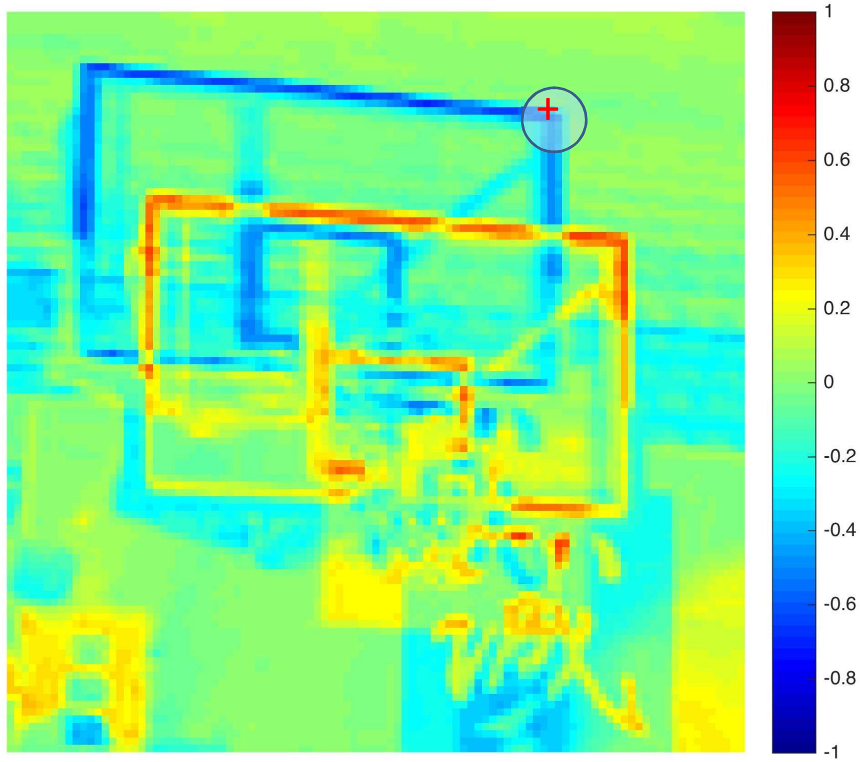
$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{T}\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta\mathbf{d}=\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{T}\big(I(\mathbf{x})-J(\mathbf{x})\big)$$



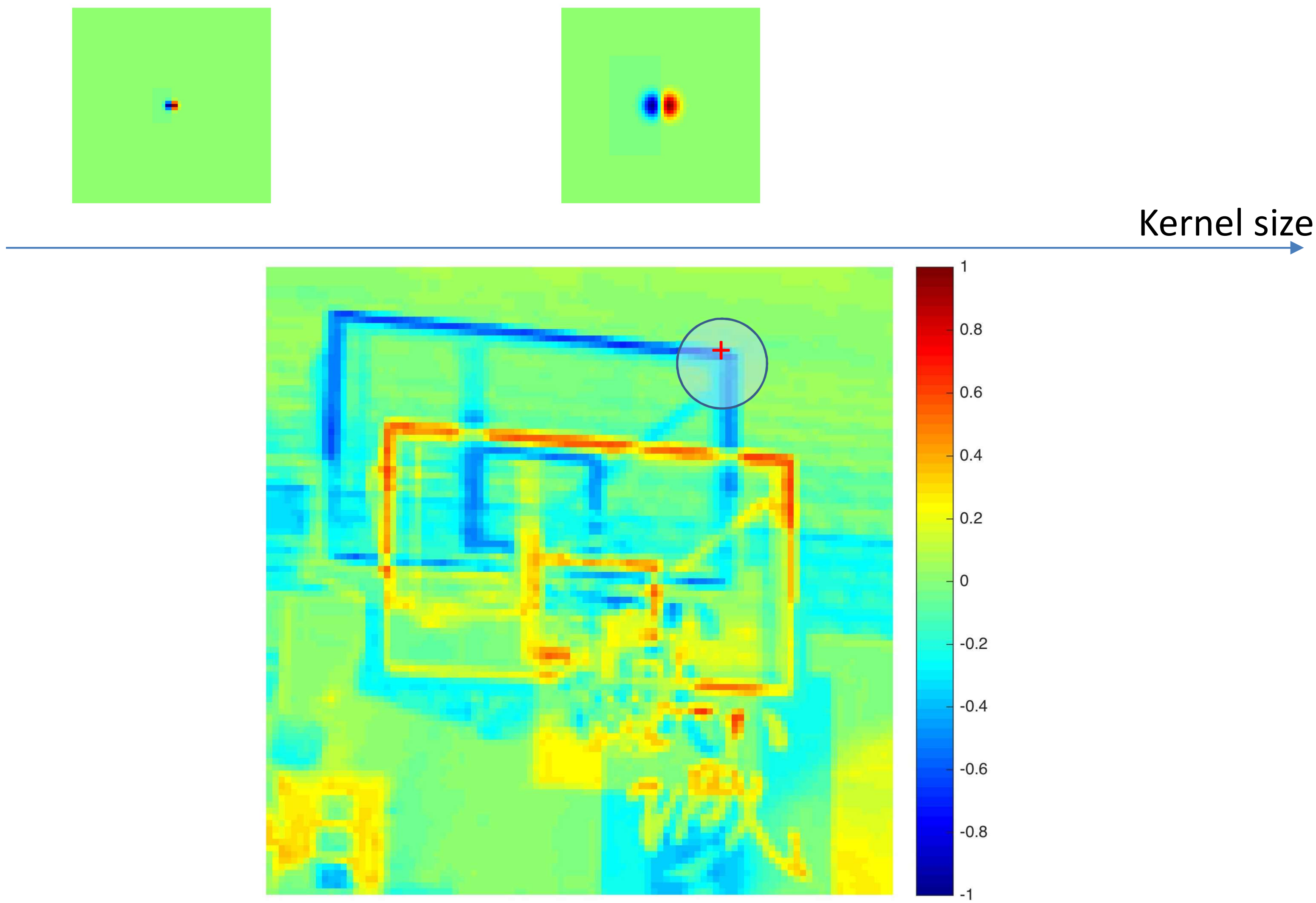
Solution 2: increase the kernel size of gradient operator



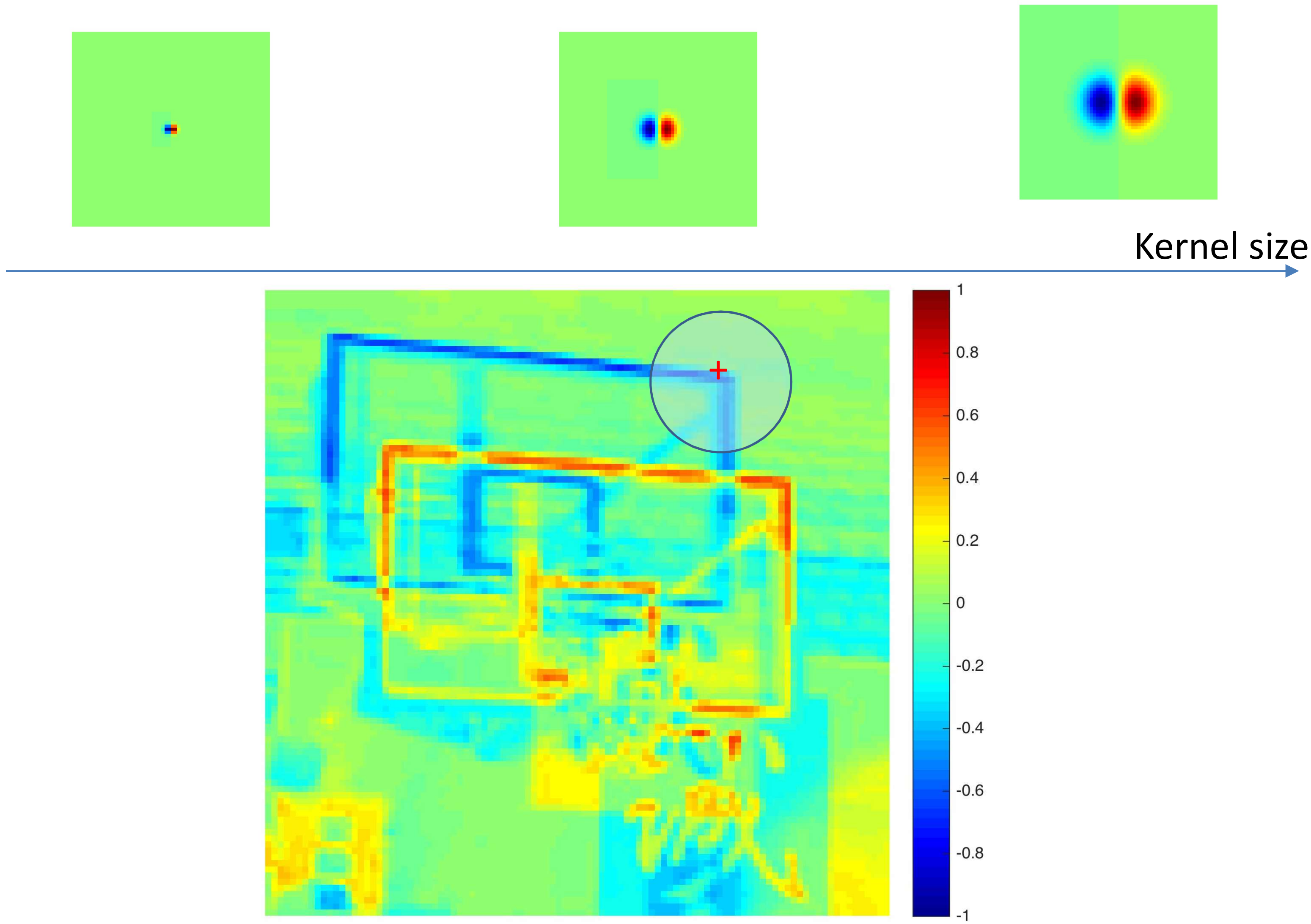
Kernel size



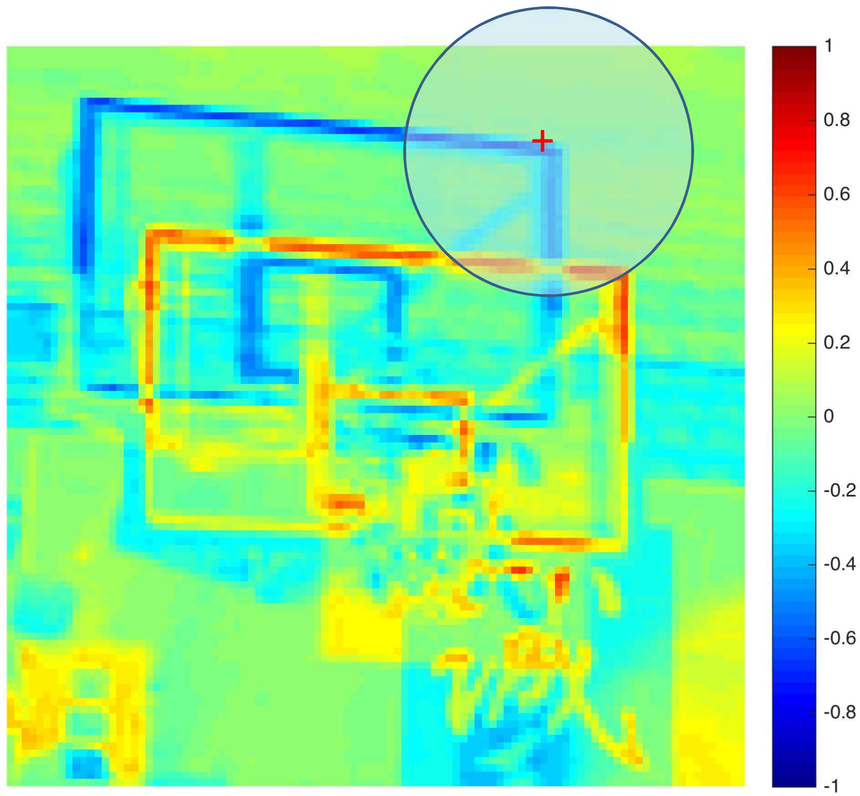
Solution 2: increase the kernel size of gradient operator



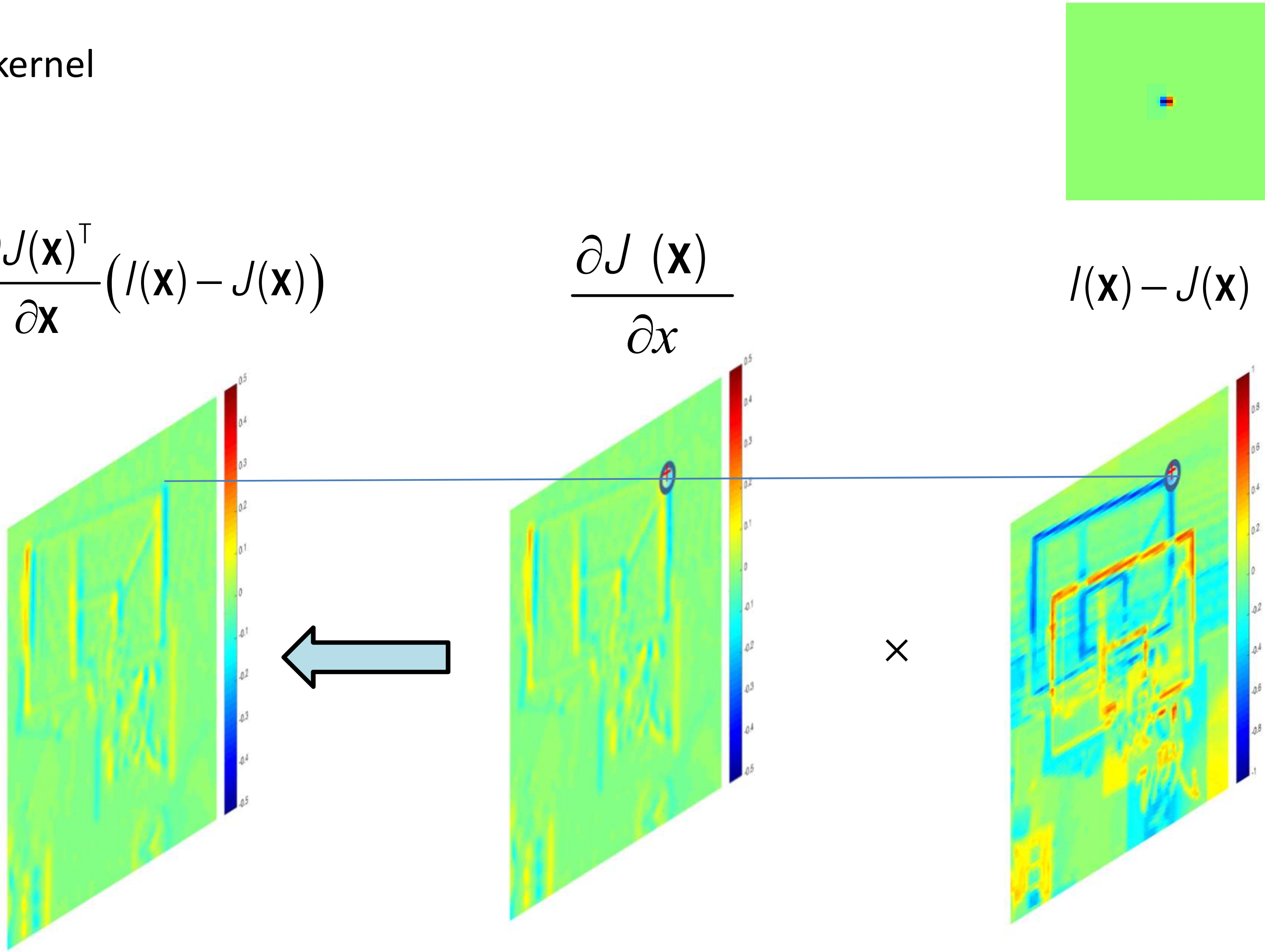
Solution 2: increase the kernel size of gradient operator



Solution 2: increase the kernel size of gradient operator

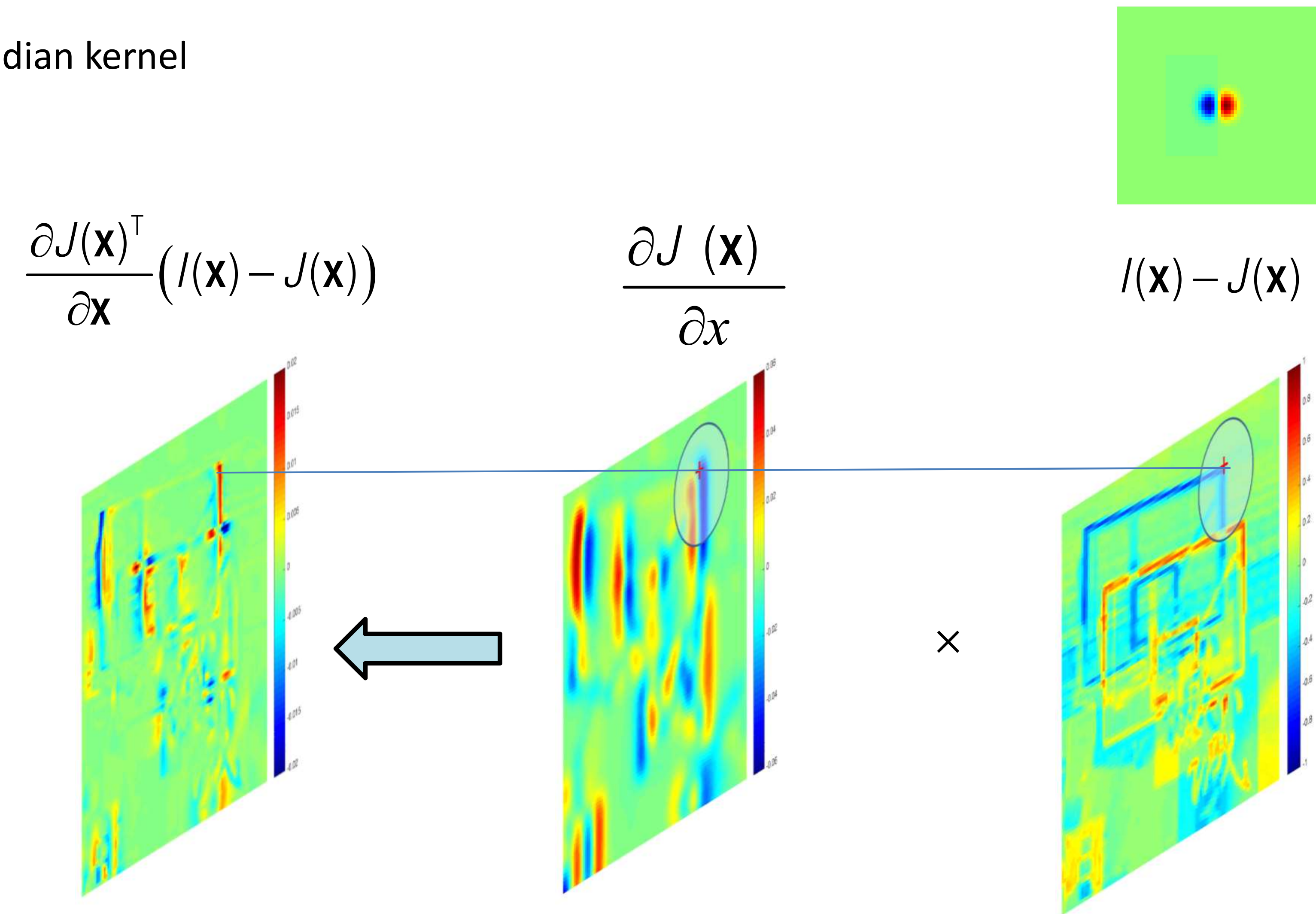


small kernel

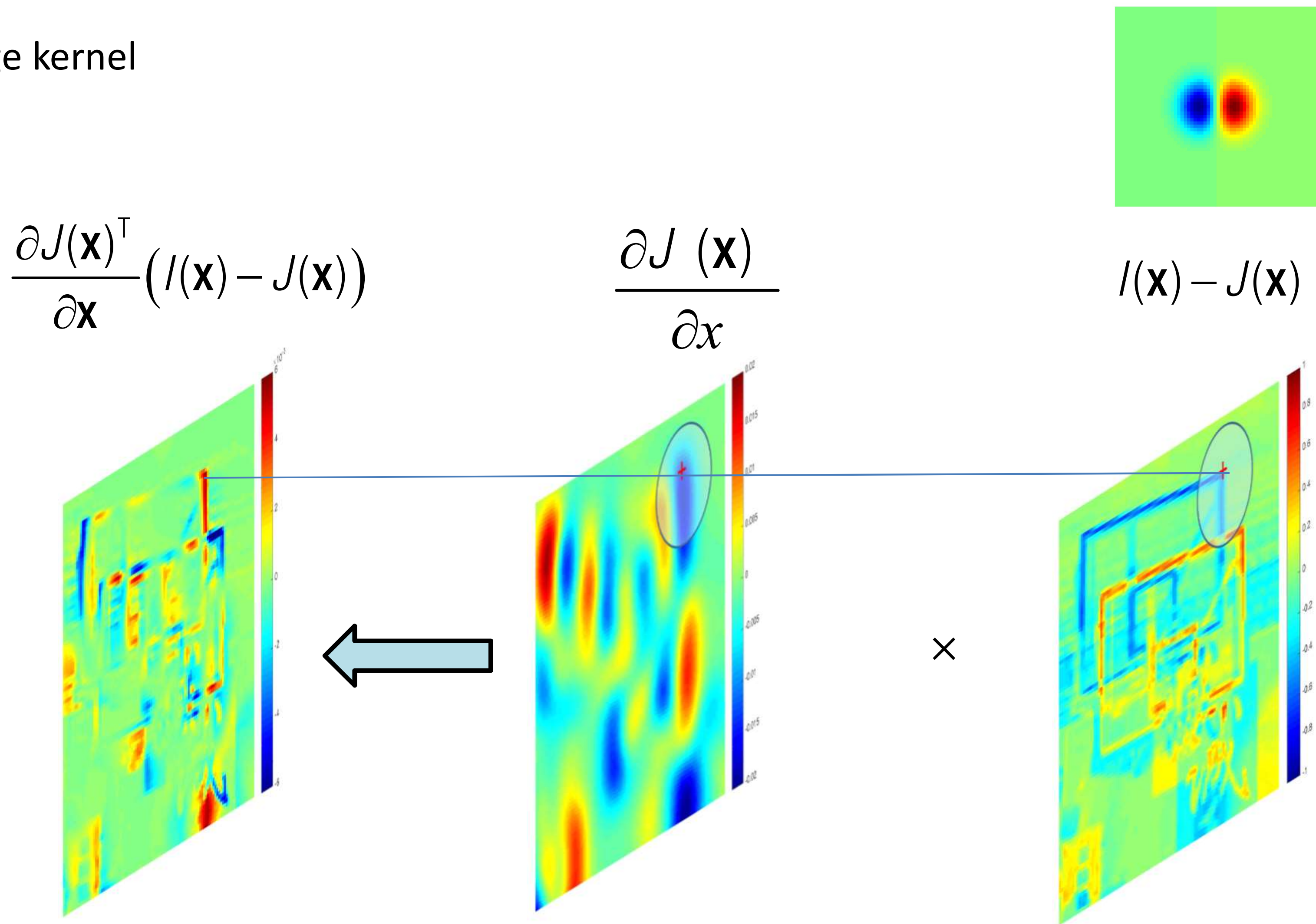


$$\frac{\partial J(\mathbf{x})^{\top}}{\partial \mathbf{x}}\left(I(\mathbf{x})-J(\mathbf{x})\right) \quad \Leftarrow \quad \frac{\partial J\ (\mathbf{x})}{\partial x} \times I(\mathbf{x})-J(\mathbf{x})$$

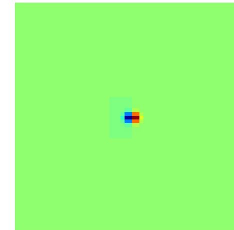
Median kernel

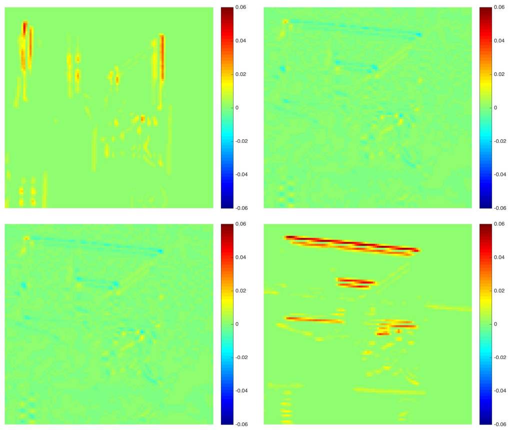


Large kernel

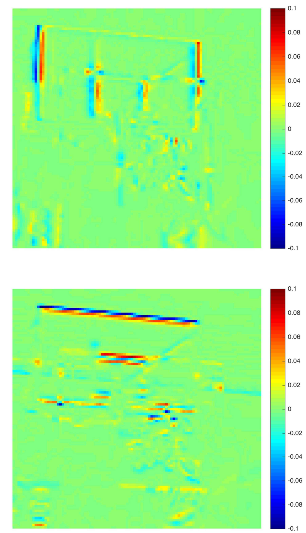


$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right) \Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \left(I(\mathbf{x}) - J(\mathbf{x}) \right)$$



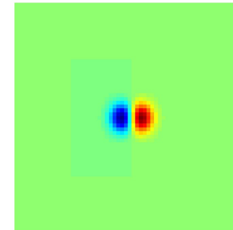


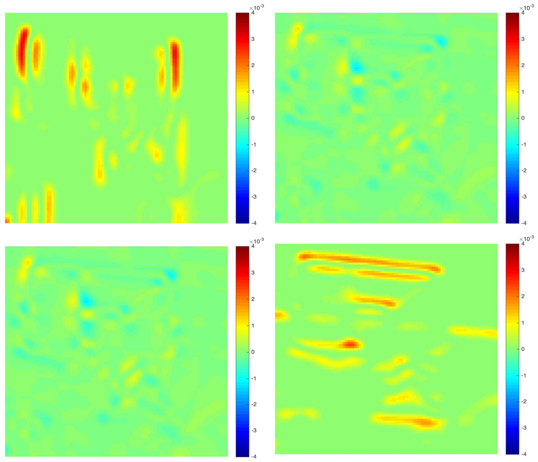
$$\begin{bmatrix} \mathbf{d_x} \\ \mathbf{d_y} \end{bmatrix} =$$



d=(-0.6, 1.1)

$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

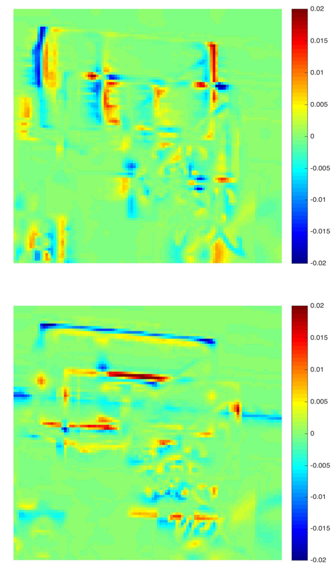




$\mathbf{d_x}$

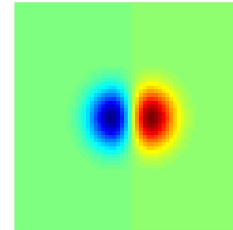
$\mathbf{d_y}$

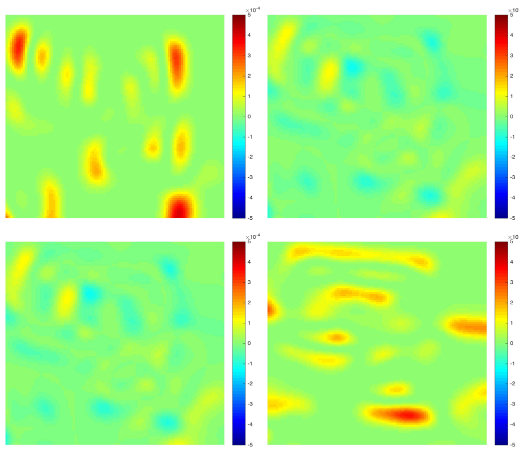
$=$



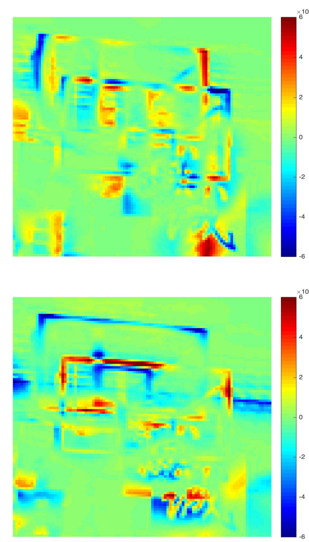
d=(-2.9, -3.0)

$$\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}} \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\Delta \mathbf{d} = \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}^{\mathsf{T}}\big(I(\mathbf{x}) - J(\mathbf{x})\big)$$

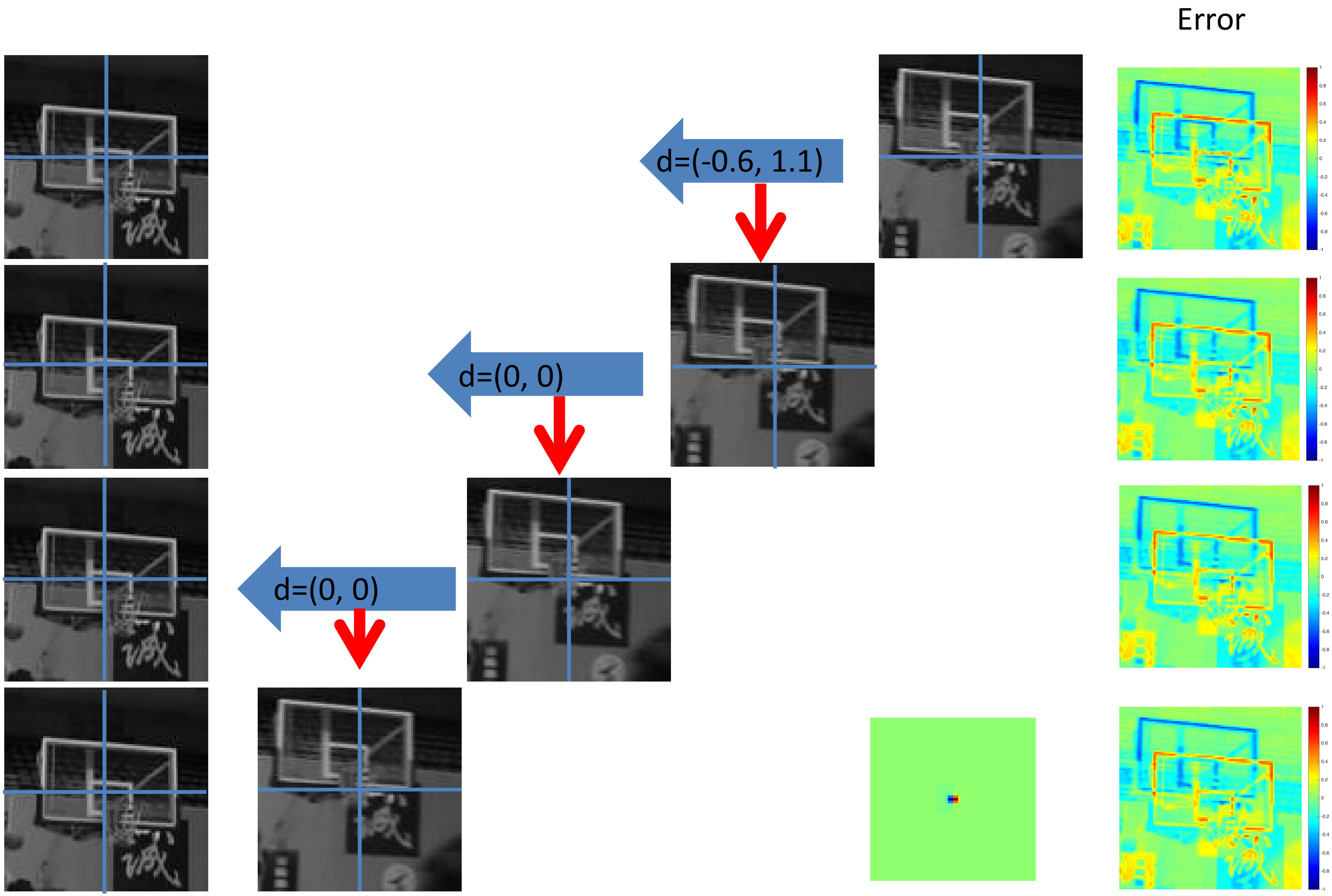




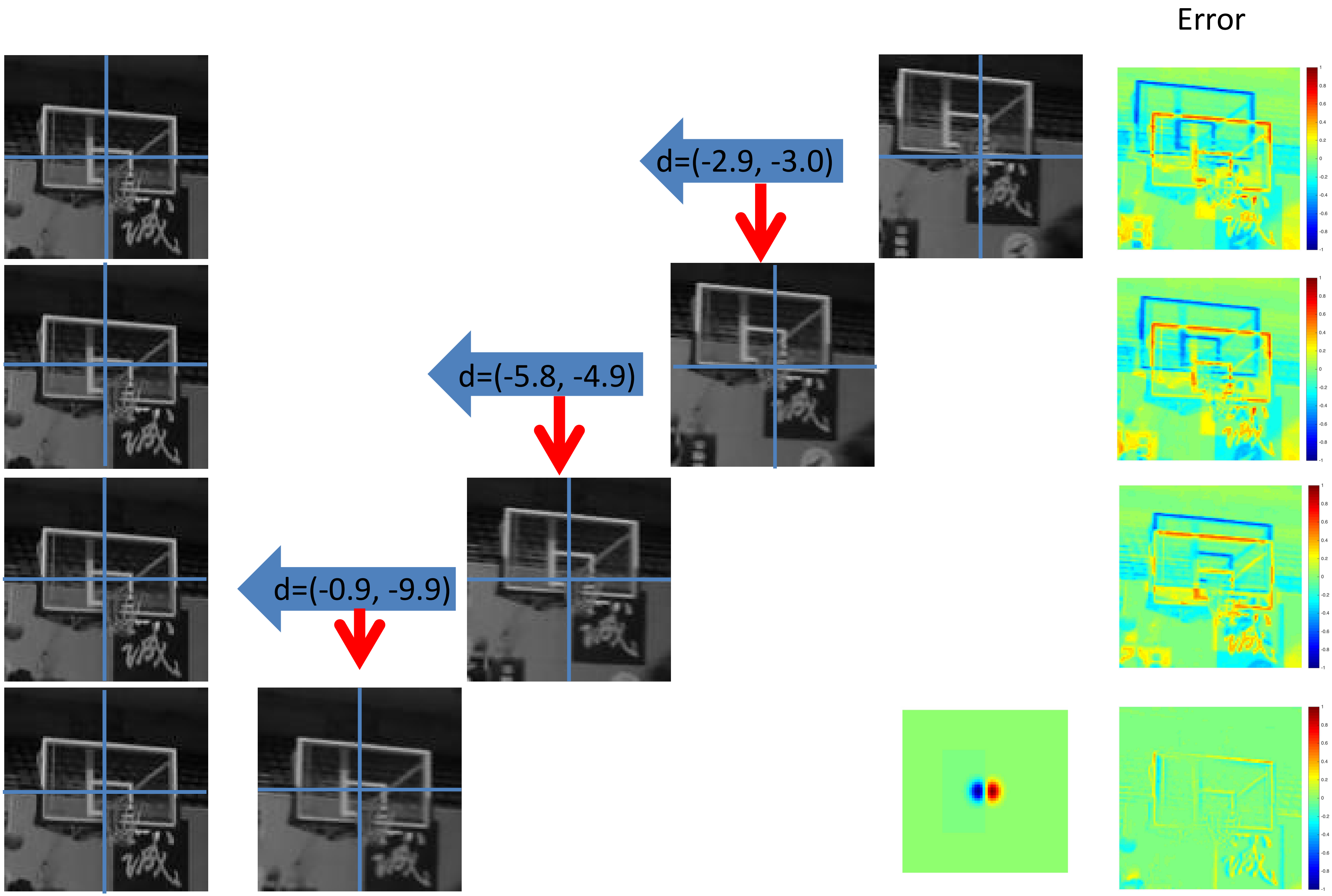
$$\begin{bmatrix} \mathbf{d_x} \\ \mathbf{d_y} \end{bmatrix} =$$



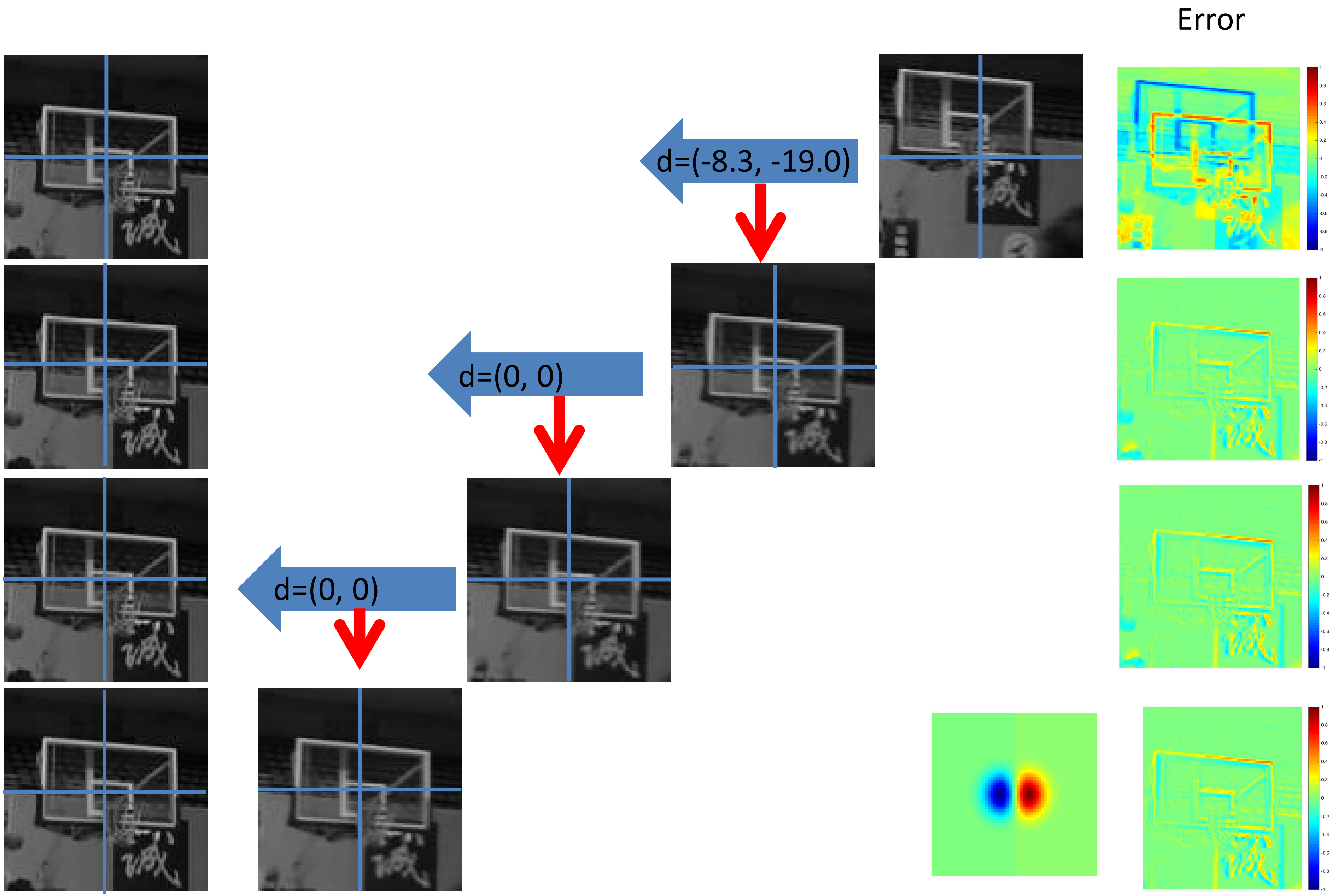
d=(-8.3, 19.0)



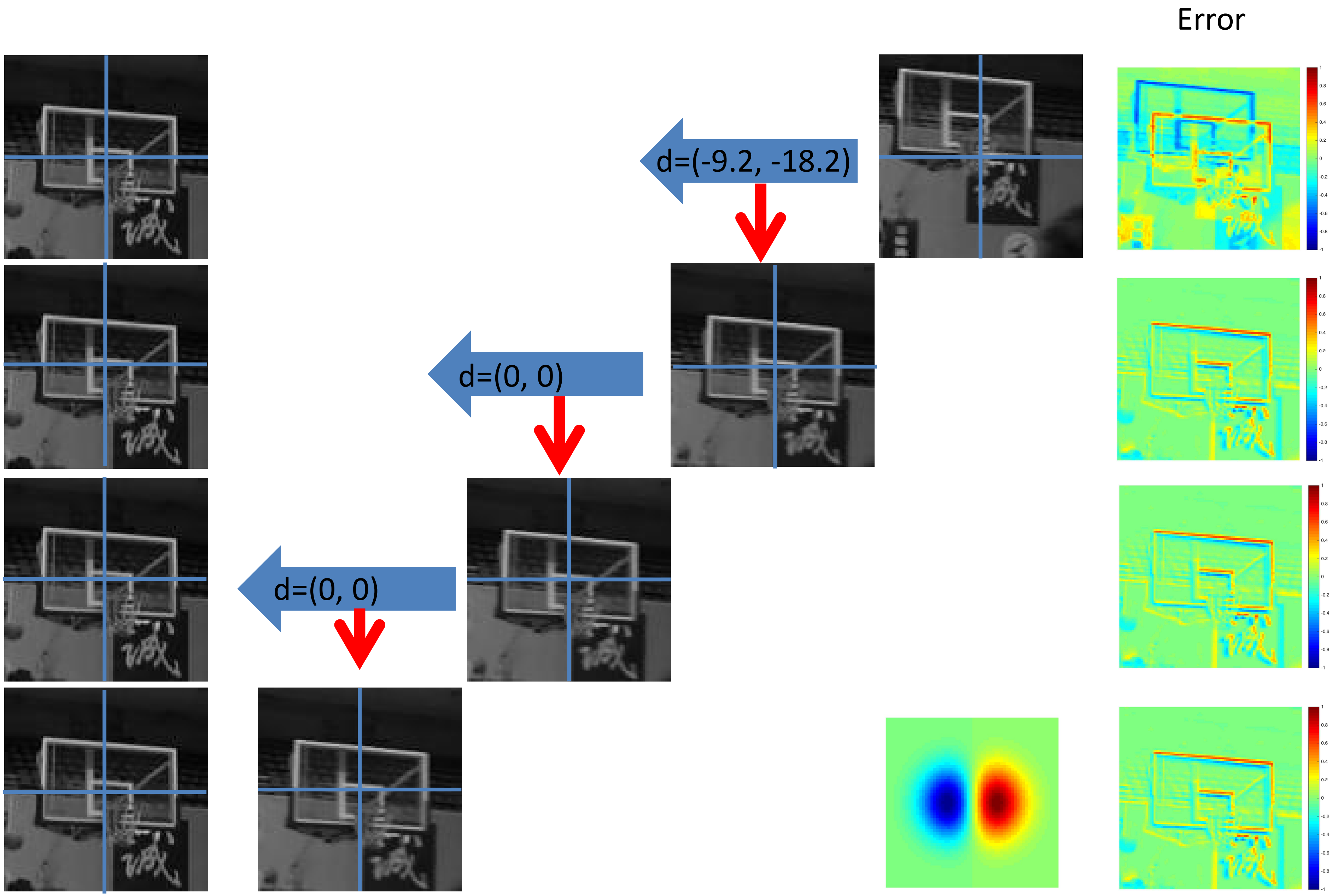
Error
d=(-0.6, 1.1)
d=(0, 0)
d=(0, 0)



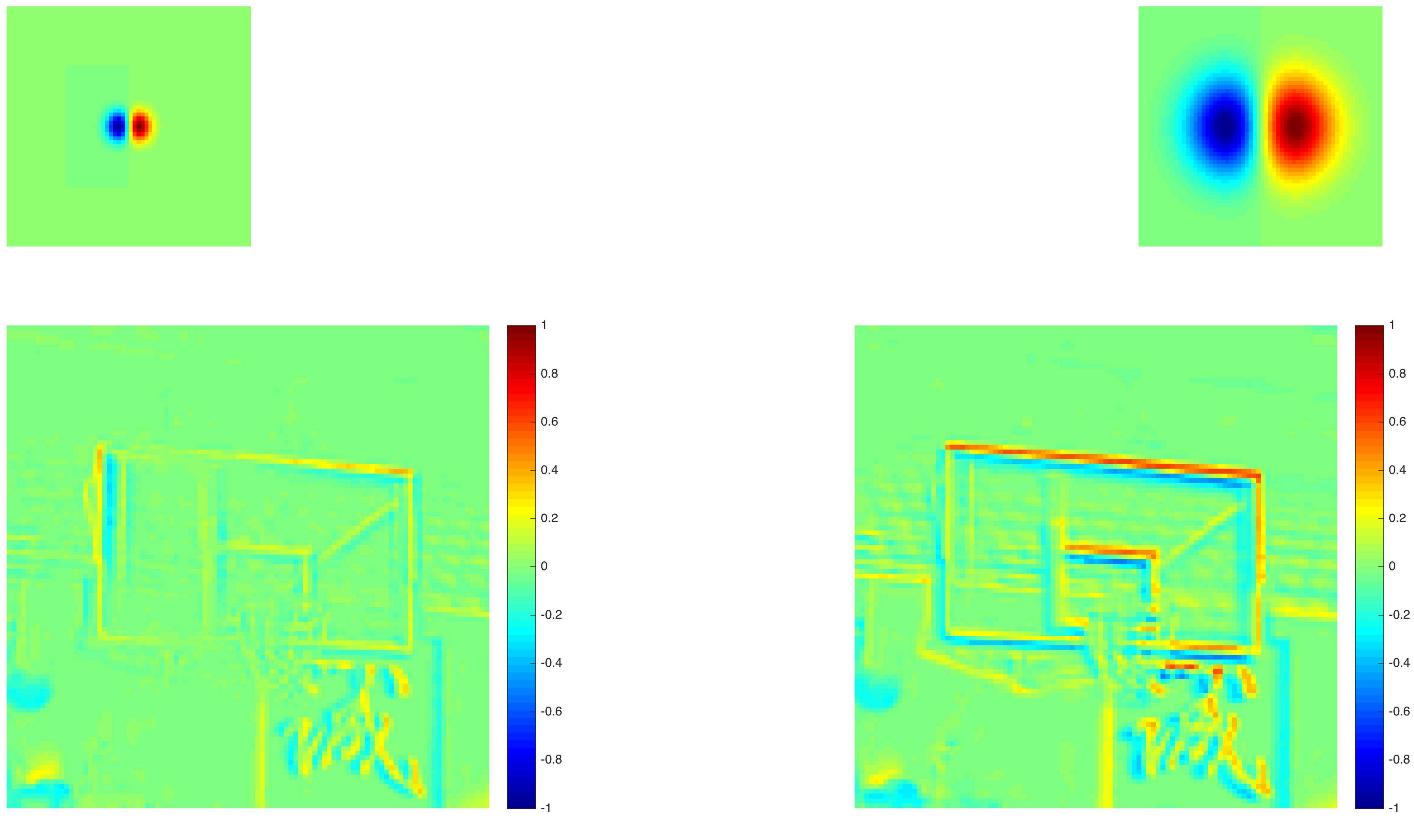
Error
d=(-2.9, -3.0)
d=(-5.8, -4.9)
d=(-0.9, -9.9)


Error
d=(-8.3, -19.0)
d=(0, 0)
d=(0, 0)

Larger kernel does not mean the better kernel!



Error
d=(-9.2, -18.2)
d=(0, 0)
d=(0, 0)


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

Proper kernel size is important!