Creating Video and Images

Stretching the sense of reality
Facade, Paul Debevec
Computational Photography
Field of View (Zoom)
Focal length

10mm 18mm 35mm 50mm 70mm 100mm 135mm 200mm 300mm

Wide Angle  Normal  Medium Telephoto  Telephoto

View angle

18mm 50mm 100mm 200mm 300mm

Architecture, Landscape  Street, Documentary  Portraiture  Sports, Birds, Wildlife
Large Focal Length compresses depth

400 mm   200 mm   100 mm   50 mm   28 mm   17 mm

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FOV depends of Focal Length

Size of field of view governed by size of the camera retina:

\[ \phi = \tan^{-1} \left( \frac{d}{2f} \right) \]

Smaller FOV = larger Focal Length
Field of View (Zoom)

From London and Upton
Field of View (Zoom)

From London and Upton
Fisheye lens distortion
Camera Model
3D object

Lens

CCD sensor

Pixel

3D object
3D Point Projection (Metric Space)

$\mathbf{3D \ point \ (X,Y,Z)} \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$

2D projection onto CCD plane
3D Point Projection (Metric Space)

\[(X, Y, Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\]

2D projection onto CCD plane
3D Point Projection (Metric Space)

\[ (X, Y, Z) \rightarrow (u_{\text{ccd}}, v_{\text{ccd}}) = \left( f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \]

2D projection onto CCD plane
3D Point Projection (Pixel Space)

$$u_{ccd} = \frac{u_{img} - p_x}{w_{img}}$$
$$v_{ccd} = \frac{v_{img} - p_y}{h_{img}}$$

$$u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x$$
$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y$$

CCD sensor (mm)

Image principal point

Image (pixel)
3D Point Projection (Pixel Space)

\[(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\]

\[u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img} X}{w_{ccd} Z} + p_x\]

Focal length in pixel

\[v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img} Y}{h_{ccd} Z} + p_y\]

Focal length in pixel
3D Point Projection (Pixel Space)

\[(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\]

\[u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img}}{w_{ccd}} \frac{X}{Z} + p_x\]

\[v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y\]

Focal length in pixel

CCD sensor (mm)

Image (pixel)
3D Point Projection (Pixel Space)

\[(X, Y, Z) \rightarrow (u_{\text{img}}, v_{\text{img}}) = \left( \frac{f_m}{w_{\text{img}}} X, \frac{f_m}{h_{\text{img}}} Y \right) \]
Homogeneous Coordinate

2D point $\rightarrow$ 3D ray $\lambda(x, y, 1)$

$(x, y) \rightarrow (x, y, 1)$: A point in Euclidean space ($\mathbb{R}^2$) can be represented by a homogeneous representation in Projective space ($\mathcal{P}^2$) (3 numbers).

$= f(x, y, 1)$

$= \lambda(x, y, 1)$
Homogeneous Coordinate

2D point $=\lambda$ 3D ray

$\lambda(x, y, 1) = (X, Y, Z)$ 3D point lies in the 3D ray passing 2D image point.

Homogeneous coordinate
3D Point Projection (Metric Space)

2D point $=\ 3D$ ray

Projection plane

$(x, y, 1) = (f_m x, f_m y, f_m) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}, f_m)$
3D Point Projection (Pixel Space)

\[(X,Y,Z) \rightarrow (u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})\]

\[u_{img} = f_x \frac{X}{Z} + p_x \quad v_{img} = f_y \frac{Y}{Z} + p_y\]

\[
\begin{bmatrix}
    u_{img} \\
    v_{img} \\
    1
\end{bmatrix} =
\begin{bmatrix}
    f_x & p_x \\
    f_y & p_y \\
    1 & 1
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix}

Homogeneous representation
Camera Intrinsic Parameter

Pixel space

\[ \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & p_x & X \\ k_y & p_y & Y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \]

Metric space

Camera intrinsic parameter: metric space to pixel space
2D Inverse Projection

2D point \( \rightarrow \) 3D ray

\[
\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \rho_x \\ f_y & \rho_y \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

The 3D point must lie in the 3D ray passing through the origin and 2D image point.
3D Point Projection (Pixel Space)

\[ (X, Y, Z) \rightarrow (U^{\text{img}}, V^{\text{img}}) = \left( \frac{f_m}{w^{\text{img}}}, \frac{X}{w^{\text{ccd}}}, \frac{h^{\text{img}}}{h^{\text{ccd}}}, \frac{Y}{Z} \right) \]
Exercise

What f to make the height of Eifel tower appear 960 pixel distance?

\[ y_{img} = f \frac{Y}{Z} = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} \]

\[ 960 = f_m \frac{1280}{0.0218} \frac{324}{1500} \rightarrow f_m = 0.0757m \]
Exercise

What f to make the height of Eifel tower appear 960 pixel distance?

\[
y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}
\]

\[
960 = 0.05 \frac{1280}{0.0218} \frac{324}{Z} \rightarrow Z = 990.826 \text{m}
\]
Exercise

What $Z_p$ to make the height of Eifel tower appear twice of the person?

\[ h_e = f \frac{Y}{Z} \quad h_p = f \frac{Y_p}{Z_p} \quad \text{s.t.} \quad h_p = \frac{h_e}{2} \]

\[ f \frac{Y_p}{Z_p} = f \frac{Y}{2Z} \quad Z_p = 2 \cdot 1500 \cdot \frac{1.7}{234} = 157.41 \text{m} \]

$f_m = 50 \text{ mm}$
Where Was I?

Circa 1984

\[ f_m = 50 \text{ mm} \]

\[ \frac{y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{ m} \]

\[ \frac{y_2}{Z_2} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079 \text{ m} \]
Where Was I?

\[ y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079\text{m} \]
Focal Length
Focal Length
Focal Length

\[ f \]
Focal Length

Diagonal viewing angle for 35mm film

Ultra wide-angle | Wide-angle | Standard | Telephoto | Super telephoto

14 mm | 20 mm | 24 mm | 35 mm | 50 mm | 70 mm | 135 mm | 200 mm | 400 mm | 600 mm

Normal view seen by the human eye
Dolly Zoom (Vertigo Effect)

(Jaws 1975)
Dolly Zoom

Given focal length \( f_m = 100\text{mm} \), what \( Z_{100} \) to make the height of the person remain the same as \( f_m = 50\text{mm} \)?

\[ Z_{100} \]

\[ Z_{50} = 157.41\text{m} \]
Given focal length \( f_m = 100\text{mm} \), what \( Z_{100} \) to make the height of the person remain the same as \( f_m = 50\text{mm} \)?

\[
f_{100} = 100 \text{ mm}
\]

\[
Z_{100} ?
\]

\[
Z_{50} = 157.41 \text{ m}
\]

\[
h_{50} = f_{50} \frac{Y}{Z_{50}}
\]

\[
h_{100} = f_{100} \frac{Y}{Z_{100}}
\]

s.t.

\[
h_{100} = h_{50}
\]
Dolly Zoom

Given focal length ($f_m=100\text{mm}$), what $Z_{100}$ to make the height of the person remain the same as $f_m=50\text{mm}$?

\[ Z_{100} = \frac{f_{100}}{f_{50}} Z_{50} \]

\[ Z_{100} = \frac{100}{50} \times 157.41 = 314.8\text{m} \]
Dolly Zoom (Vertigo Effect)