Putting it together

What we see

What we really see
Object Segmentation
Basic Shape Comparison
Let $p, q$ be two edge sets to be compared.

$$\text{ShapeDiff}(p, q) = \sum_{x \in p} \min_{y \in q} \|x - y\|^2$$

Distance transform: $D_q(x)$
Distance Transform Definition

Set of points, $P$, some distance $\| \bullet \|

\[
D_P(x) = \min_{y \in P} \| x - y \|
\]

- For each location $x$ distance to nearest $y$ in $P$
- Think of as cones rooted at each point of $P$
Two pass O(n) algorithm for 1D $L_1$ norm (for simplicity just distance)

1. **Initialize:** For all $j$
   \[ D[j] \leftarrow 1_p[j] \]

2. **Forward:** For $j$ from 1 up to $n-1$
   \[ D[j] \leftarrow \min(D[j], D[j-1]+1) \]

3. **Backward:** For $j$ from $n-2$ down to 0
   \[ D[j] \leftarrow \min(D[j], D[j+1]+1) \]

\[ \infty 0 \infty 0 \infty \infty \infty \infty 0 \infty \]

\[ \infty 0 1 0 1 2 3 0 1 \]

\[ 1 0 1 0 1 2 1 0 1 \]
- 2D case analogous to 1D
  - Initialization
  - Forward and backward pass
    - Fwd pass finds closest above and to left
    - Bwd pass finds closest below and to right
- Note nothing depends on $0, \infty$ form of initialization
  - Can “distance transform” arbitrary array
Deformable Shape
Applying chamfer matching directly
Deformable part model detection with 6 parts
Applying chamfer matching directly
Deformable part model detection with 4 parts
Voting based shape detection
Simplified
Construct a code book for each model points: (green) nodes

Code: \( = \)

(Hog or Shape Context + offset to center \( \))
scan over image points, find the top k matches in model
Create vote map in Input image, based on the top k matches in model.
scan over image points, find the top k matches in model
1) Create vote map in Input image, based on the top k matches in model
2) Summing up the map
Using this ‘score map’, we can choose hypotheses centers on it (green stars).
For each hypothesis position, trace back to find its voters.

Note: the numbers inside the rectangles are scores for each hypothesis after enforce one-2-one match, so they are a bit lower than voting scores.
Pictorial Structure
Simplified
Generate part score map in image
Combine multiple part score function into one score map
construct a ‘star’ graph, with parts as nodes, pick one node as “root”
For each non-root node:

Shift score map for Left eye onto center(nose)
Shift score map for Right eye onto center(nose)
Shift score map for Left mouth onto center(nose)
Shift score map for Right mouth onto center(nose)
Add up all the part vote score maps
Object Representation

Pictorial Structure
Object Representation

- Object with n parts labeled 1 through n

- Object configuration given by: \( L = (l_1, \ldots, l_n) \)

  - Location of each part

\[
(L_1, L_2, L_3, L_4) = (300,200), (300,250), (330,230), (360,230)
\]
Find the most probable configuration of the object,

\[ P(L|I) \propto P(I|L)P(L) \]

Geometrical model: \( P(L) \)

Appearance model: \( P(I|L) \propto \prod g_i(I, l_i) \)
Part-based Object Representation

**Geometrical model:** $P(L)$

measuring “goodness” of the part configuration

**Appearance model:** $P(I|L) \propto \prod g_i(I, l_i)$

image $\rightarrow$ Label

measuring “goodness” of the part appearance
Part-based Object Representation

Find the most probable configuration of the object,

\[ P(L|I) \propto P(I|L)P(L) \]

- Size of configuration space is exponential
  - \( n \) parts, \( m \) locations - \( O(m^n) \) configurations
  - Use implicit search techniques
Solution

1) Reduce number of possible feature locations, by feature detection.
   -- a possible solution is use shape context features

2) Find efficient way of dealing large number of features, each of which has a goodness measure
   -- we will cover this story here...
Geometrical model: $P(L)$

measuring “goodness” of the part configuration

Simplifying “goodness” measure using k-fan model

1) we only check if the parts configuration between the reference node (nose in this case), with all other nodes
Dealing with “soft” features

- Recognition without feature detection
  - Single overall inference problem
  - Parts have a match quality at each location
A simplified object of two parts (front & back wheel)

- Back wheel (reference)
- Front wheel

Distance between the wheels in a known range

Soft object detection map
Soft object detection map

\[ L(p,q|I) = f(p) + h(p-q) + f(q) \]

- \( p \): location of back wheel
- \( q \): location of front wheel
- distance between the wheels in a known range
- Soft part detection measure for \( p \)
- configuration goodness \((p,q)\)
- Soft part detection measure for \( q \)
In this case \( p, q \) each has \( n \) (image size=1 million) possible locations, \( L(p, q \mid I) \) has \( n^2 \) (Trillion) possible solutions. Fast solution is needed!
The distance between the wheels in a known range is given by:

\[ L(p, q | I) = f(p) + h(p-q) + f(q) \]

Minimizing over \( p \) and \( q \):

\[ \min_{(p,q)} L(p,q | I) = \min_{(p,q)} f(p) + h(p-q) + f(q) \]

\[ = \min_{(p)}( f(p) + \min_{(q)} (f(q) + h(p-q)) ) \]

\[ D_q(p) : \text{generalized distance transform} \]

This can be computed in linear time!
Generalized distance transform

Given a function $f: \mathcal{G} \rightarrow \mathbb{R}$,

$$D_f(q) = \min_{p \in \mathcal{G}} \left( \|q - p\|^2 + f(p) \right)$$

- for each location $q$, find nearby location $p$ with $f(p)$ small.

- equals DT of points $P$ if $f$ is an indicator function.

$$f(p) = \begin{cases} 0 & \text{if } p \in P \\ \infty & \text{otherwise} \end{cases}$$
1D case: \[ D_f(q) = \min_{p \in G} \left( (q - p)^2 + f(p) \right) \]

For each \( p \), \( D_f(q) \) is below the parabola rooted at \((p, f(p))\).

\( D_f(q) \) is defined by the lower envelope of \( h \) parabolas.
There is an efficient exact inference for graph without loops.

Procedure:

Step 1, order tree
determine a root of the tree, and order
the nodes according to its depth

Step 2-3: Gather information.
processing from the bottom of the tree (nodes
with max. depth) backward to the root of the
tree

Step 4-5: Decide at root, and propagate
Make decision at the tree root, and recursively
propagate the information down
Step 2: Gather Information for leaves nodes

for the leaf nodes, \( j \), (nodes with max. depth)

Compute the following table, indexed by its possible parent node assignment:

\[
B_j(l_i) = \min_{l_j} (m_j(l_j) + d_{ij}(l_i, l_j)),
\]

Given a parent node label, find the best label for itself
Step 2: Gather Information for leaves nodes

For the leaf nodes, j, (nodes with max. depth)

Compute the following table, indexed by its possible parent node assignment:

\[ B_j(l_i) = \min_{l_j} (m_j(l_j) + d_{ij}(l_i, l_j)) , \]

Important: we need to store both the optimal value \( l_j \), as well the cost at the optimal label \( l_j \).
Step 3: Gather Information at inside node

for inside nodes, $j$, (not root, not leaves)

Compute the following table, indexed by its possible parent node assignment:

$$B_j(l_i) = \min_{l_j} \left( m_j(l_j) + d_{ij}(l_i, l_j) + \sum_{v_c \in C_j} B_c(l_j) \right).$$

- parent node label
- do the best for itself
- considering parent’s (i) preference
- considering votes from all its children (c)
Step 3: Make decision at the root node

\[ l^*_r = \arg\min_{l_r} \left( m_r(l_r) + \sum_{v_c \in C_r} B_c(l_j) \right) \]

- do the best for itself
- considering votes from all its children (c)

The decision at the root is purely local, no need to check with anyone else.

Good to the root, but one wrong choice, it effects the whole tree.
Step 4: recursively propagate information down

Given parent node is decided

\[ B_j(l_i) \]

current node label decision can be directly read off from the table

\[ B_j(l_i) \]

decide by read off from table
Deformable part model detection with 4 parts
Learning Pictorial Structure
A Modern Version
1) fine level with deformable parts
2) coarse level with a fixed template model