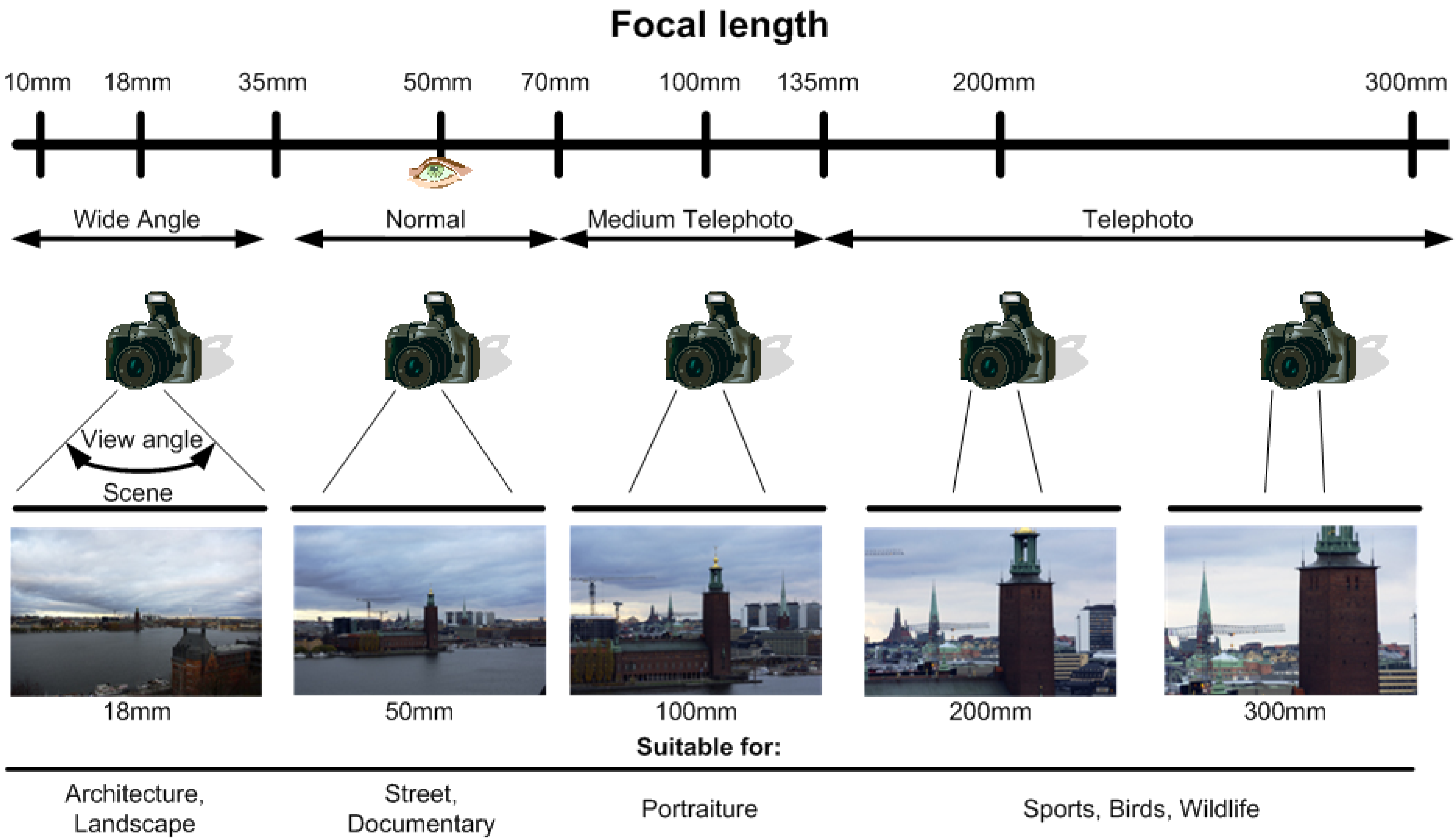
Image Projection

Field of View (Zoom)

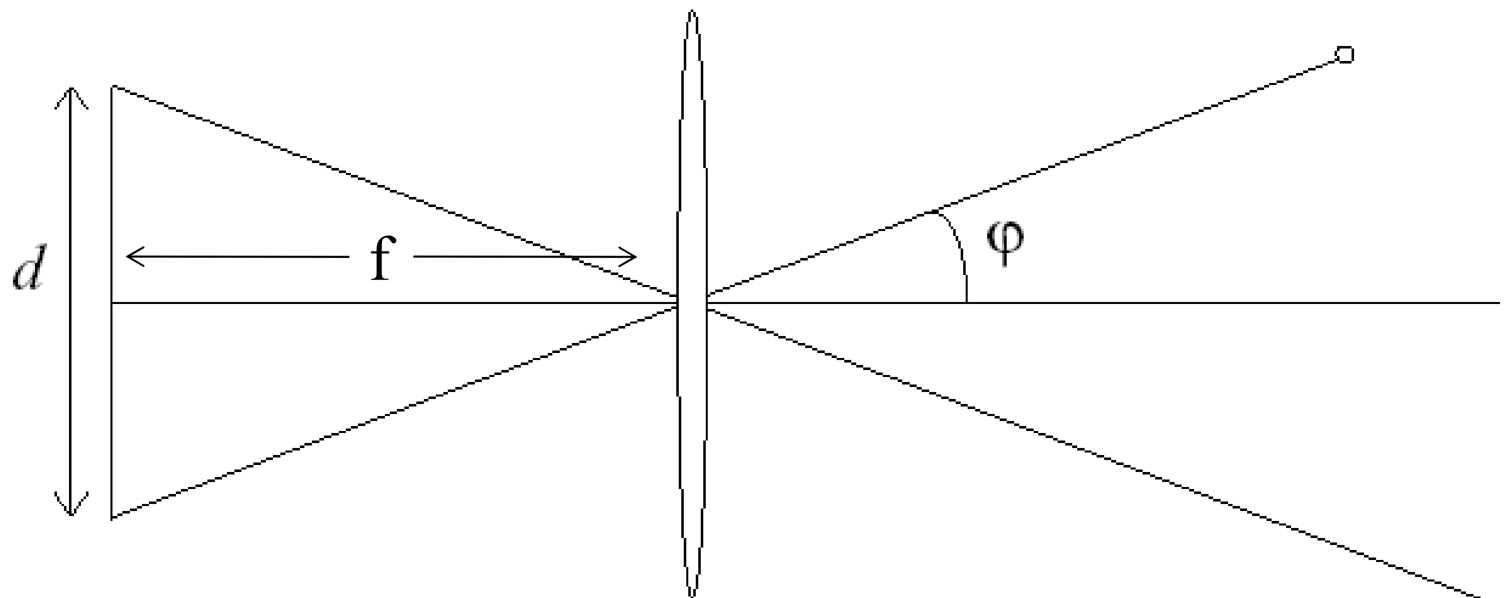


Focal length
10mm
18mm
35mm
50mm
70mm
100mm
135mm
200mm
300mm
Wide Angle
Normal
Medium Telephoto
Telephoto
View angle
Scene
18mm
50mm
100mm
200mm
300mm
Suitable for:
Architecture, Landscape
Street, Documentary
Portraiture
Sports, Birds, Wildlife

Large Focal Length compresses depth

400 mm 200 mm 100 mm 50 mm 28 mm 17 mm

FOV depends of Focal Length

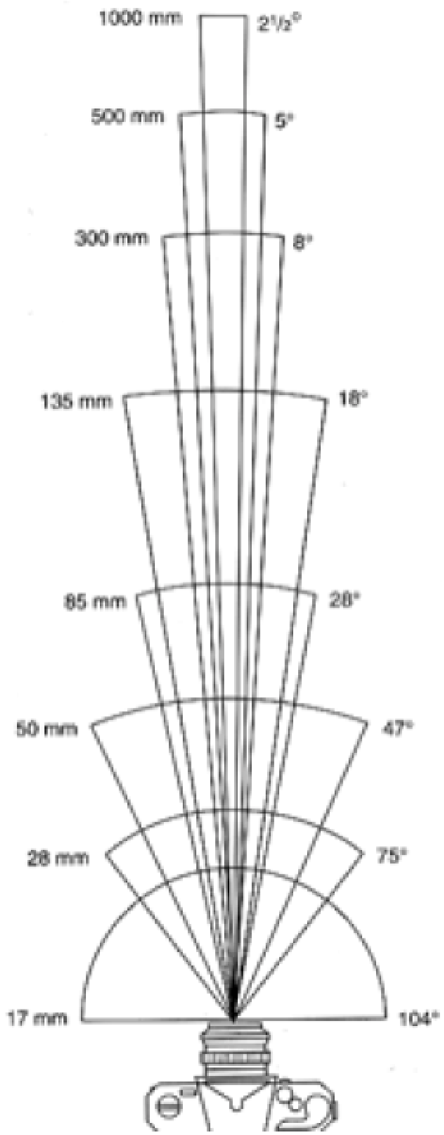


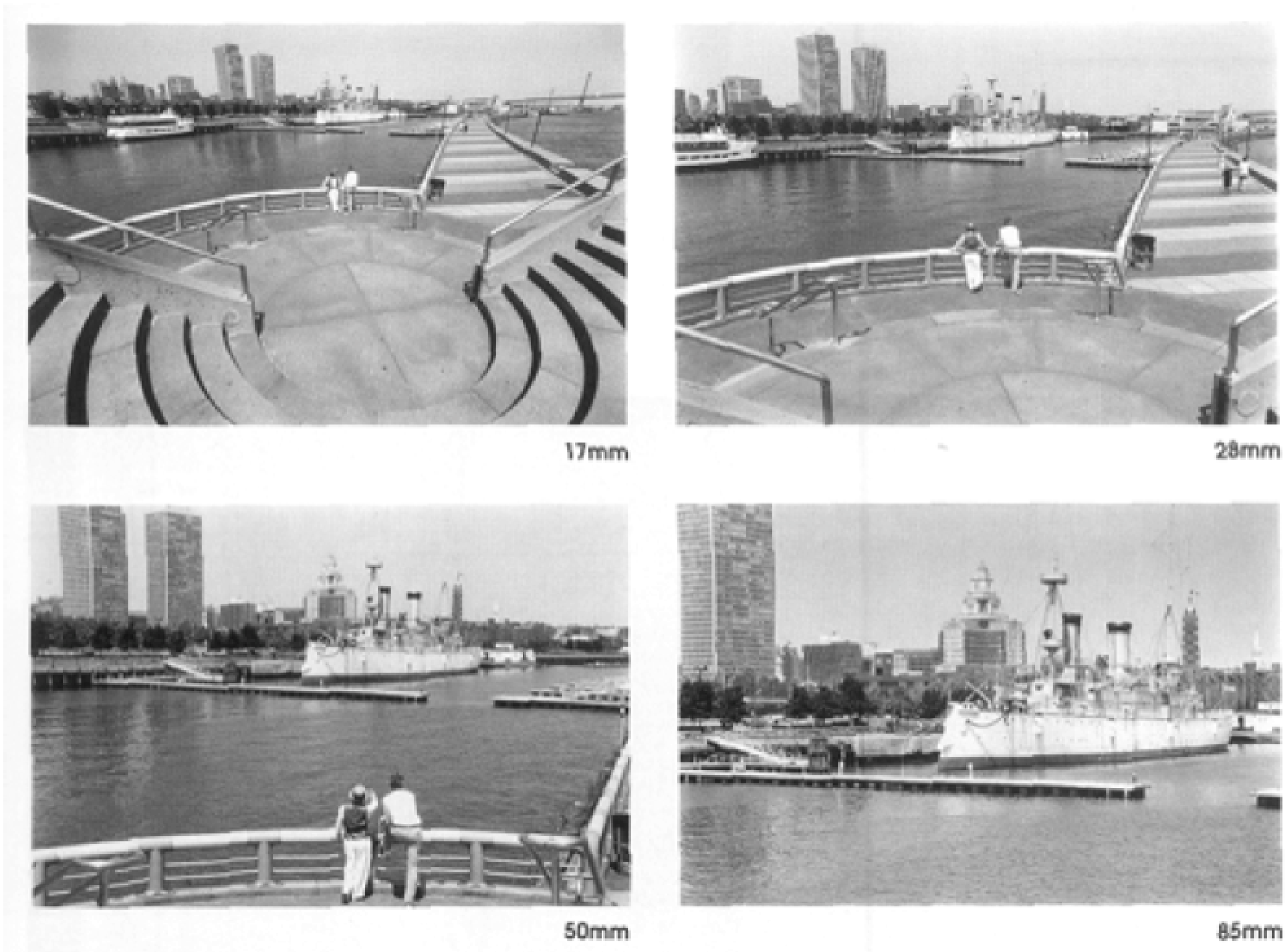
Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

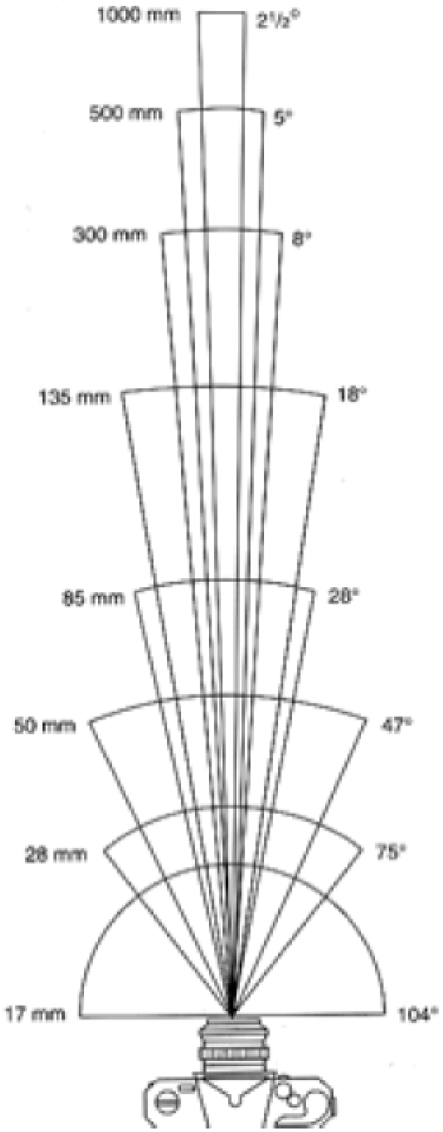
Field of View (Zoom)

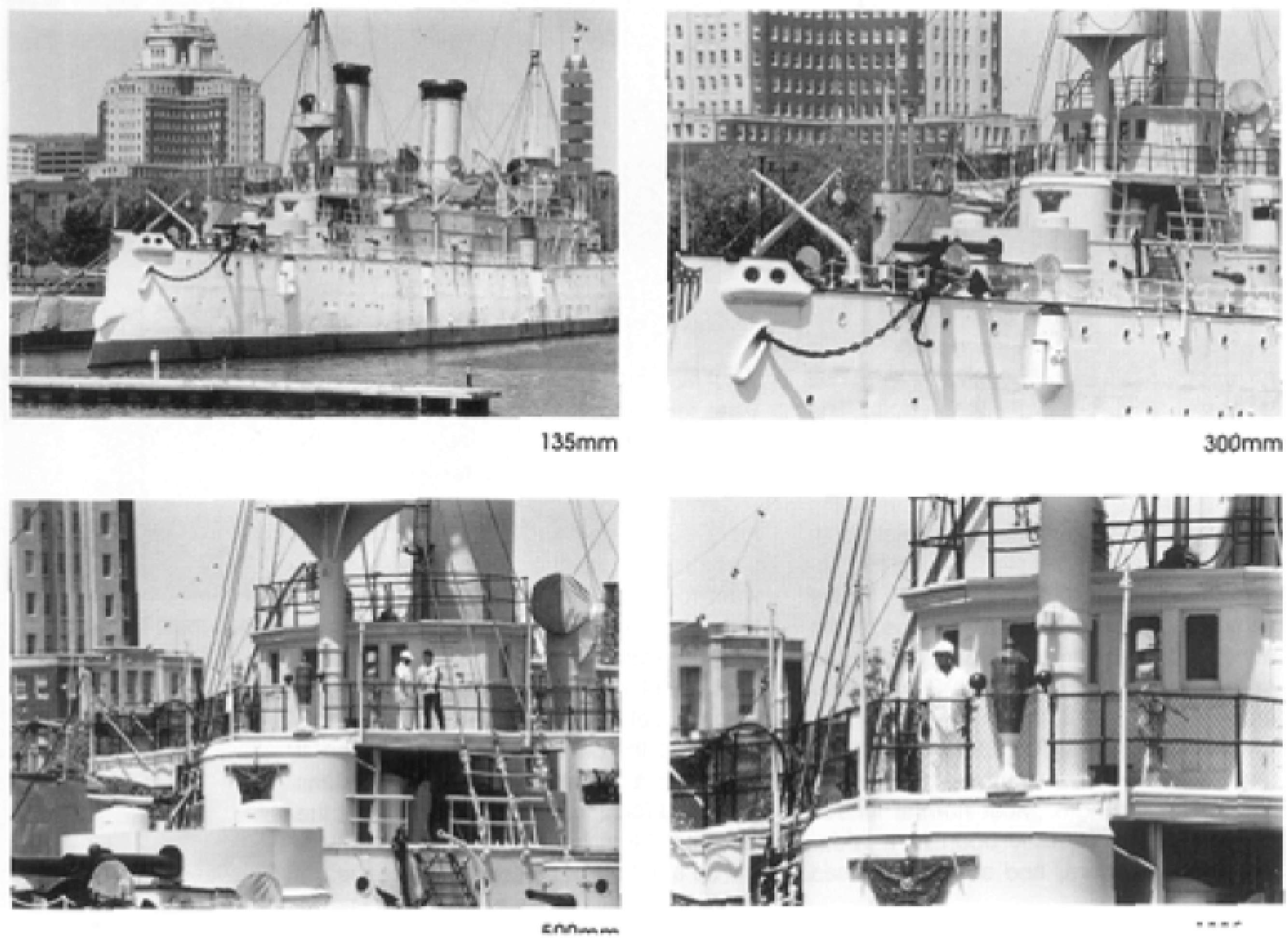




From London and Upton

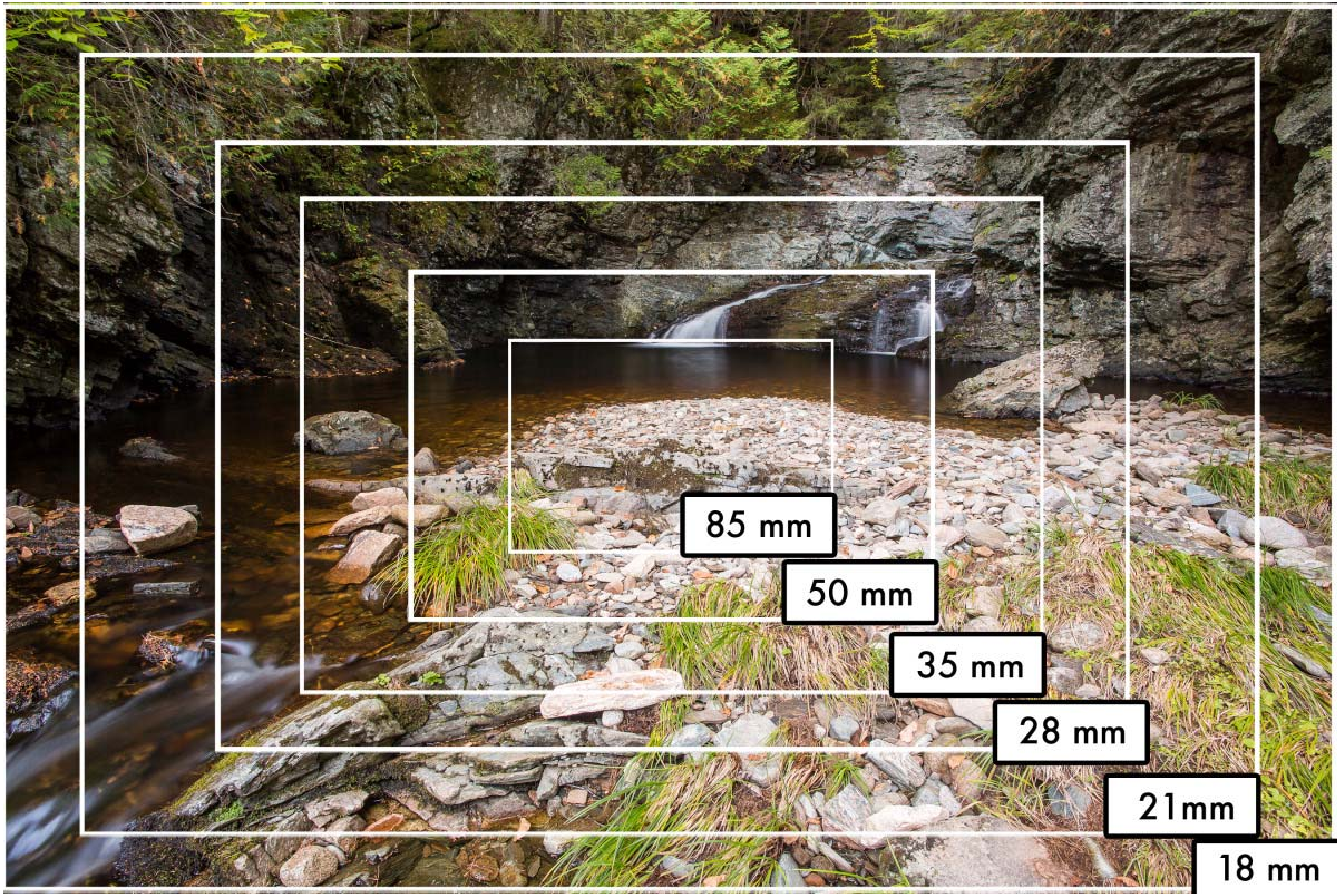
Field of View (Zoom)





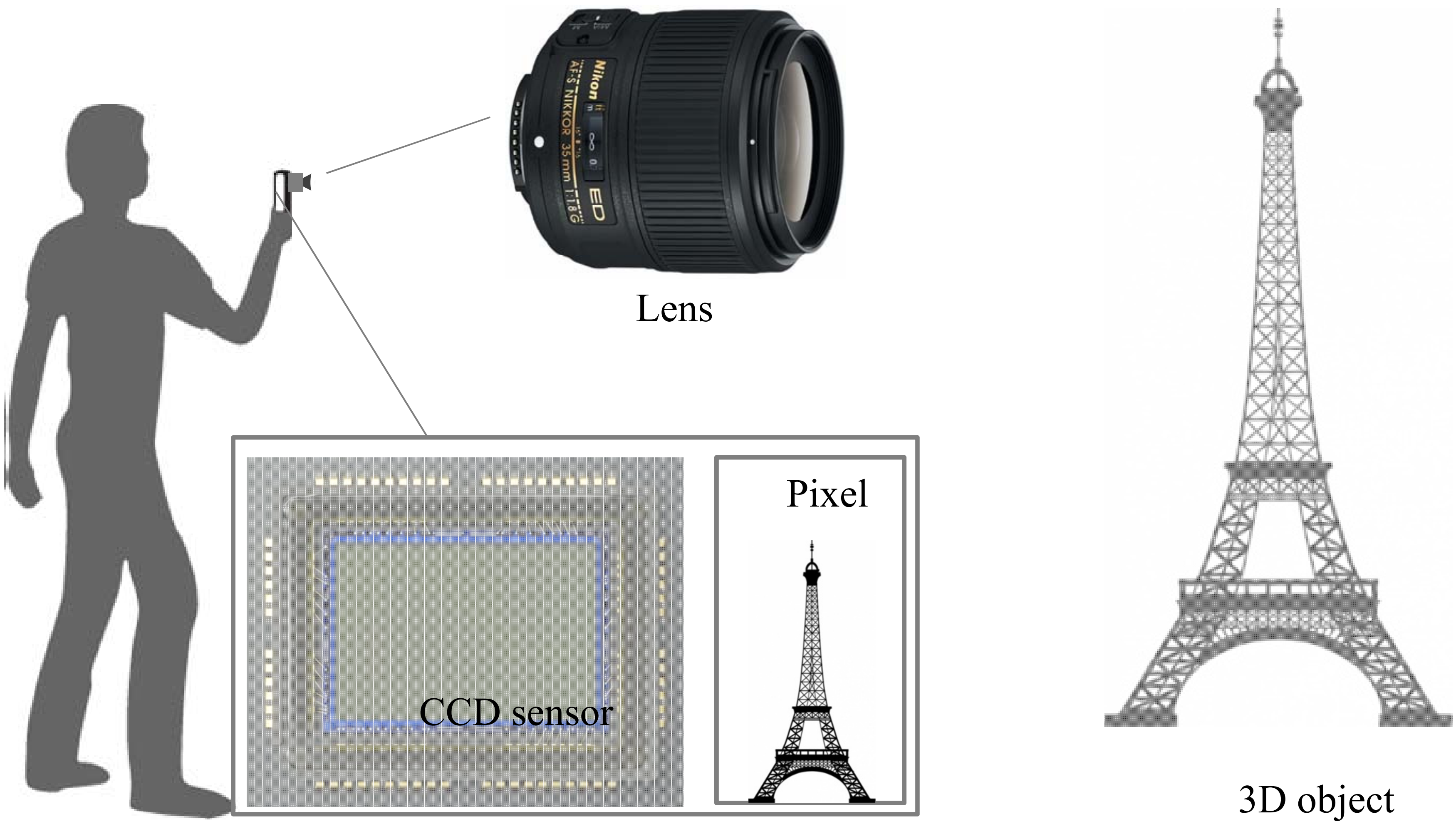
From London and Upton

http://2blowup.com/fotografia-para-egobloggers-ii/


85 mm
50 mm
35 mm
28 mm
21mm
18 mm

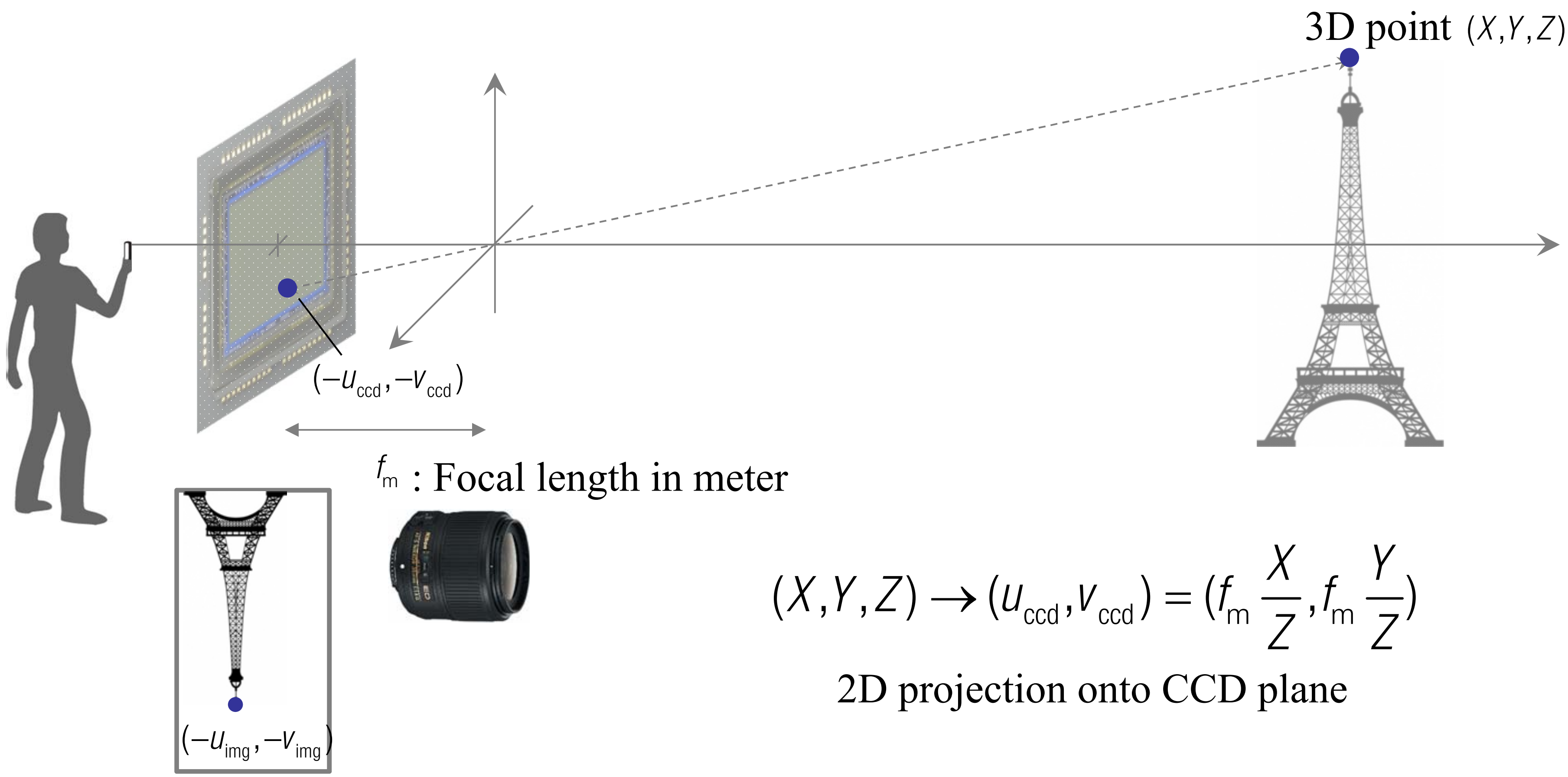
Fisheye lens distortion

Camera Model



Lens
Pixel
CCD sensor
3D object

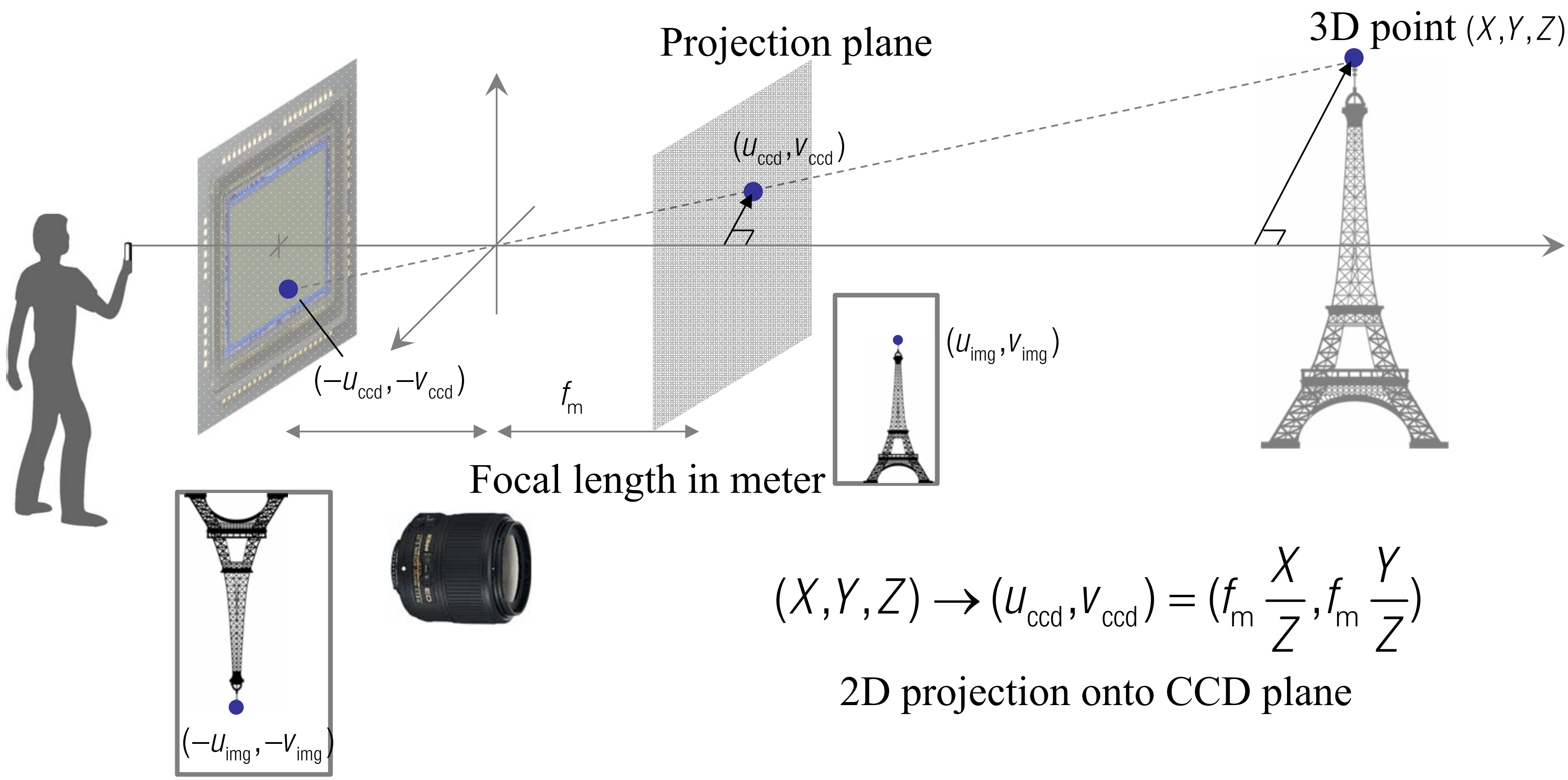
3D Point Projection (Metric Space)



$$(X,Y,Z) \rightarrow (u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

2D projection onto CCD plane

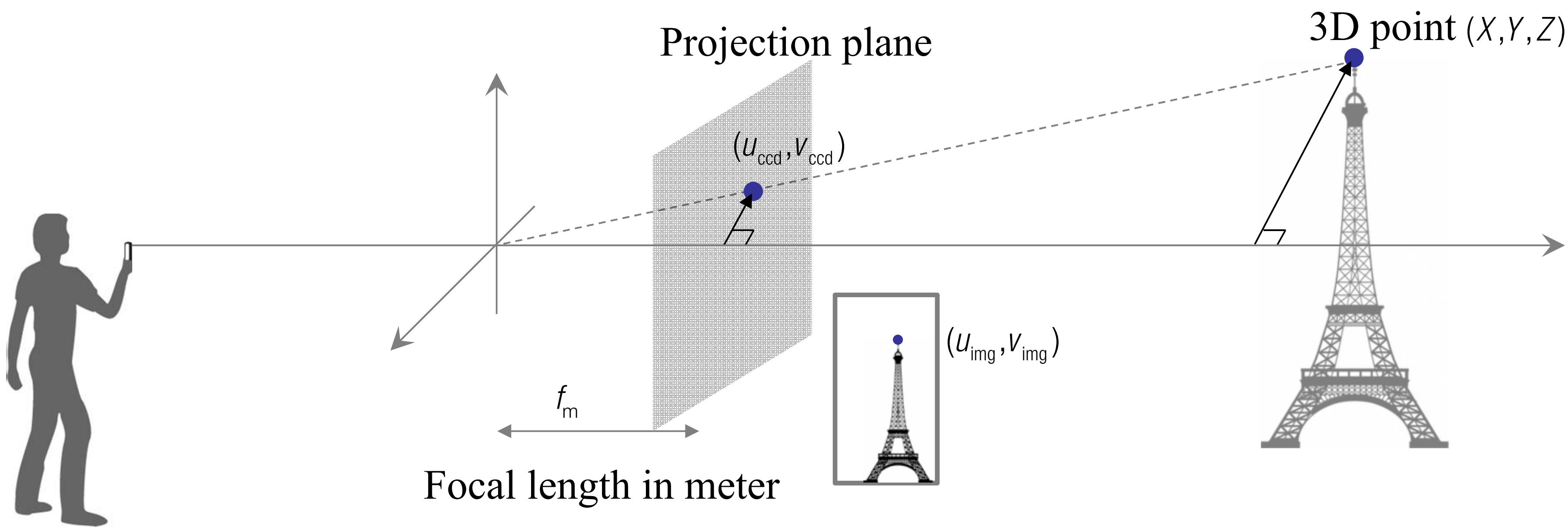
3D Point Projection (Metric Space)



$$(X,Y,Z) \rightarrow (u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

2D projection onto CCD plane

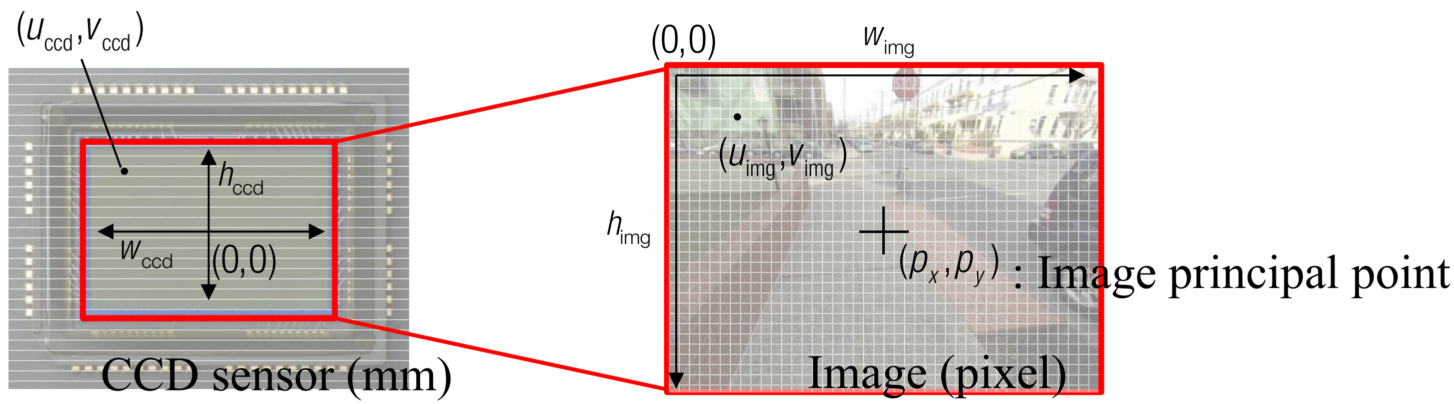
3D Point Projection (Metric Space)



$$(X,Y,Z) \rightarrow (u_{\mathrm{ccd}}, v_{\mathrm{ccd}}) = (f_{\mathrm{m}}\frac{X}{Z}, f_{\mathrm{m}}\frac{Y}{Z})$$

2D projection onto CCD plane

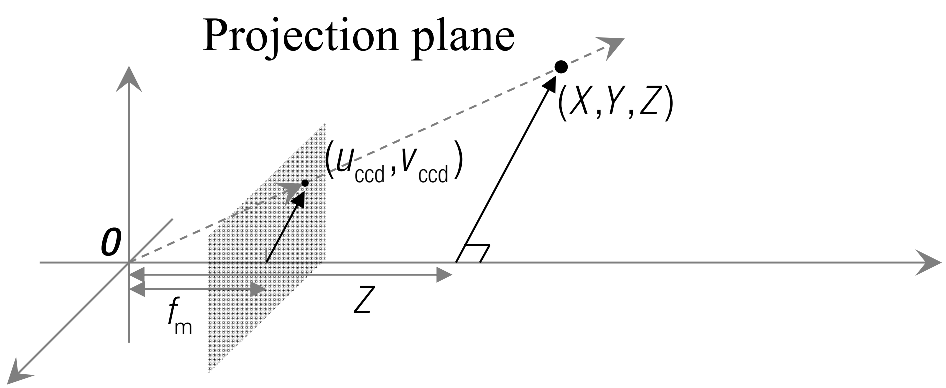
3D Point Projection (Pixel Space)

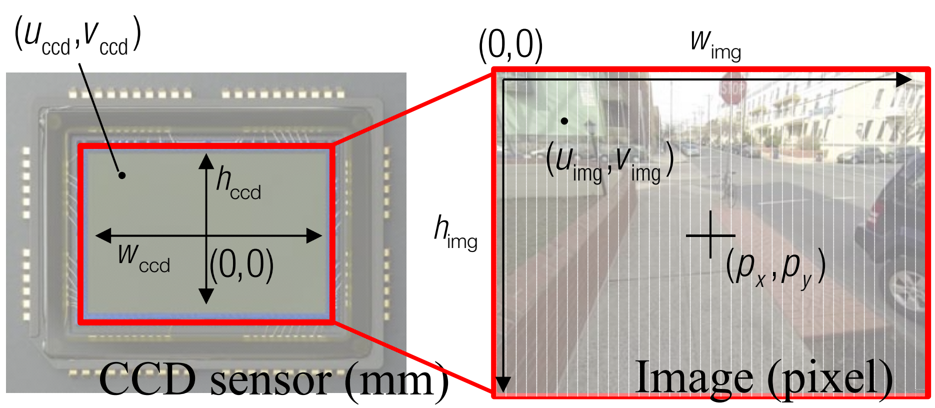


$$\frac{u_{ccd}}{w_{ccd}} = \frac{u_{img} - p_x}{w_{img}} \qquad \frac{v_{ccd}}{h_{ccd}} = \frac{v_{img} - p_y}{h_{img}}$$

$$u_{img} = u_{ccd}\frac{w_{img}}{w_{ccd}} + p_x \qquad v_{img} = v_{ccd}\frac{h_{img}}{h_{ccd}} + p_y$$

3D Point Projection (Pixel Space)



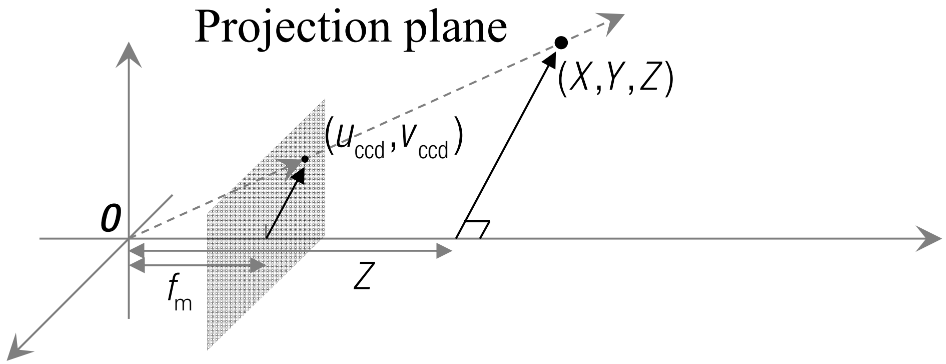


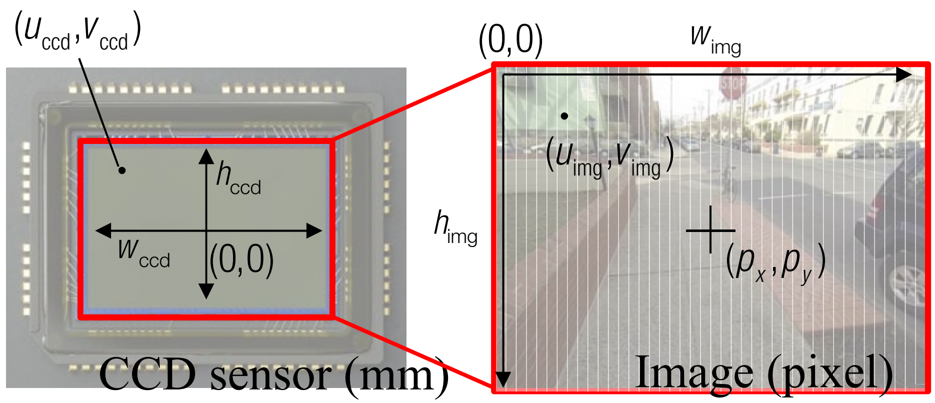
$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

$$u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = \underbrace{f_m \frac{w_{img}}{w_{ccd}}}_{\text{Focal length in pixel}} \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = \underbrace{f_m \frac{h_{img}}{h_{ccd}}}_{\text{Focal length in pixel}} \frac{Y}{Z} + p_y$$

3D Point Projection (Pixel Space)





$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

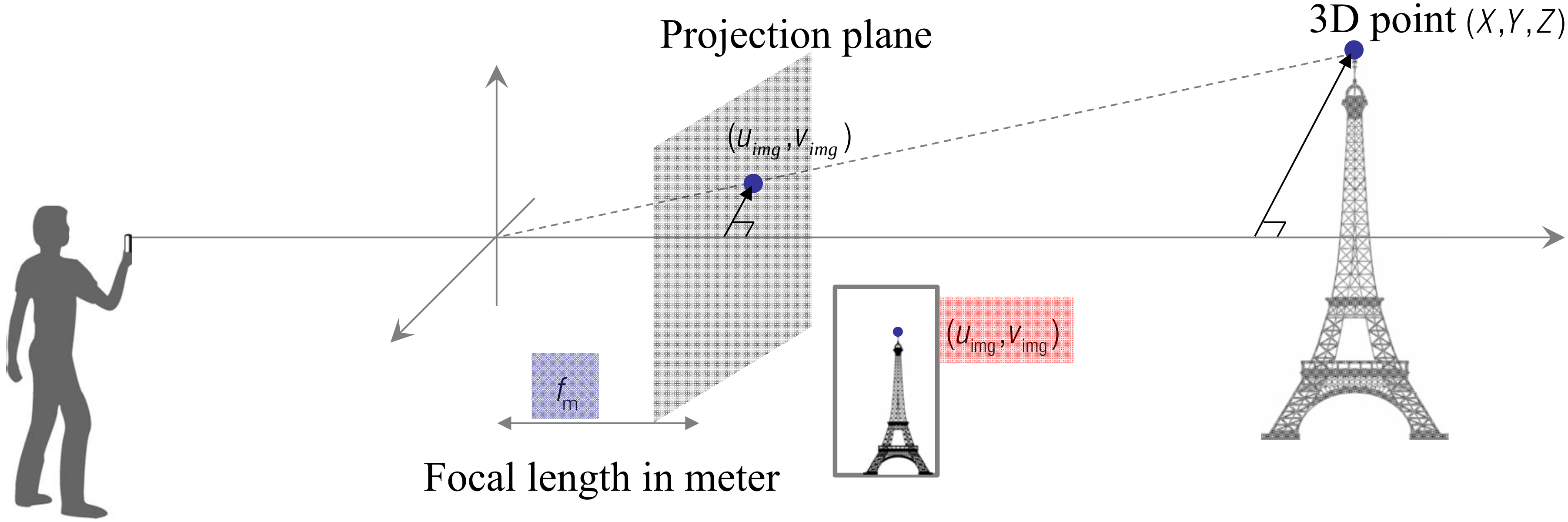
$$u_{img} = u_{ccd} \frac{W_{img}}{W_{ccd}} + p_x = f_x \frac{X}{Z} + p_x$$

Focal length in pixel

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_y \frac{Y}{Z} + p_y$$

Focal length in pixel

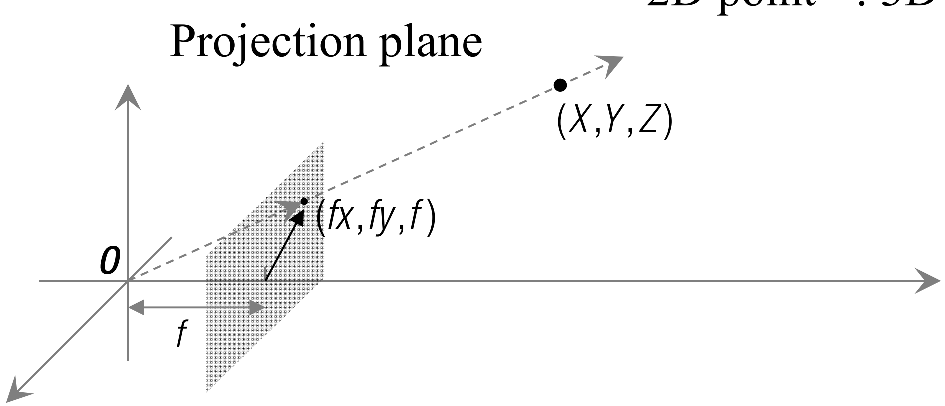
3D Point Projection (Pixel Space)



$$(X,Y,Z) \rightarrow (u_{img}, v_{img}) = \left(f_m \frac{W_{img}}{W_{ccd}} \frac{X}{Z}, f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z}\right)$$

Homogeneous Coordinate

2D point =: 3D ray $\lambda(x, y, 1)$

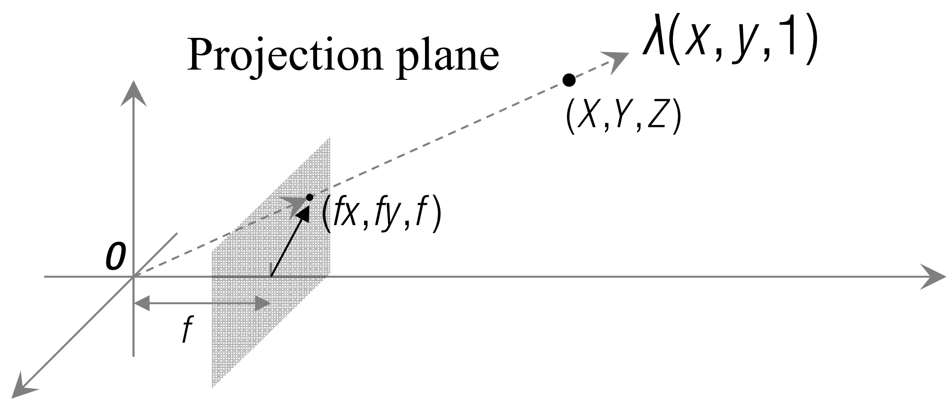


$$(x, y) \rightarrow (x, y, 1)$$
$$= f(x, y, 1)$$
$$= \lambda(x, y, 1)$$

: A point in Euclidean space ($\mathbb{R}^2$) can be represented by
a homogeneous representation in Projective space (P^2) (3 numbers).

Homogeneous Coordinate

2D point =: 3D ray

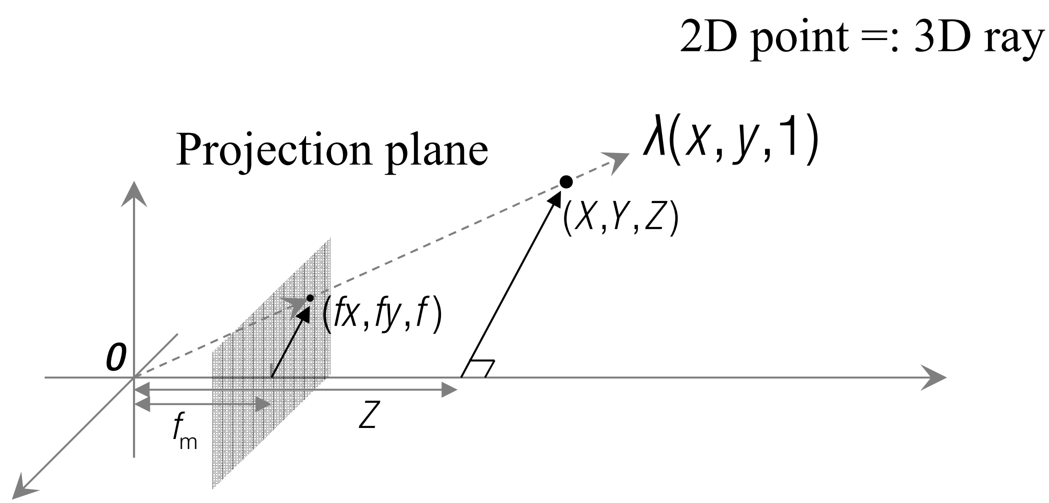


$$\underbrace{\lambda(x, y, 1)}_{\text{}} = (X, Y, Z)$$

Homogeneous coordinate

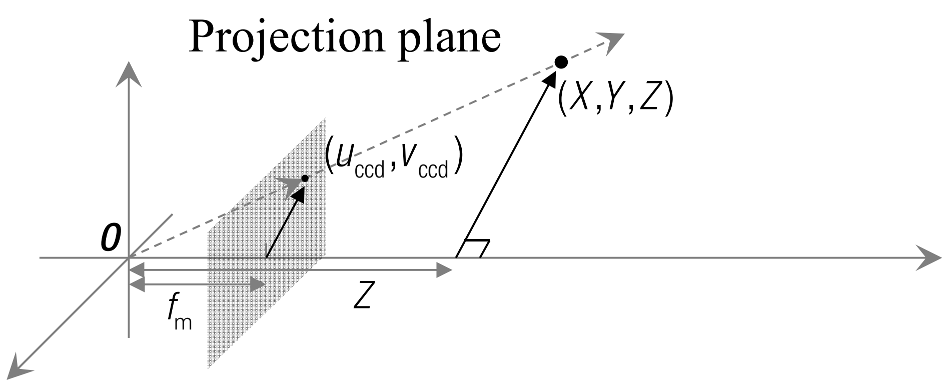
: 3D point lies in the 3D ray passing 2D image point.

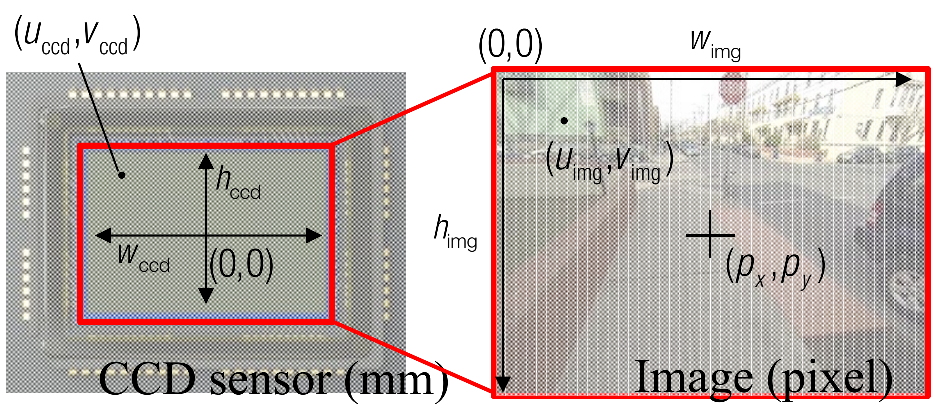
3D Point Projection (Metric Space)



$$(x, y, 1) = (f_m x, f_m y, f_m) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}, f_m)$$

3D Point Projection (Pixel Space)





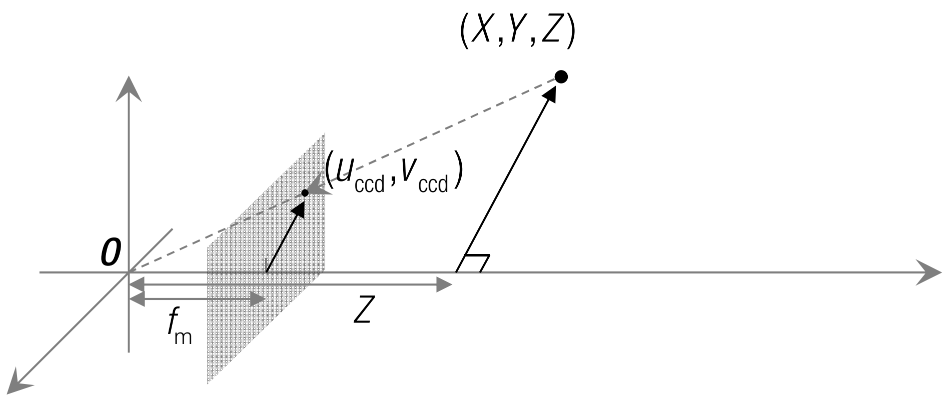
$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

$$u_{img} = f_x \frac{X}{Z} + p_x \qquad v_{img} = f_y \frac{Y}{Z} + p_y$$

$$\lambda \underbrace{\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}} = \begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Homogeneous representation

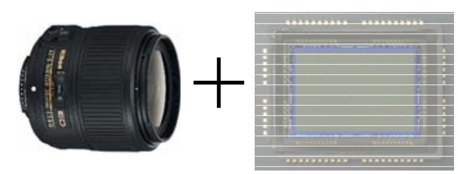
Camera Intrinsic Parameter



Projection plane

Pixel space | Metric space

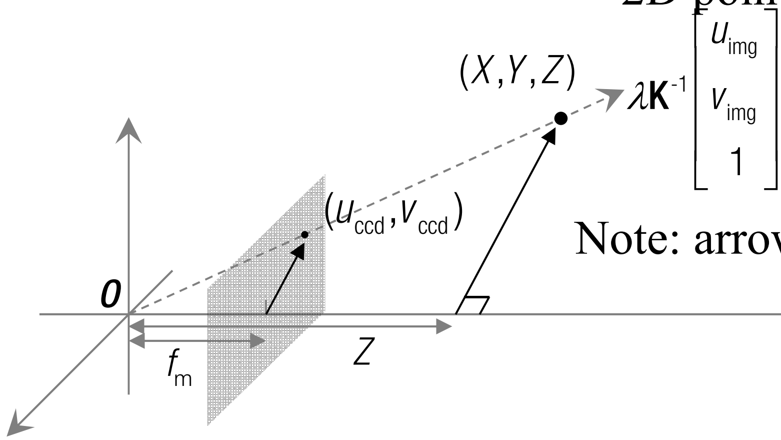
$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

2D Inverse Projection

2D point == 3D ray



Note: arrow direction

Projection plane

Pixel space | Metric space

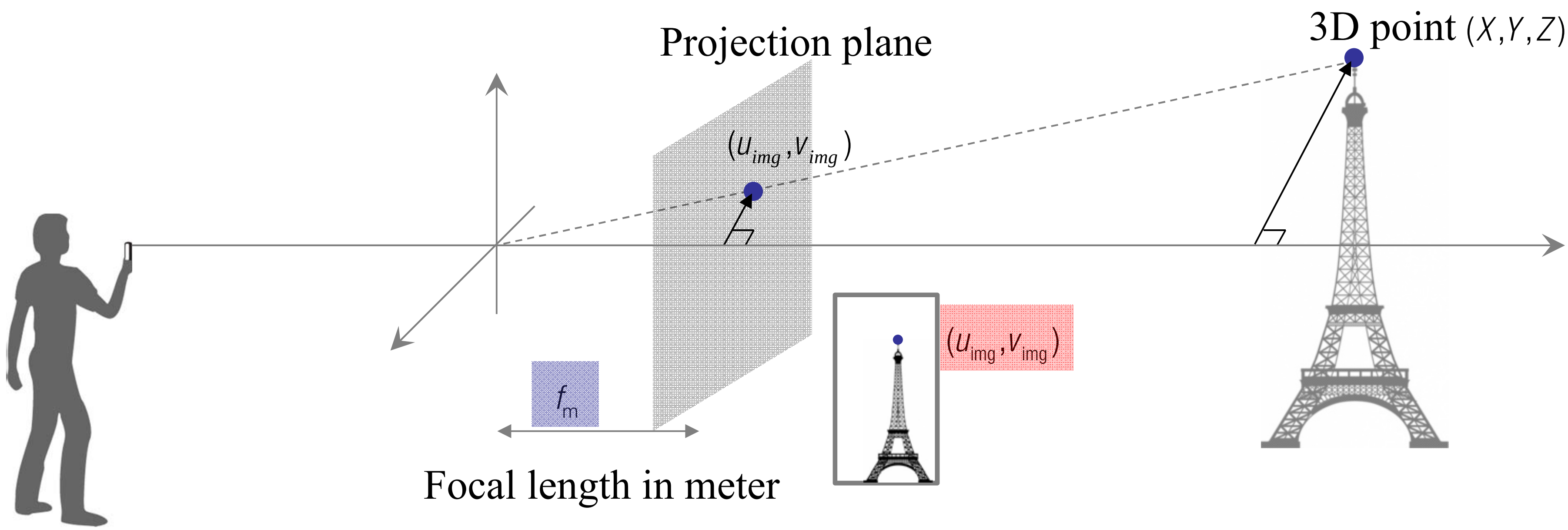
$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D ray

The 3D point must lie in
the *3D ray* passing through the origin and 2D image point.

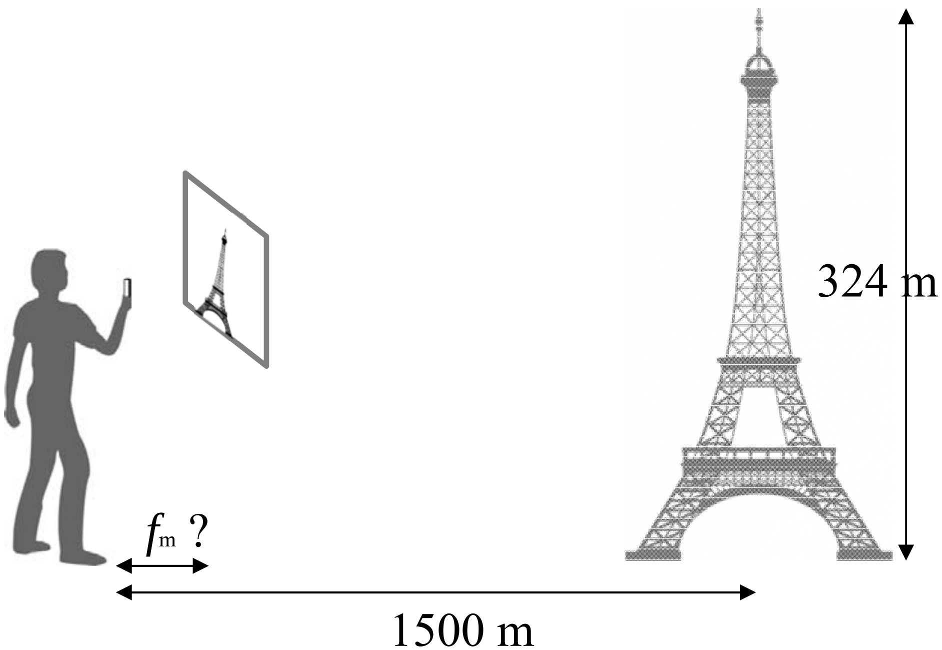
3D Point Projection (Pixel Space)

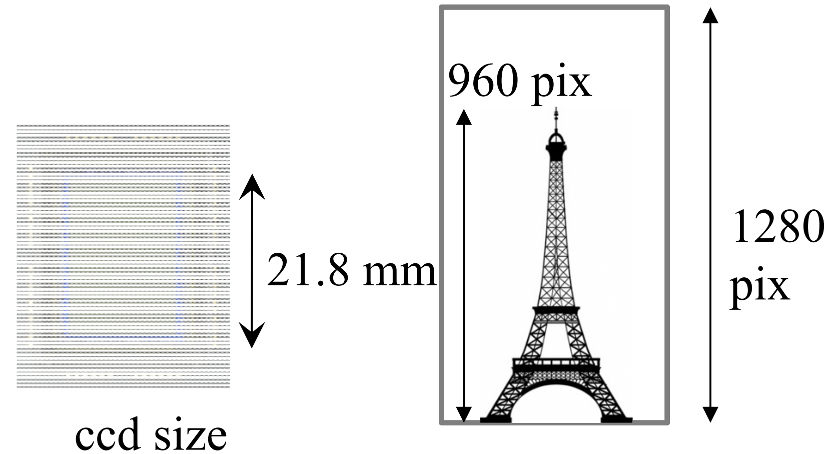


$$(X,Y,Z) \rightarrow (u_{img}, v_{img}) = \left(f_m \frac{W_{img}}{W_{ccd}} \frac{X}{Z}, f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z}\right)$$

Exercise

What f to make the height of Eifel tower appear 960 pixel distance?



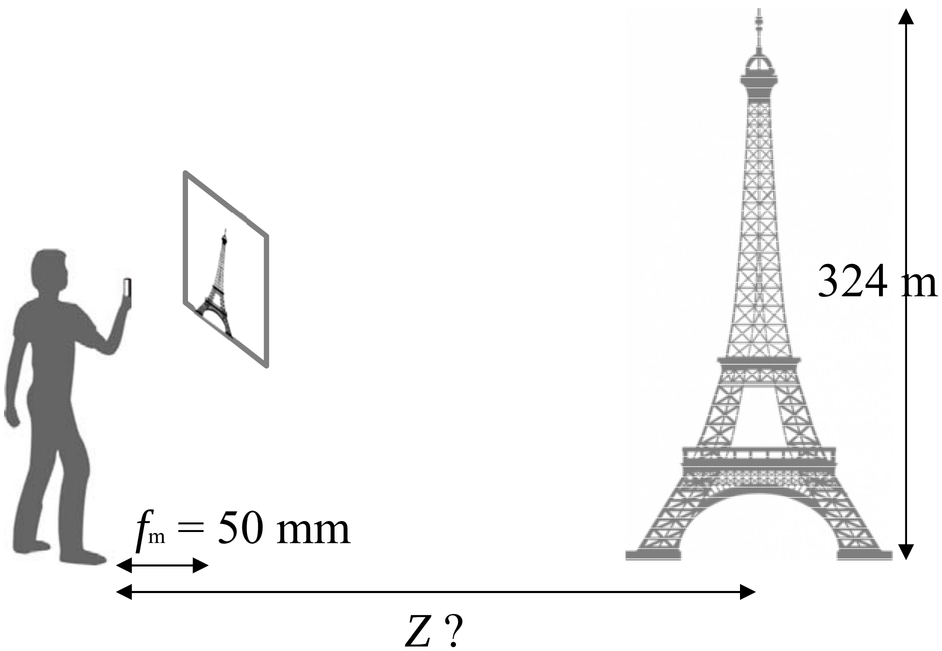


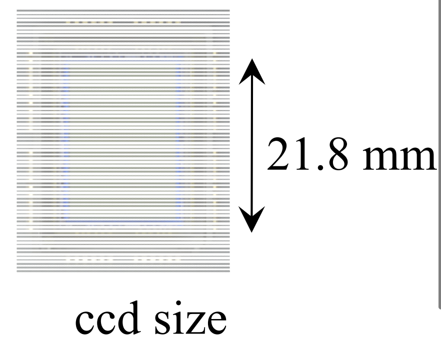
$$y_{\text{img}} = f\frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}}\frac{Y}{Z}$$

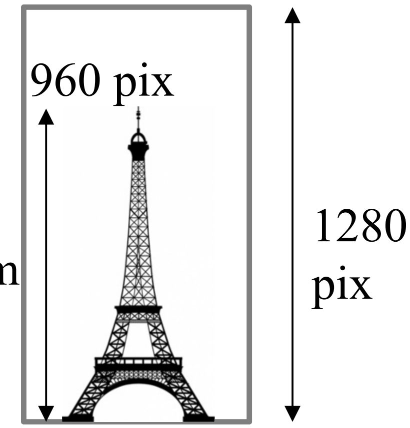
$$960 = f_m \frac{1280}{0.0218}\frac{324}{1500} \rightarrow f_m = 0.0757m$$

Exercise

What f to make the height of Eifel tower appear 960 pixel distance?





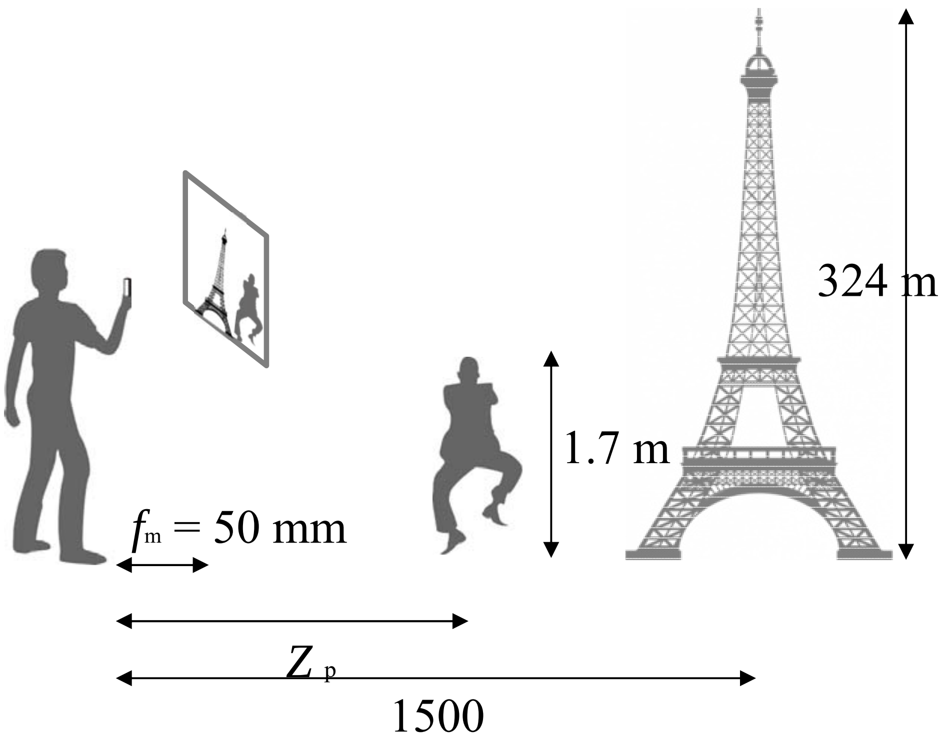


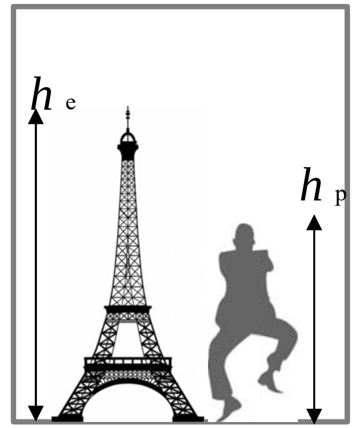
$$y_{img} = f\frac{Y}{Z} = f_m \frac{h_{img}}{h_{ccd}}\frac{Y}{Z}$$

$$960 = 0.05\frac{1280}{0.0218}\frac{324}{Z} \rightarrow Z = 990.826\text{m}$$

Exercise

What Z_p to make the height of Eifel tower appear twice of the person?

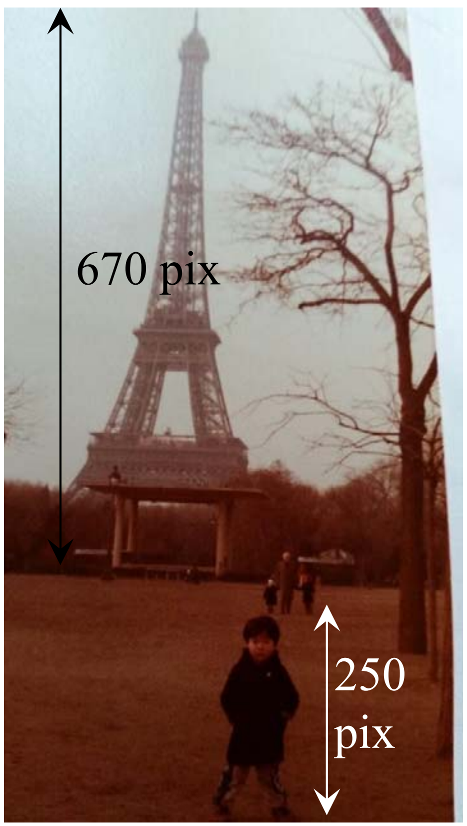




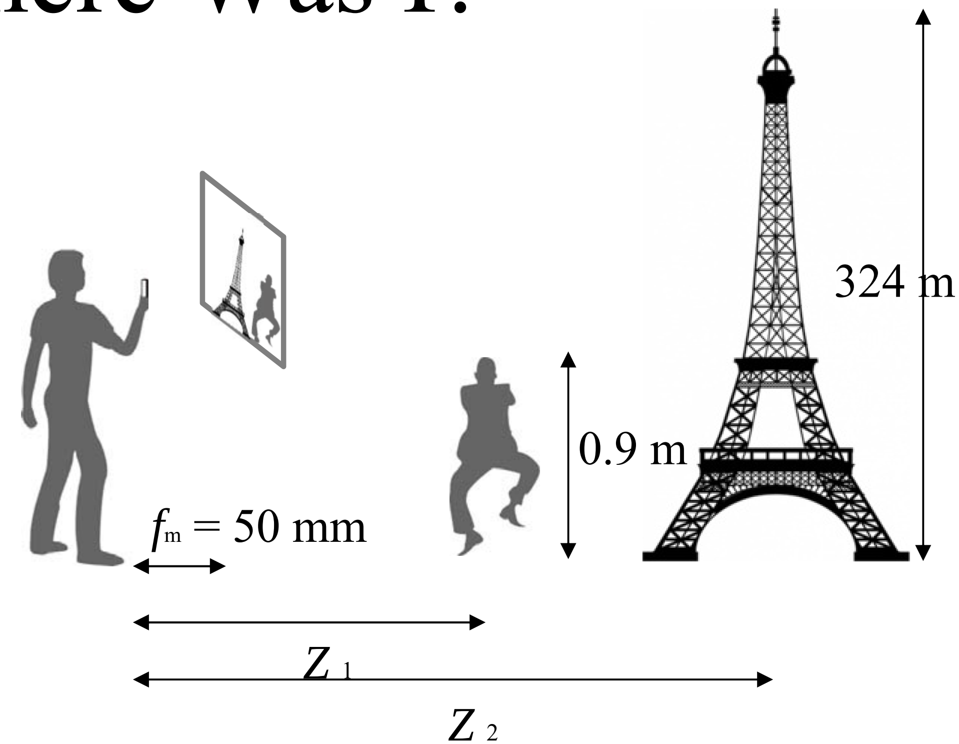
$$h_e = f\frac{Y}{Z} \qquad h_p = f\frac{Y_p}{Z_p} \qquad \text{s.t.} \qquad h_p = \frac{h_e}{2}$$

$$f\frac{Y_p}{Z_p} = f\frac{Y}{2Z} \rightarrow \qquad Z_p = 2 \cdot 1500\frac{1.7}{234} = 157.41\text{m}$$

Where Was I?



Circa 1984

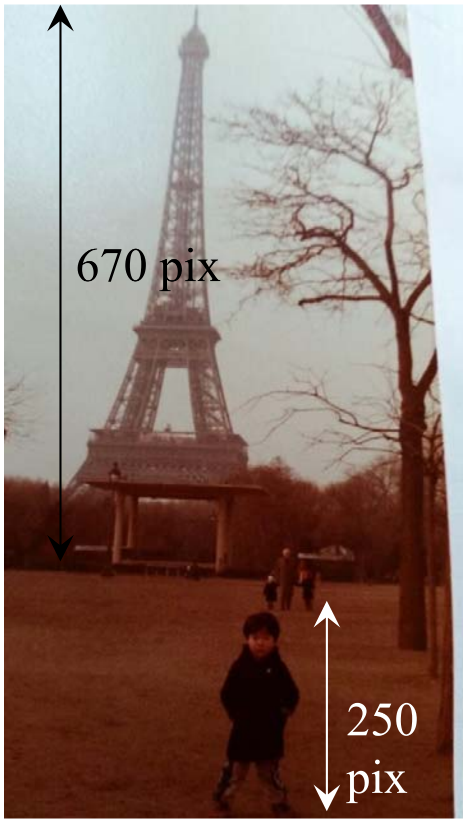


$$y_1 = f\frac{Y}{Z} = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_1}{y_1} = 0.05\frac{1280}{0.0218}\frac{0.9}{250} = 8.03\text{m}$$

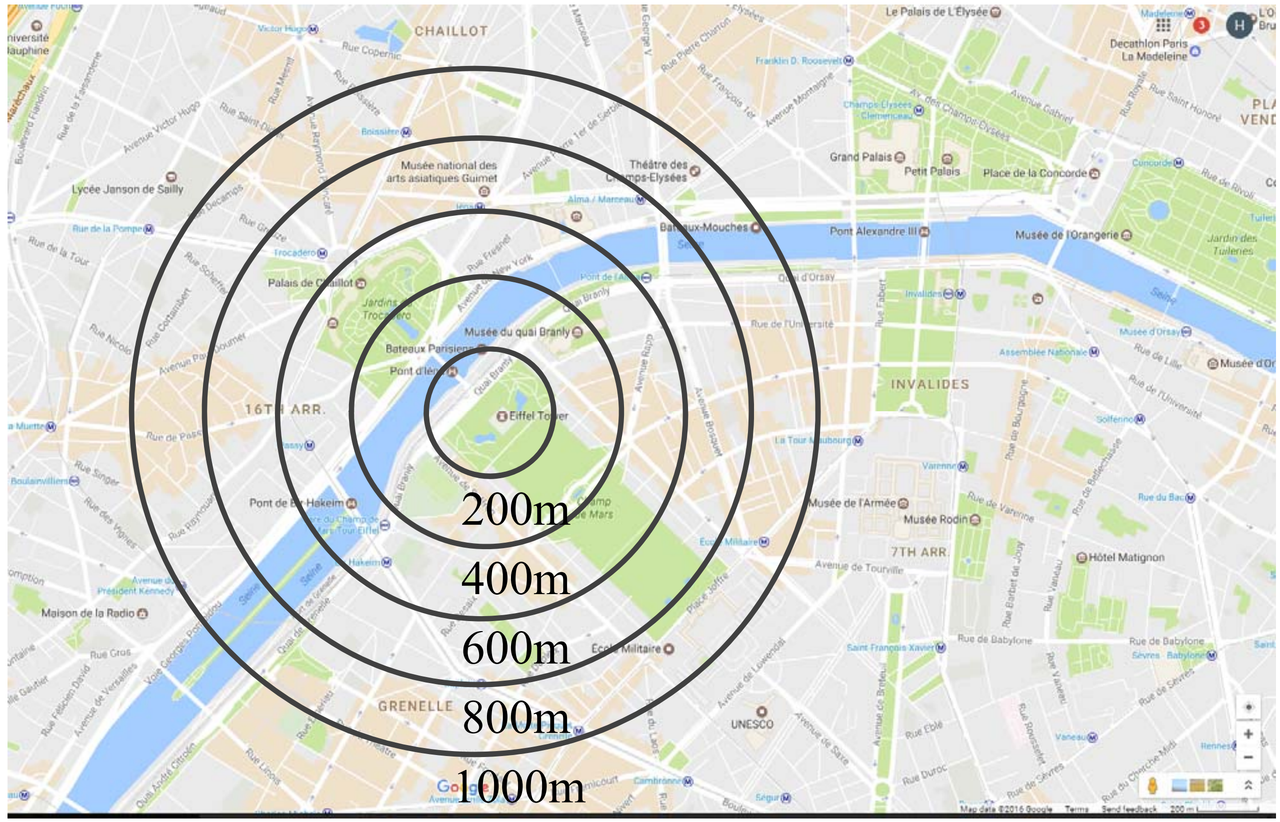
$$y_2 = f\frac{Y_2}{Z_2} = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_2}{y_2} = 0.05\frac{1280}{0.0218}\frac{324}{670} = 1079\text{m}$$

Where Was I?

$$y_2 = f\frac{Y_2}{Z_2} = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{img}}{h_{ccd}}\frac{Y_2}{y_2} = 0.05\frac{1280}{0.0218}\frac{324}{670} = 1079\text{m}$$

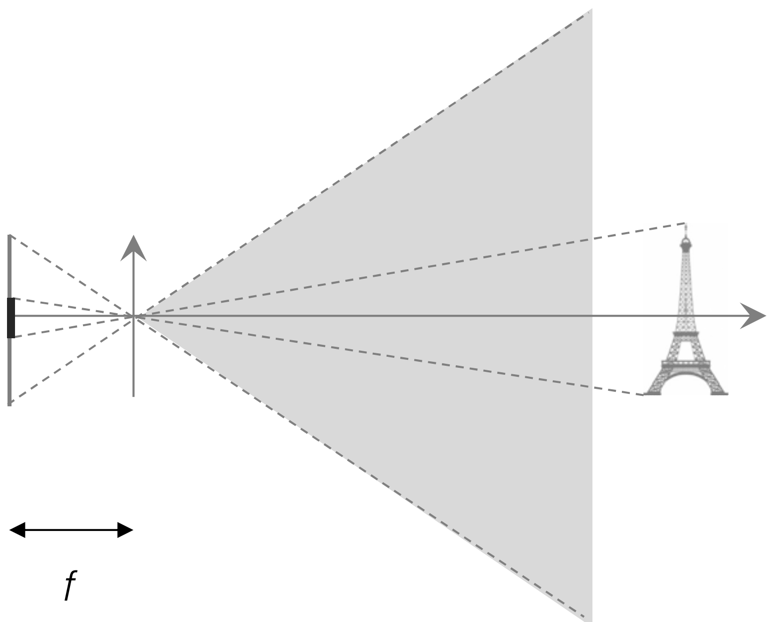


Circa 1984

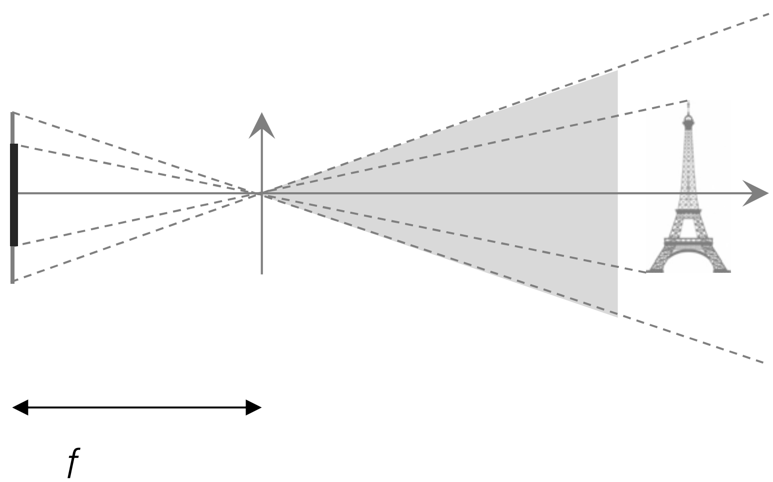


Where Was I?

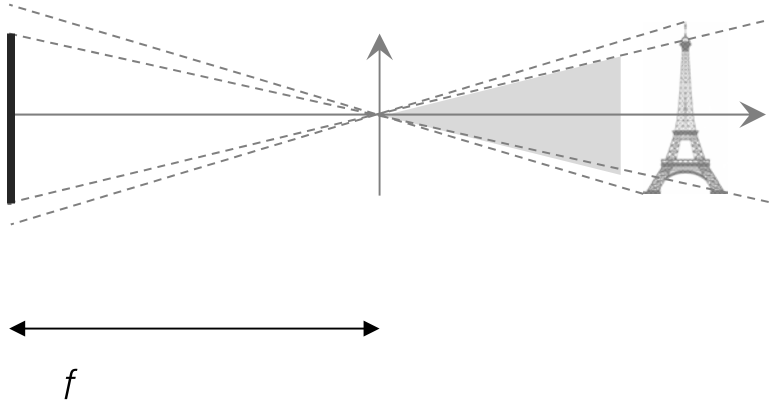
Focal Length

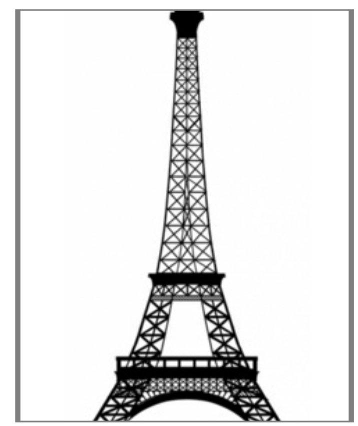


Focal Length

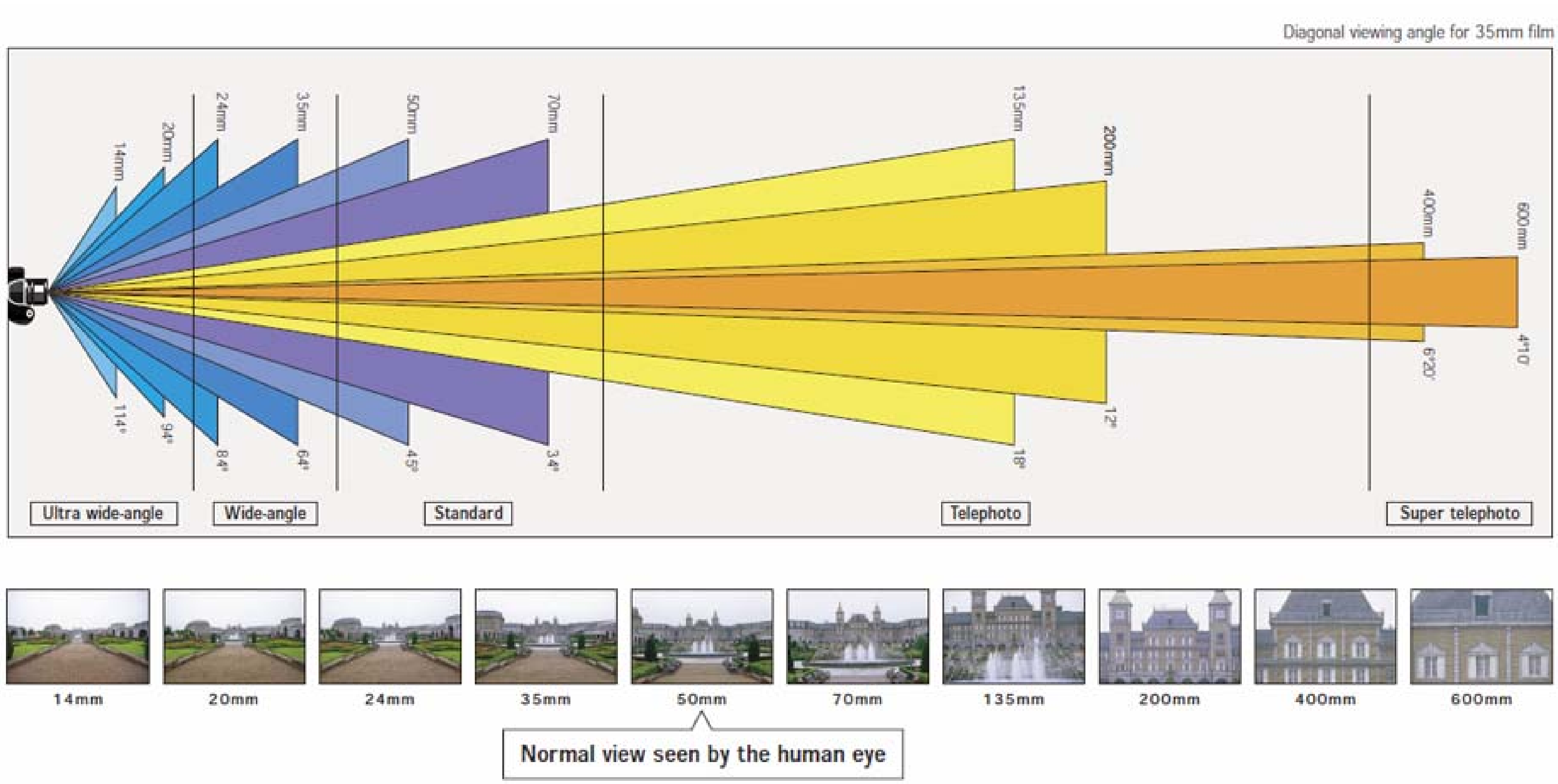


Focal Length





Focal Length

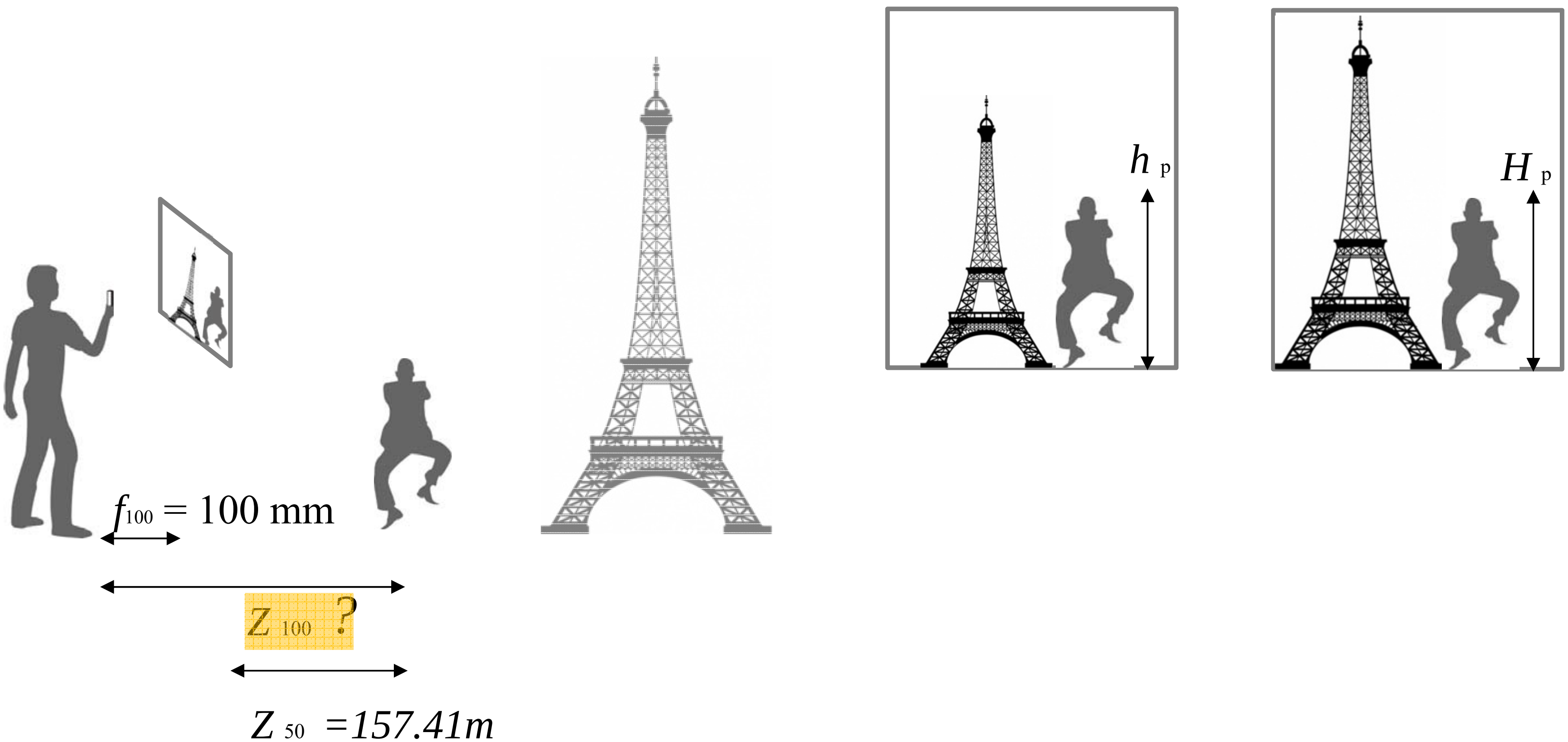


Dolly Zoom (Vertigo Effect)

(Jaws 1975)

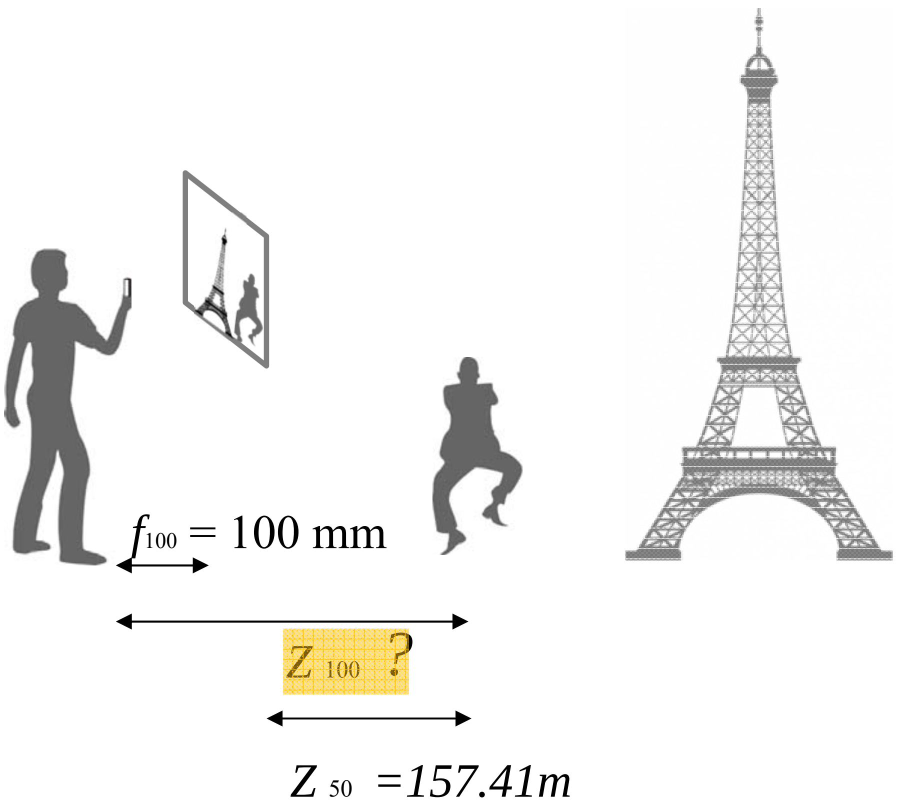
Dolly Zoom

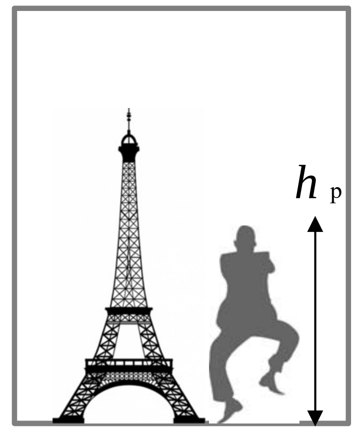
Given focal length (f_m=100mm),
what Z_{100} to make the height of the person remain the same as f_m=50mm?

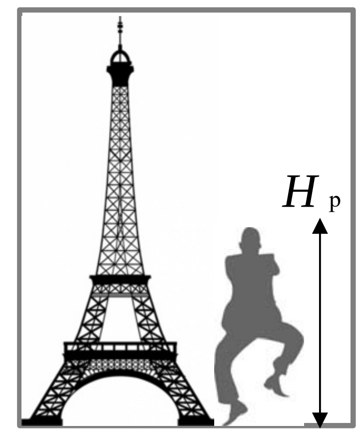


Dolly Zoom

Given focal length (f_m=100mm),
what Z_{100} to make the height of the person remain the same as f_m=50mm?







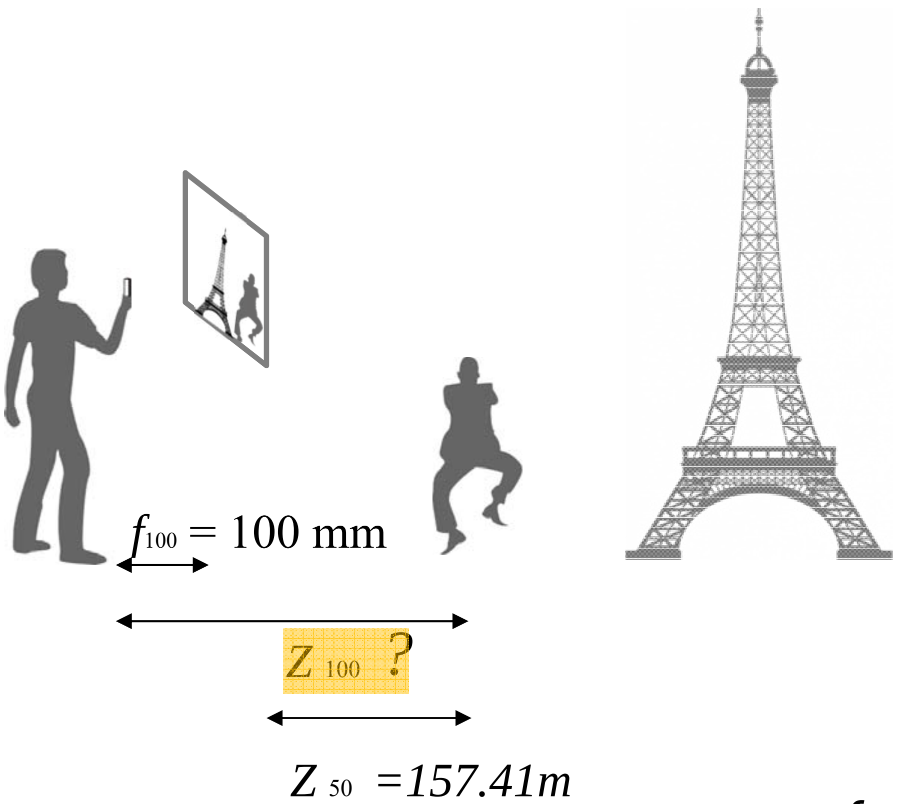
$$h_{50} = f_{50}\frac{Y}{Z_{50}}$$

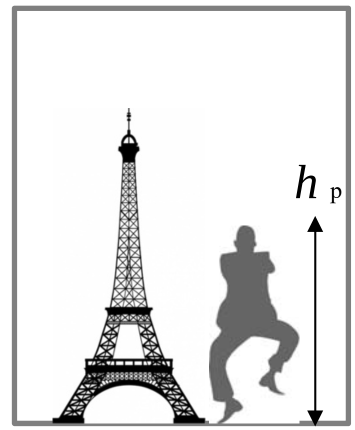
$$h_{100} = f_{100}\frac{Y}{Z_{100}}$$

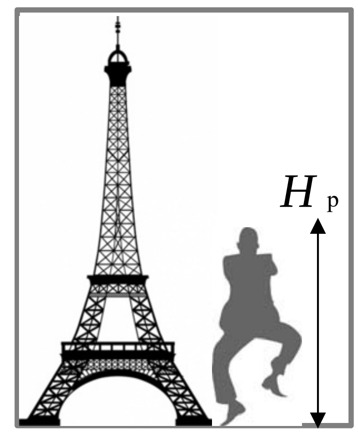
s.t. $$h_{100} = h_{50}$$

Dolly Zoom

Given focal length (f_m=100mm),
what Z_{100} to make the height of the person remain the same as f_m=50mm?







$$h_{50} = f_{50} \frac{Y}{Z_{50}}$$

$$h_{100} = f_{100} \frac{Y}{Z_{100}}$$

s.t. $h_{100} = h_{50}$

$$Z_{100} = \frac{f_{100}}{f_{50}} Z_{50}$$

$$Z_{100} = \frac{100}{50} 157.41 = 314.8\text{m}$$

Dolly Zoom (Vertigo Effect)