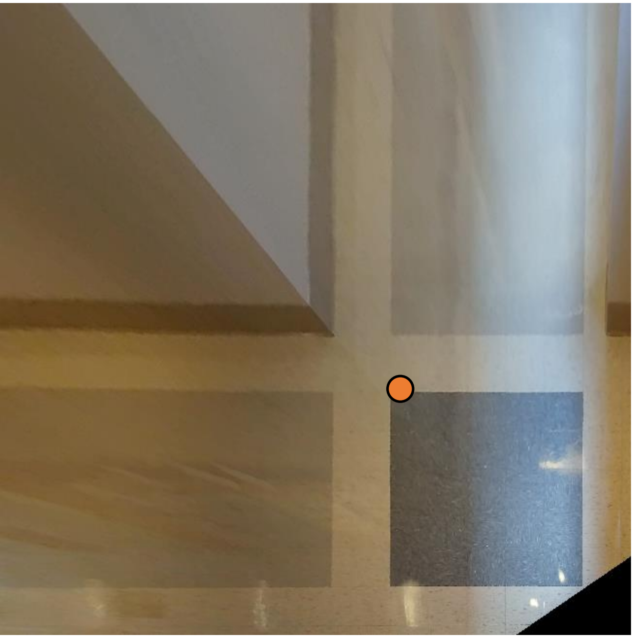
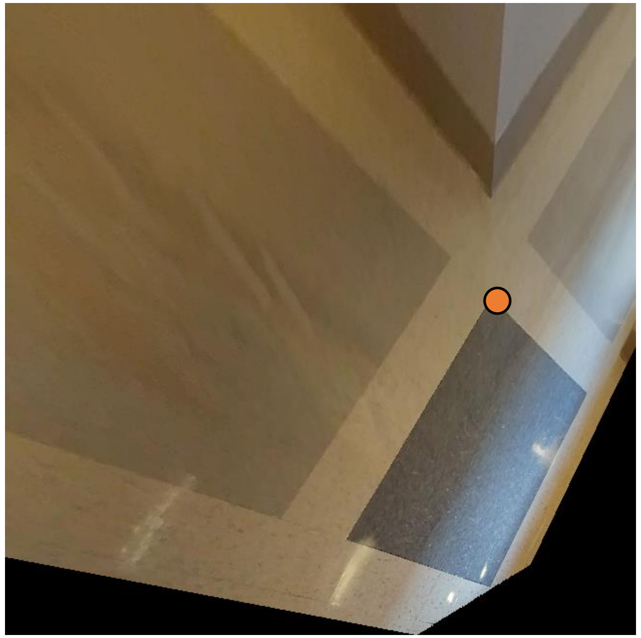
Homography Computation



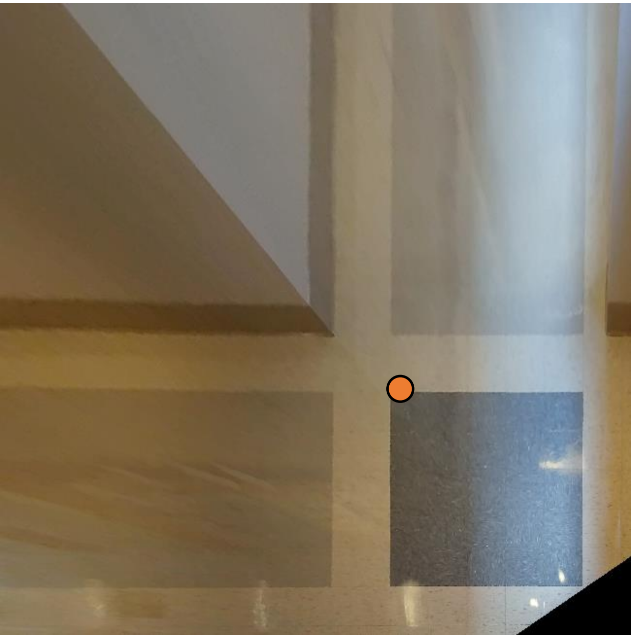


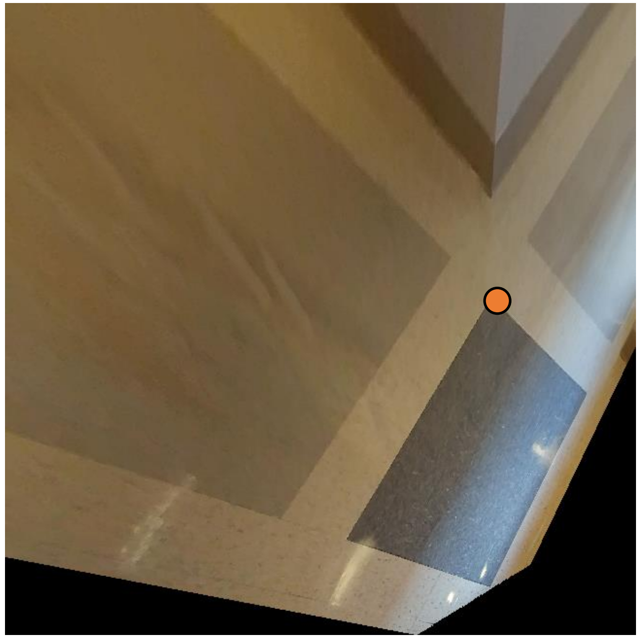
$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Homography Computation





$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

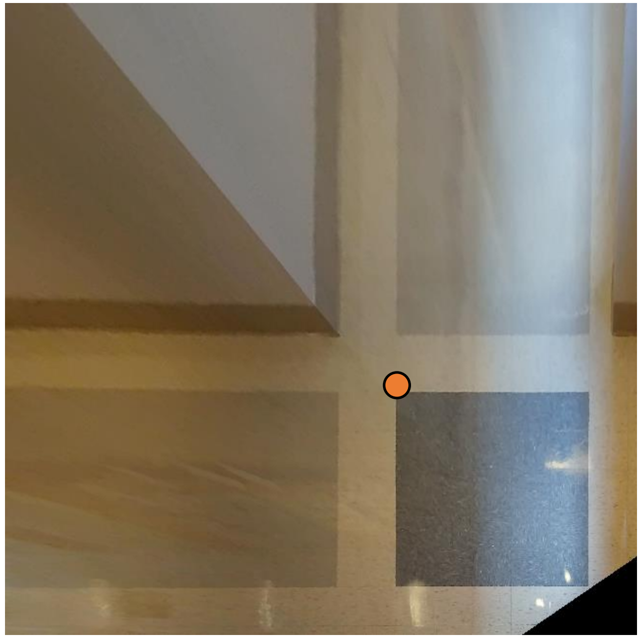
$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

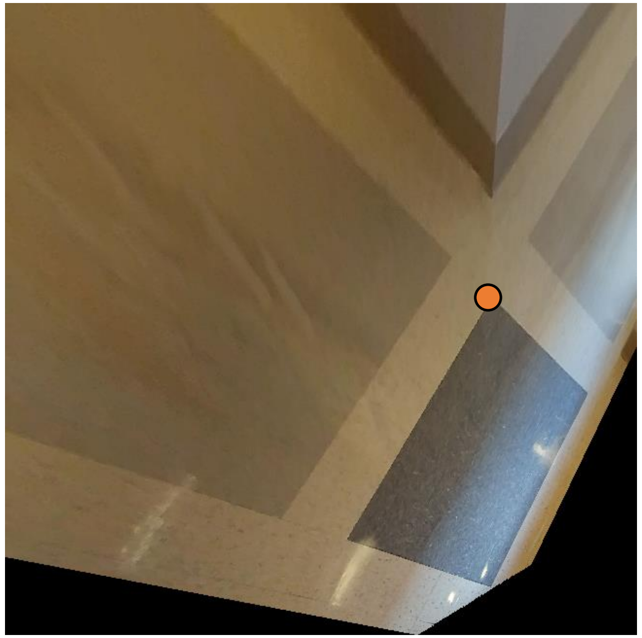
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\longrightarrow \begin{array}{l} h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0 \\ h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0 \end{array}$$

$$\longrightarrow \begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & 1 & -u_xv_y & -u_yv_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Homography Computation





$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\rightarrow \quad h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0$$

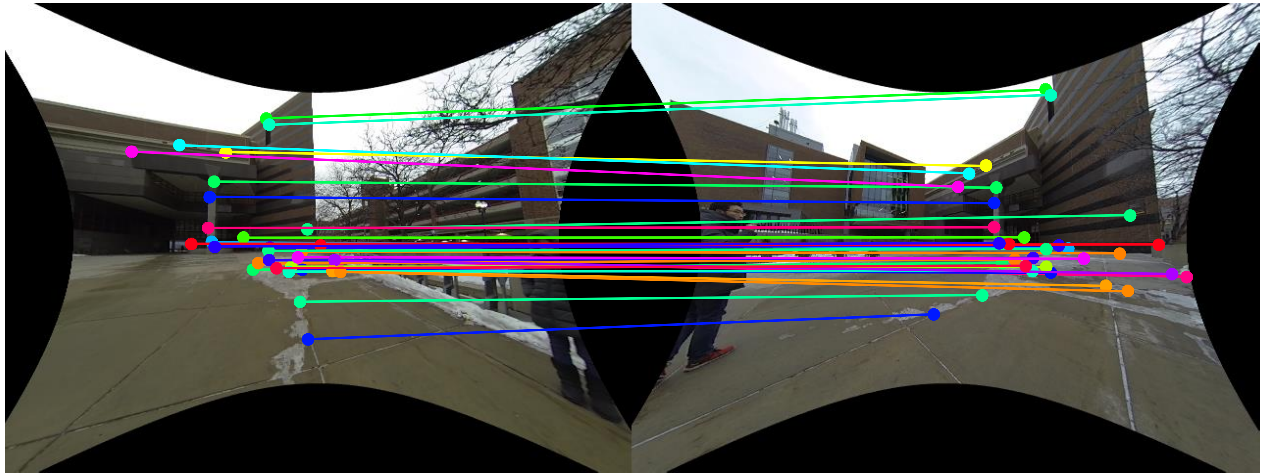
$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0$$

$$\rightarrow \quad \begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & 1 & -u_xv_y & -u_yv_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

2x9

Linear System for Homography Matrix

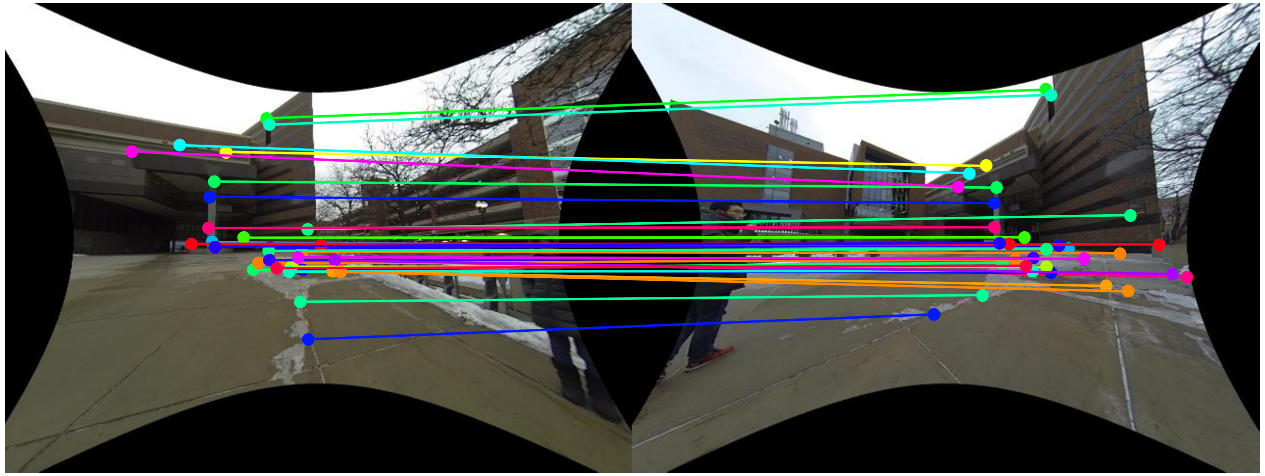


$$\begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{0}$$

2x9

How Many Correspondences?



$$\begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{0}$$

2x9

What is minimum m?

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\begin{Bmatrix} \mathbf{v}_1 \leftrightarrow \mathbf{u}_1 \\ \mathbf{v}_2 \leftrightarrow \mathbf{u}_2 \\ \mathbf{v}_3 \leftrightarrow \mathbf{u}_3 \\ \mathbf{v}_4 \leftrightarrow \mathbf{u}_4 \end{Bmatrix} \rightarrow \mathbf{H}$$

Homography computation

$$\begin{bmatrix} u^1_x & u^1_y & 1 & & & & -u^1_x v^1_x & -u^1_y v^1_x & -v^1_x \\ & & & u^1_x & u^1_y & 1 & -u^1_x v^1_y & -u^1_y v^1_y & -v^1_y \\ u^2_x & u^2_y & 1 & & & & -u^2_x v^2_x & -u^2_y v^2_x & -v^2_x \\ & & & u^2_x & u^2_y & 1 & -u^2_x v^2_y & -u^2_y v^2_y & -v^2_y \\ u^3_x & u^3_y & 1 & & & & -u^3_x v^3_x & -u^3_y v^3_x & -v^3_x \\ & & & u^3_x & u^3_y & 1 & -u^3_x v^3_y & -u^3_y v^3_y & -v^3_y \\ u^4_x & u^4_y & 1 & & & & -u^4_x v^4_x & -u^4_y v^4_x & -v^4_x \\ & & & u^4_x & u^4_y & 1 & -u^4_x v^4_y & -u^4_y v^4_y & -v^4_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{\begin{matrix} \mathbf{v}_1 \leftrightarrow \mathbf{u}_1 \\ \mathbf{v}_2 \leftrightarrow \mathbf{u}_2 \\ \mathbf{v}_3 \leftrightarrow \mathbf{u}_3 \\ \mathbf{v}_4 \leftrightarrow \mathbf{u}_4 \end{matrix}\right\} \rightarrow \mathbf{H}$$

Homography computation

$$\mathbf{A} = \begin{bmatrix} u^1_x & u^1_y & 1 & & & & -u^1_x v^1_x & -u^1_y v^1_x & -v^1_x \\ & & & u^1_x & u^1_y & 1 & -u^1_x v^1_y & -u^1_y v^1_y & -v^1_y \\ u^2_x & u^2_y & 1 & & & & -u^2_x v^2_x & -u^2_y v^2_x & -v^2_x \\ & & & u^2_x & u^2_y & 1 & -u^2_x v^2_y & -u^2_y v^2_y & -v^2_y \\ u^3_x & u^3_y & 1 & & & & -u^3_x v^3_x & -u^3_y v^3_x & -v^3_x \\ & & & u^3_x & u^3_y & 1 & -u^3_x v^3_y & -u^3_y v^3_y & -v^3_y \\ u^4_x & u^4_y & 1 & & & & -u^4_x v^4_x & -u^4_y v^4_x & -v^4_x \\ & & & u^4_x & u^4_y & 1 & -u^4_x v^4_y & -u^4_y v^4_y & -v^4_y \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{\begin{matrix} \mathbf{v}_1 \leftrightarrow \mathbf{u}_1 \\ \mathbf{v}_2 \leftrightarrow \mathbf{u}_2 \\ \mathbf{v}_3 \leftrightarrow \mathbf{u}_3 \\ \mathbf{v}_4 \leftrightarrow \mathbf{u}_4 \end{matrix}\right\} \rightarrow \mathbf{H}$$

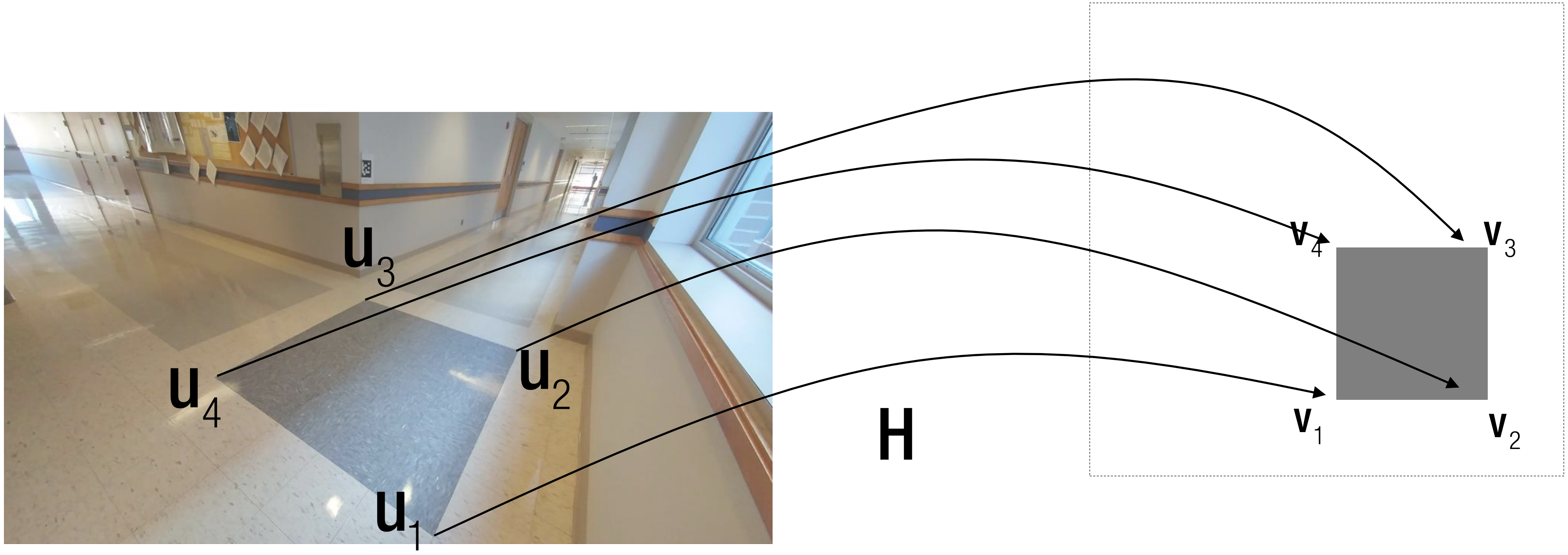
Homography computation

$$\mathbf{A}\mathbf{X} = \mathbf{0}$$

$$\begin{bmatrix} u^1_x & u^1_y & 1 & & & & -u^1_x v^1_x & -u^1_y v^1_x & -v^1_x \\ & & & u^1_x & u^1_y & 1 & -u^1_x v^1_y & -u^1_y v^1_y & -v^1_y \\ u^2_x & u^2_y & 1 & & & & -u^2_x v^2_x & -u^2_y v^2_x & -v^2_x \\ & & & u^2_x & u^2_y & 1 & -u^2_x v^2_y & -u^2_y v^2_y & -v^2_y \\ u^3_x & u^3_y & 1 & & & & -u^3_x v^3_x & -u^3_y v^3_x & -v^3_x \\ & & & u^3_x & u^3_y & 1 & -u^3_x v^3_y & -u^3_y v^3_y & -v^3_y \\ u^4_x & u^4_y & 1 & & & & -u^4_x v^4_x & -u^4_y v^4_x & -v^4_x \\ & & & u^4_x & u^4_y & 1 & -u^4_x v^4_y & -u^4_y v^4_y & -v^4_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

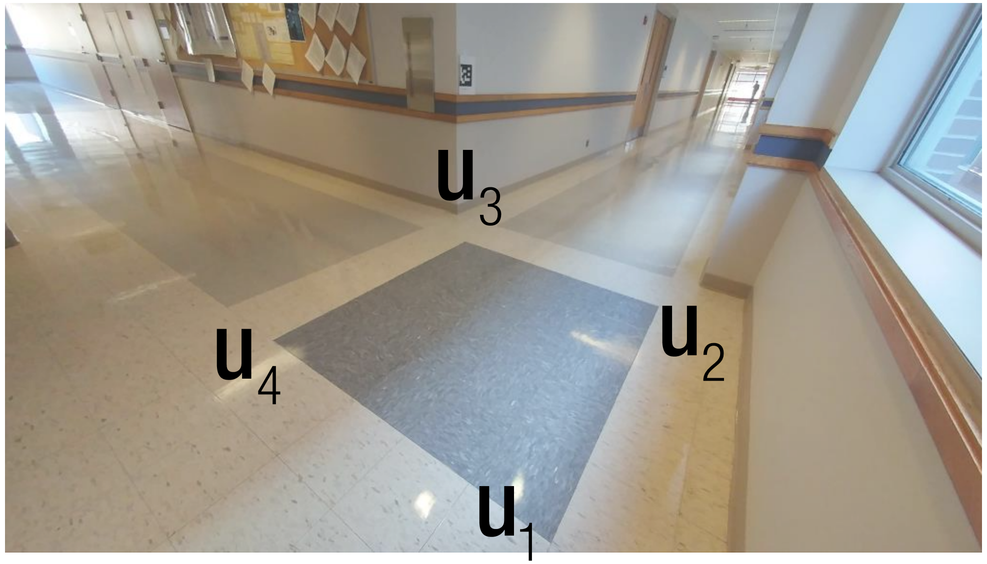
```
[u,d,v] = svd(A);
X = v(:,end)/v(end,end);
H = reshape(X,3,3)';
```

Fun with Homography



The image can be rectified as if it is seen from top view.

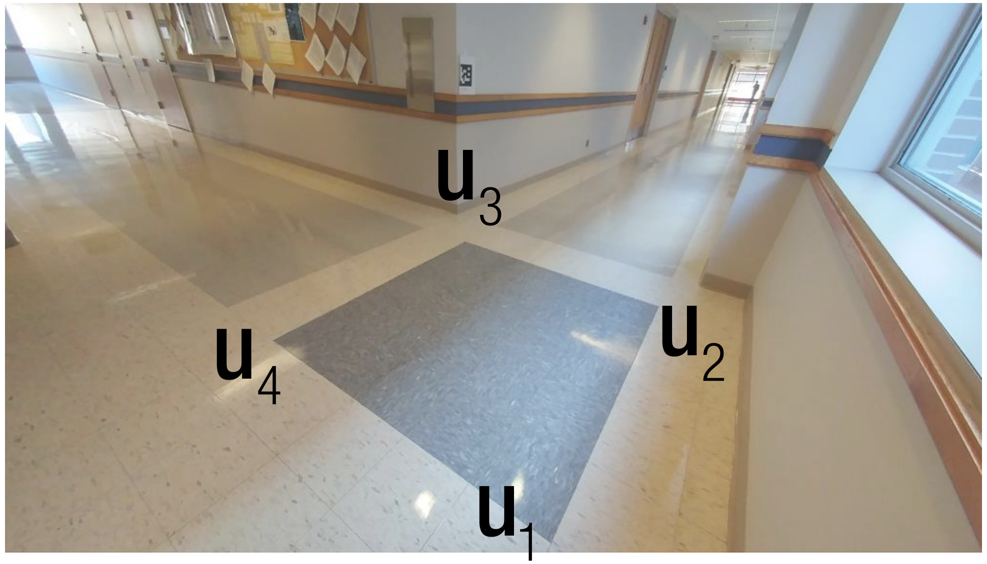
Fun with Homography



RectificationViaHomography.m

```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, inv(H));
```

Fun with Homography



Cf) ImageWarpingEuclidean.m

```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
```

RectificationViaHomography.m

```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, inv(H));
```

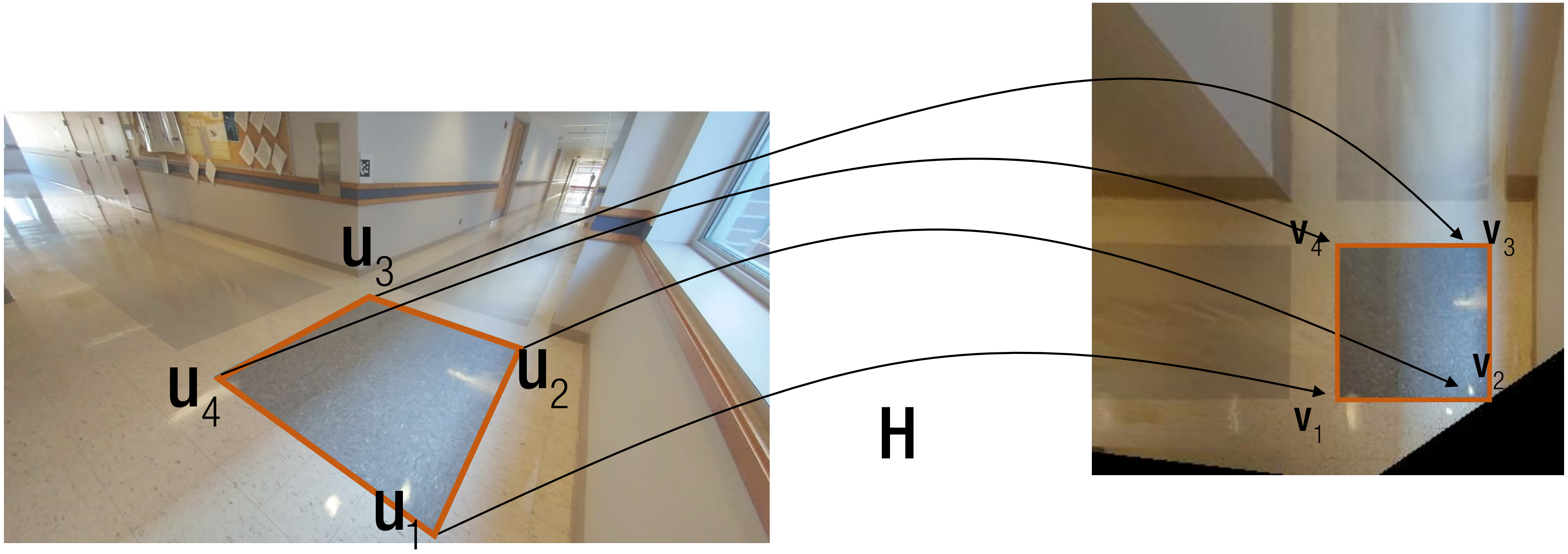
ImageWarping.m

```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
u_z = H(3,1)*v_x + H(3,2)*v_y + H(3,3);

u_x = u_x./u_z;
u_y = u_y./u_z;
```

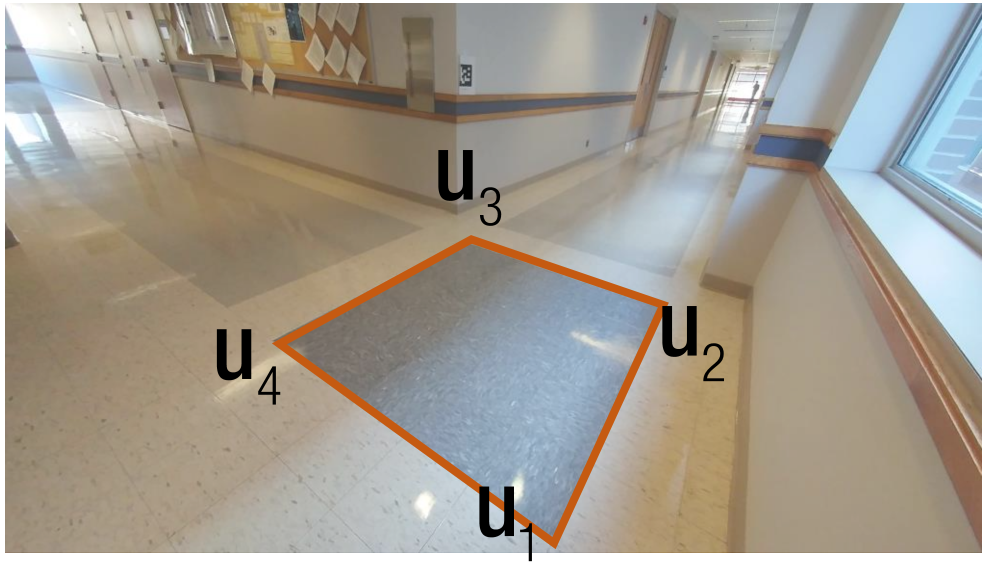
$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \mathbf{H}^{-1} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

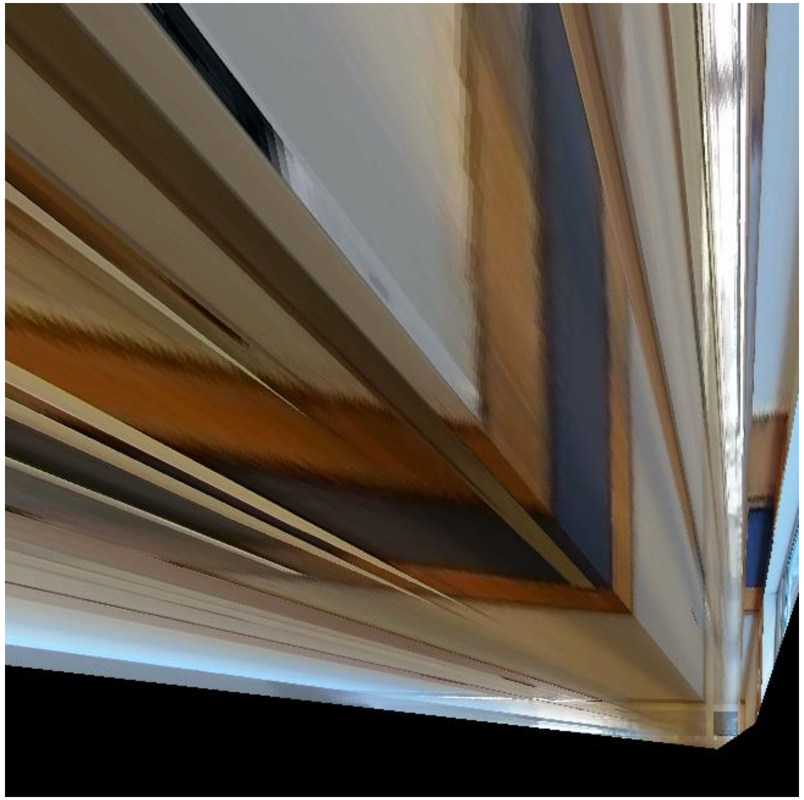
```
im_warped(:,:,1) = reshape(interp2(im(:,:,1), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,2) = reshape(interp2(im(:,:,2), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,3) = reshape(interp2(im(:,:,3), u_x(:), u_y(:)), [h, w]);

im_warped = uint8(im_warped);
```

Fun with Homography



Fun with Homography





Feature Matching


u

v

Local Patch

Local Patch (Orientation)

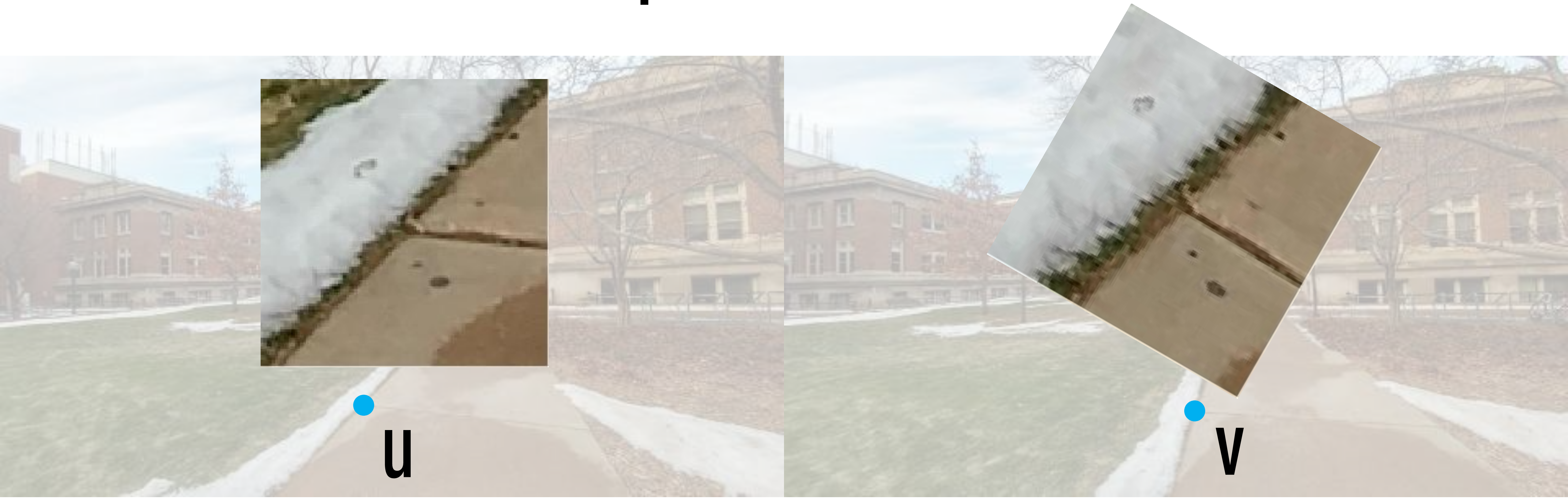
Local Patch (Scale)

Local Visual Descriptor

Desired properties:

- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.

Local Visual Descriptor



Desired properties:

- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.
- Orientation aware

Local Scale Invariant Feature Transform (SIFT)

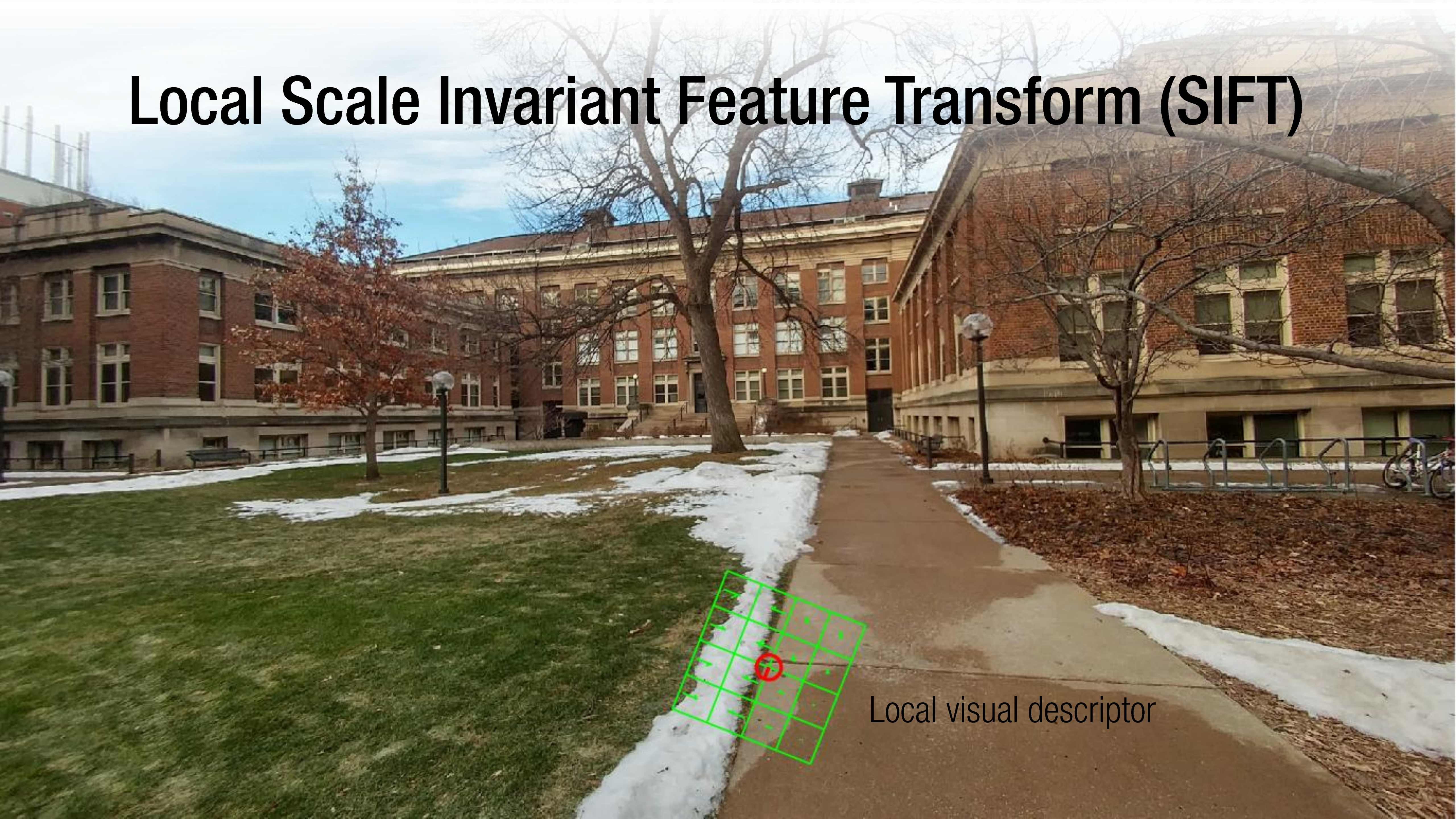
SIFT automatically finds the optimal scale of feature point and its orientation.

Desired properties:

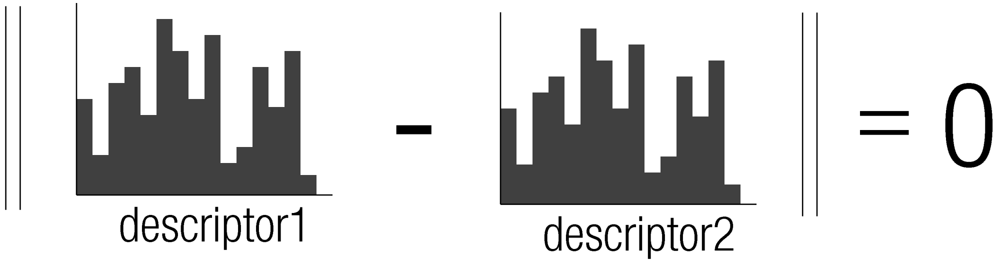
- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.
- Orientation aware

Local Scale Invariant Feature Transform (SIFT)

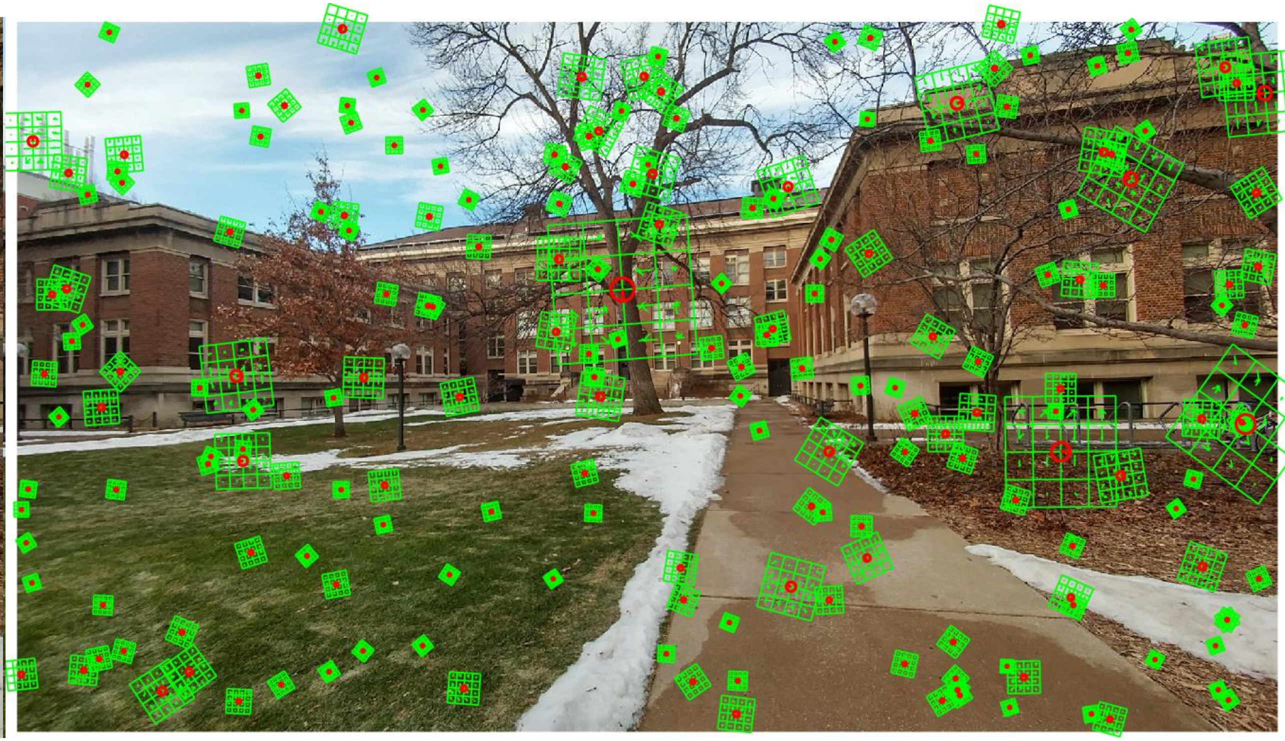
Local Scale Invariant Feature Transform (SIFT)



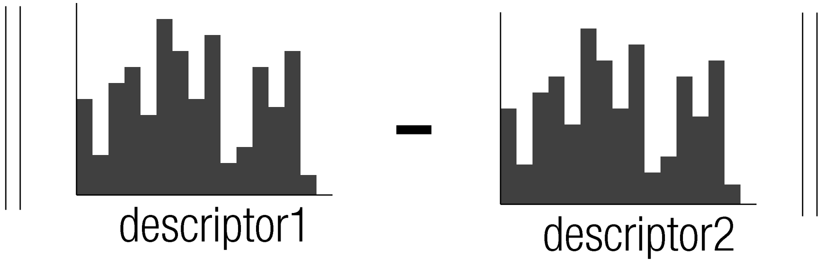
Local Scale Invariant Feature Transform (SIFT)



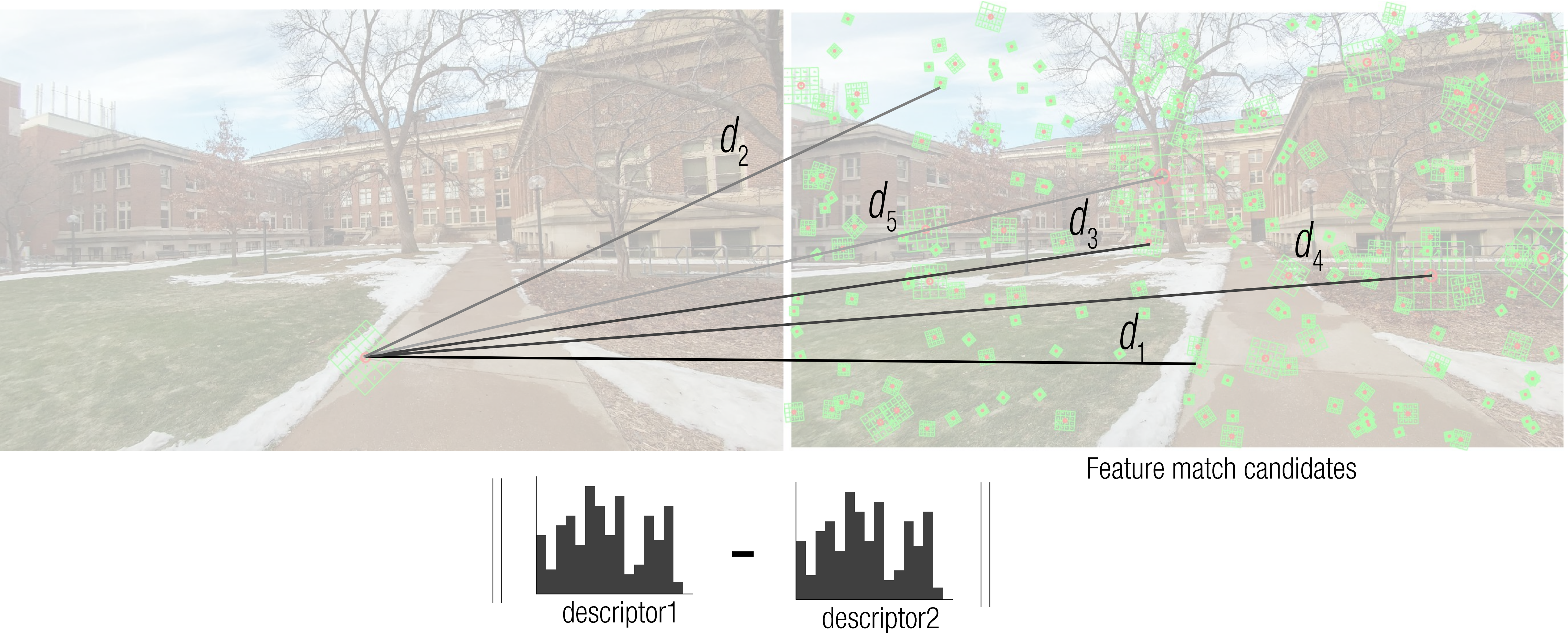
Local Scale Invariant Feature Transform (SIFT)



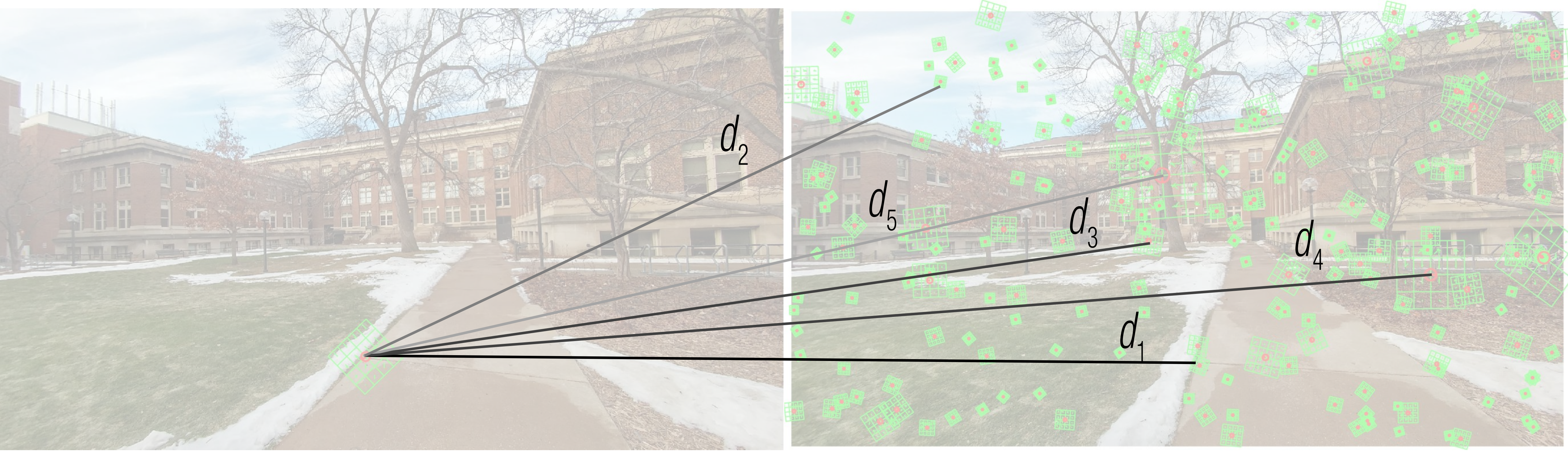
Feature match candidates



Nearest Neighbor Search



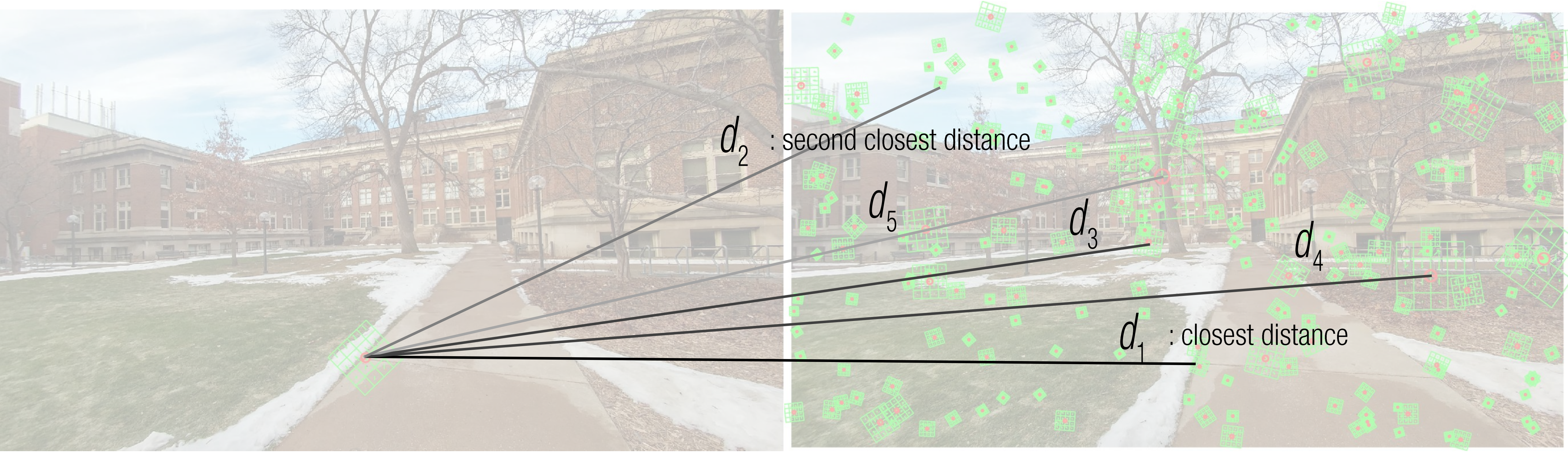
Nearest Neighbor Search



Feature match candidates

Discriminativity: how is the feature point unique?

Nearest Neighbor Search w/ Ratio Test

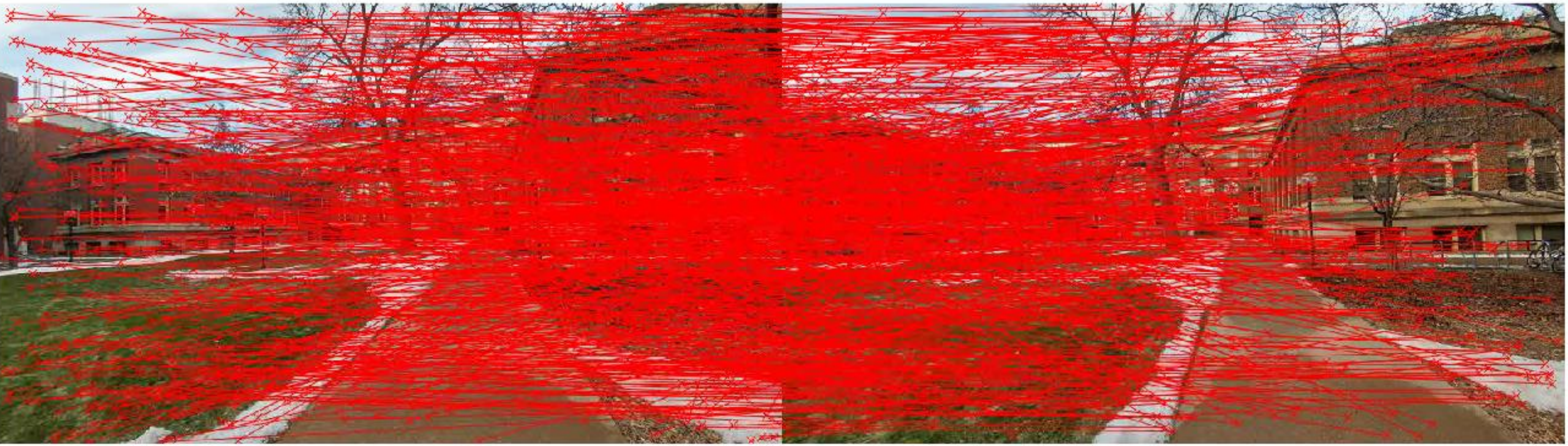


Feature match candidates

Discriminativity: how is the feature point unique?

$$\frac{d_1}{d_2} < 0.7$$

Nearest Neighbor Search w/o Ratio Test

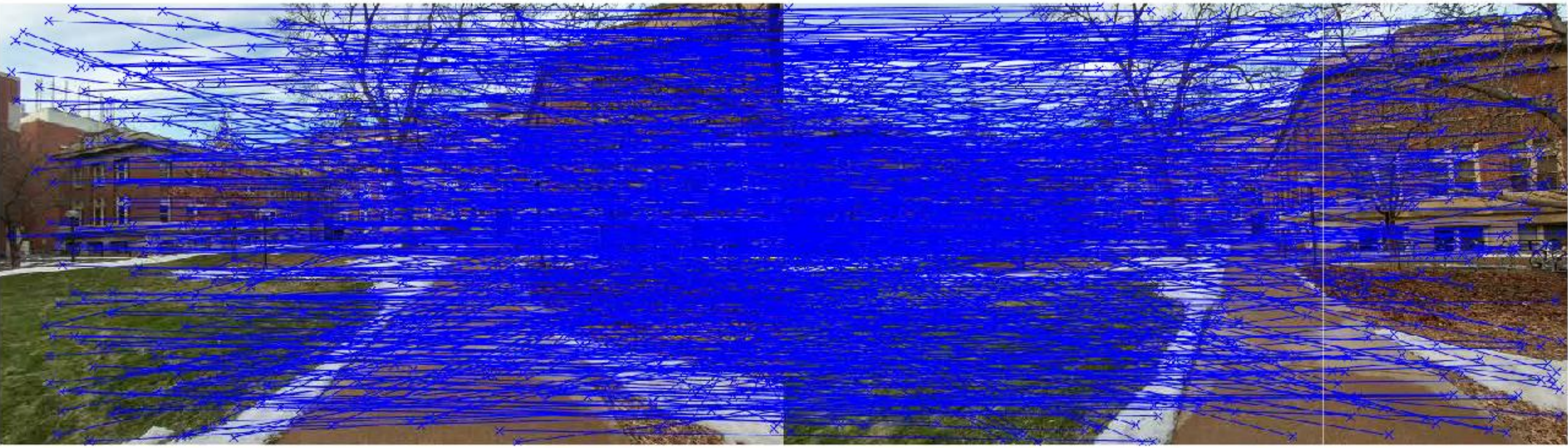


Left image→right image

Nearest Neighbor Search w/ Ratio Test

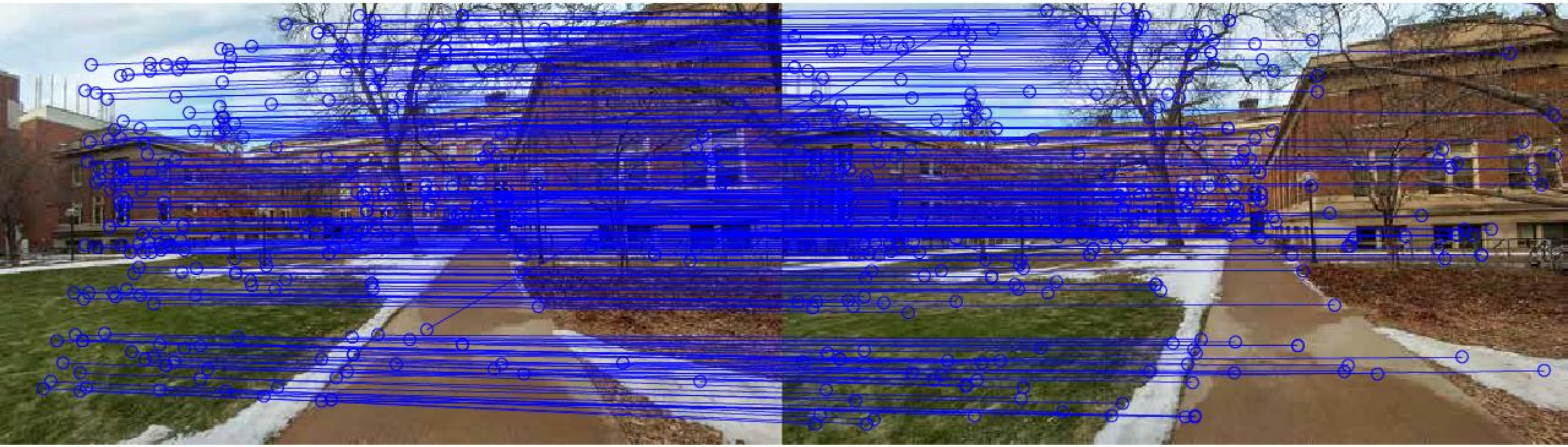
Left image→right image

Nearest Neighbor Search w/o Ratio Test



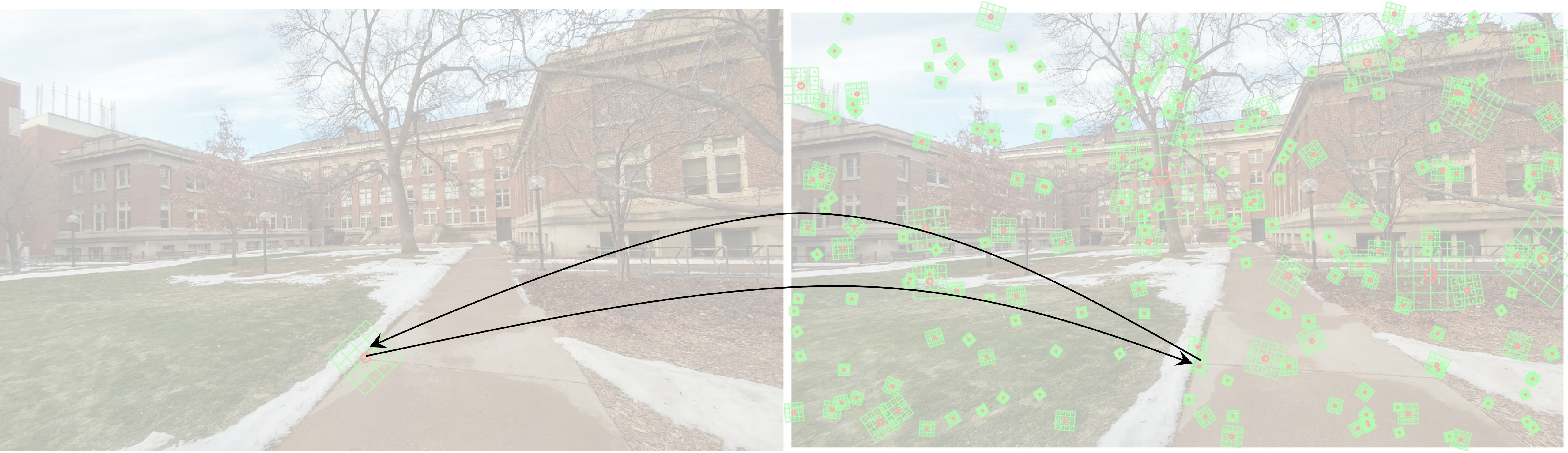
Left image←right image

Nearest Neighbor Search w/ Ratio Test



Left image←right image

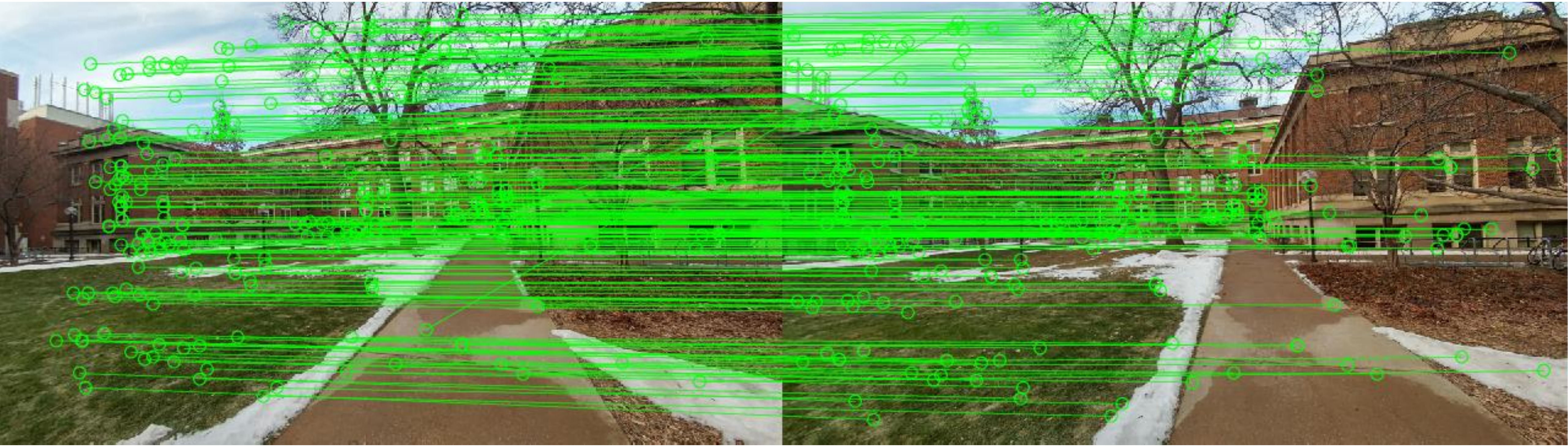
Bi-directional Consistency Check



Feature match candidates

Consistency: would a feature match correspond to each other?

Bi-directional Consistency Check

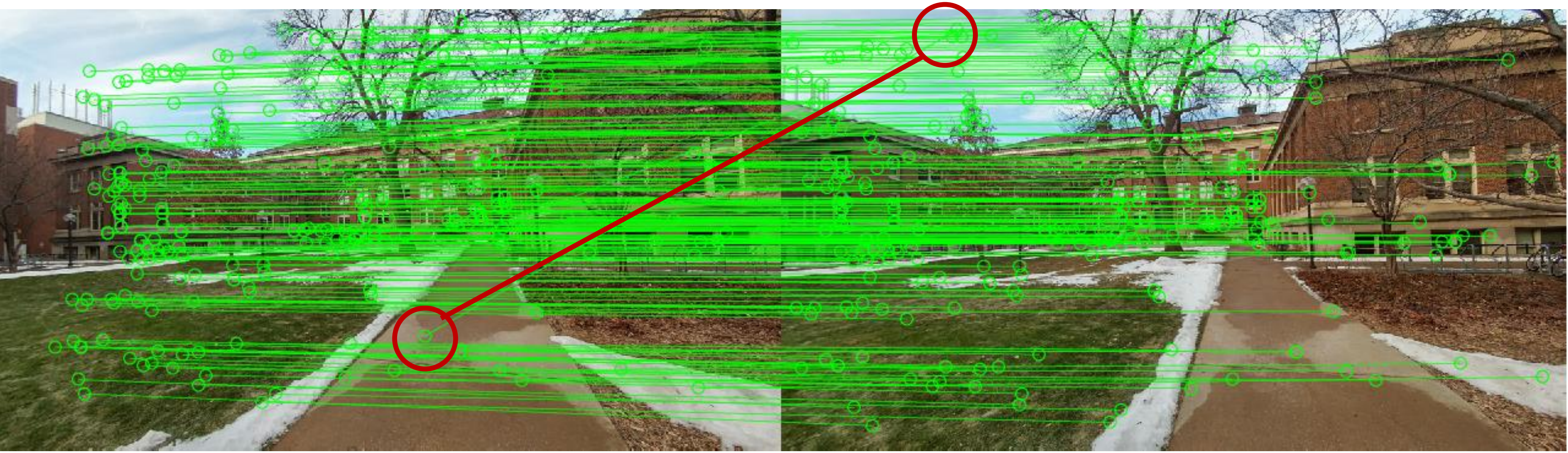


RANSAC: Random Sample Consensus: Linear Least Squares

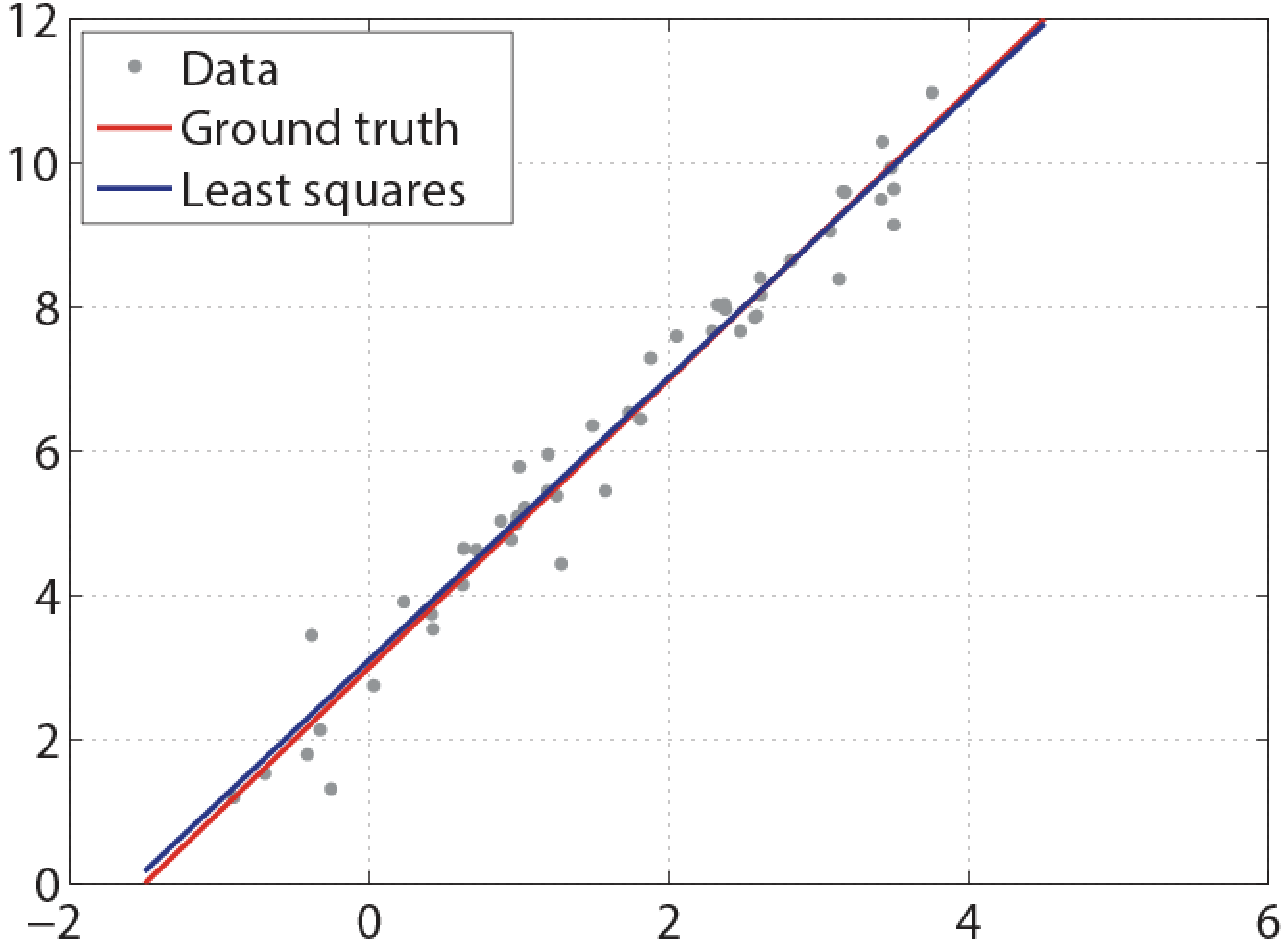
$$\underbrace{\begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & 1 & -u_xv_y & -u_yv_y & -v_y \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

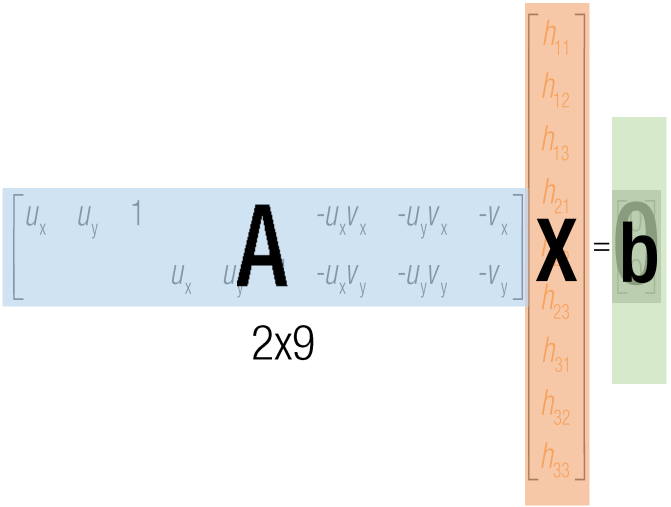
2x9

Outlier?

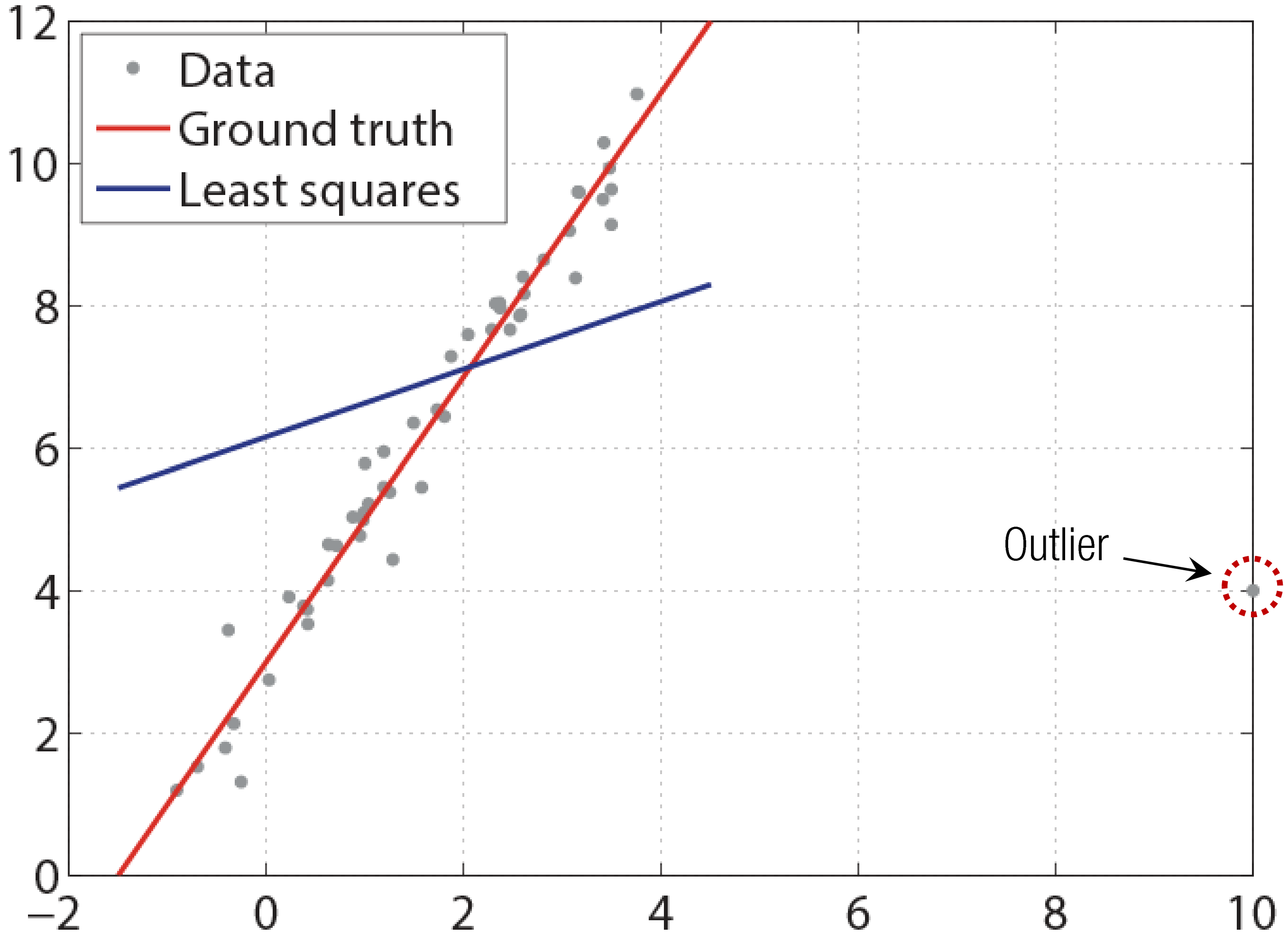


Recall: Line Fitting (Ax=b)

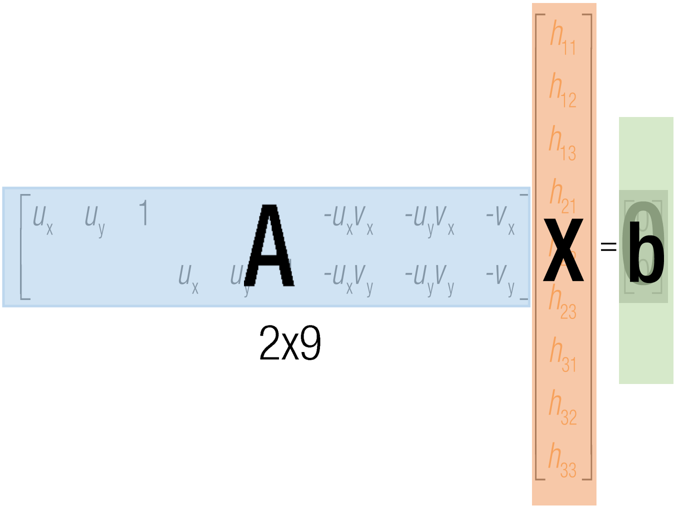




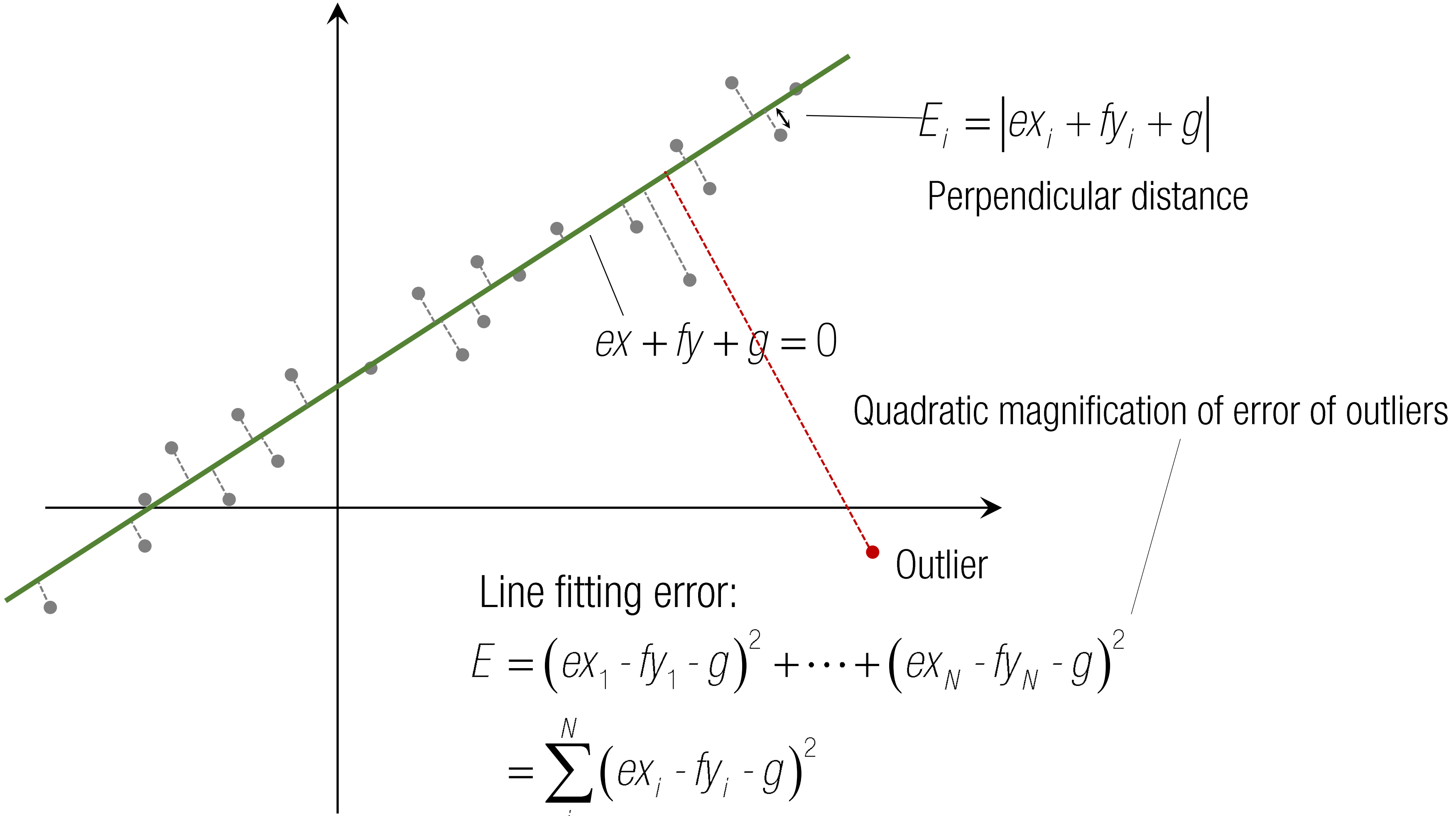
Outlier



Data
Ground truth
Least squares
Outlier
12
10
8
6
4
2
0
−2
0
2
4
6
8
10



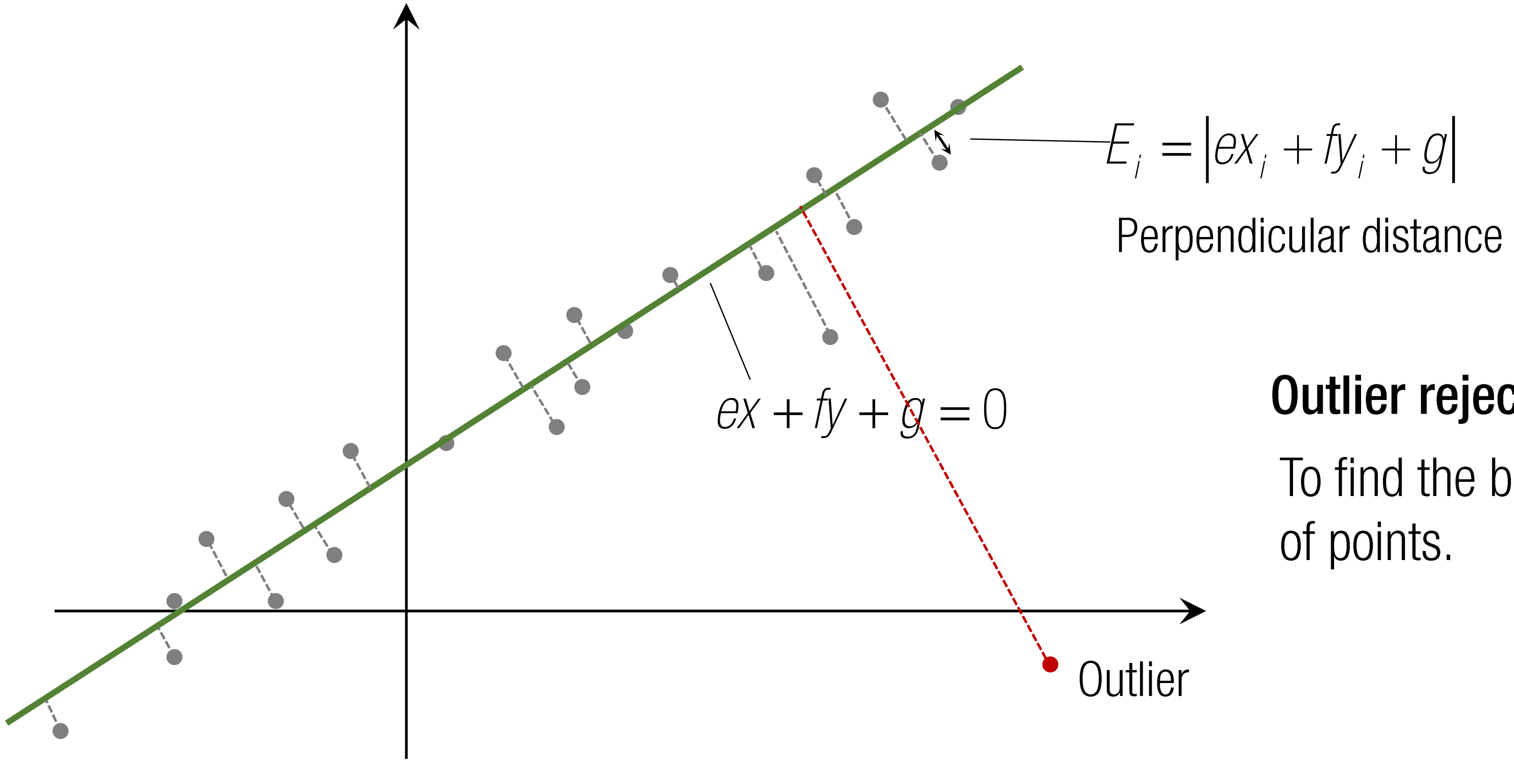
A
X
=
b
2x9



Line fitting error:

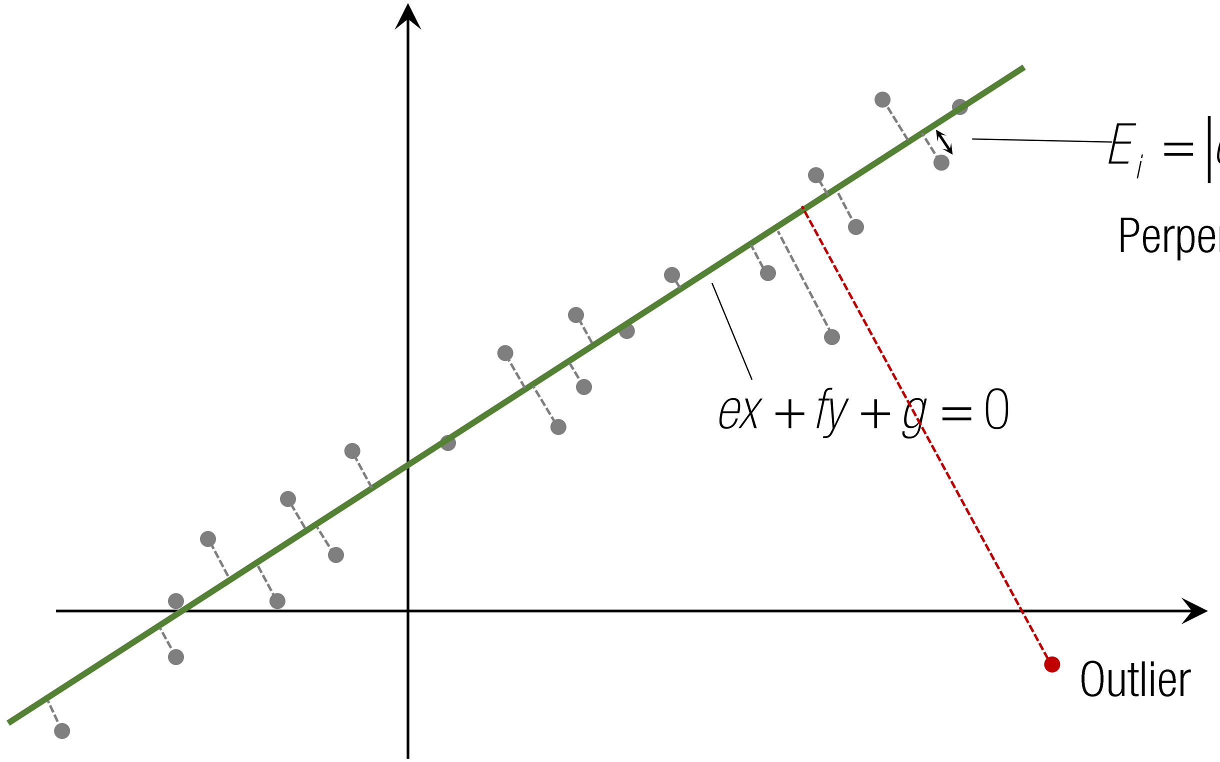
$$E = (ex_1 - fy_1 - g)^2 + \cdots + (ex_N - fy_N - g)^2$$

$$= \sum_{i}^{N} (ex_i - fy_i - g)^2$$



Outlier rejection strategy:

To find the best line that explanes the <u>maximum</u> number of points.



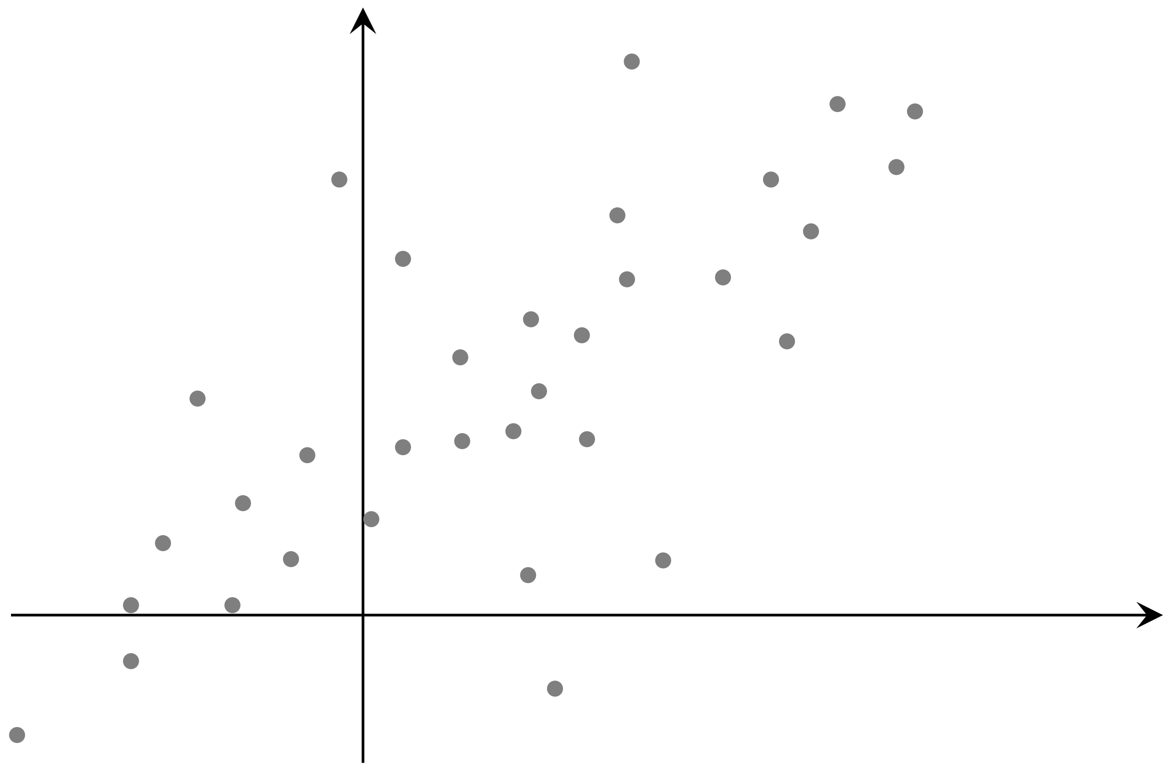
Outlier rejection strategy:

To find the best line that explanes the maximum number of points.

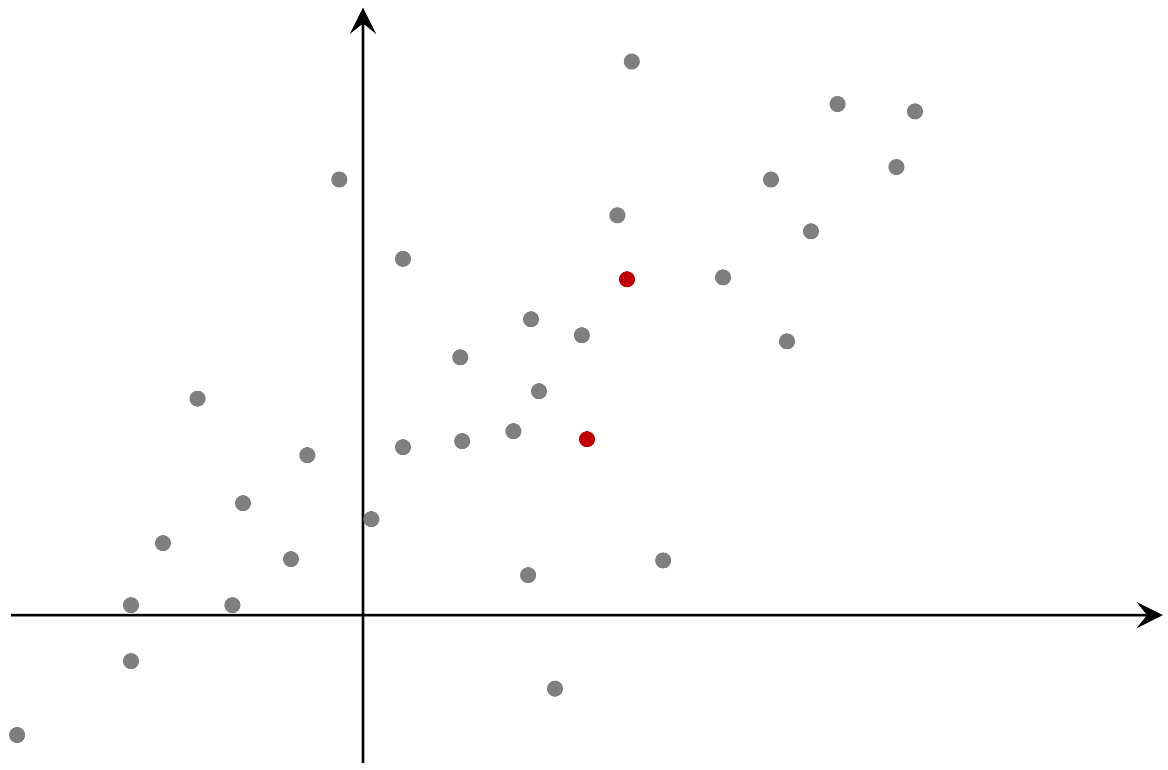
Assumptions:

1. Majority of good samples agree with the underlying model (good apples are same and simple.).

2. Bad samples does not consistently agree with a single model
(all bad apples are different and complicated.).

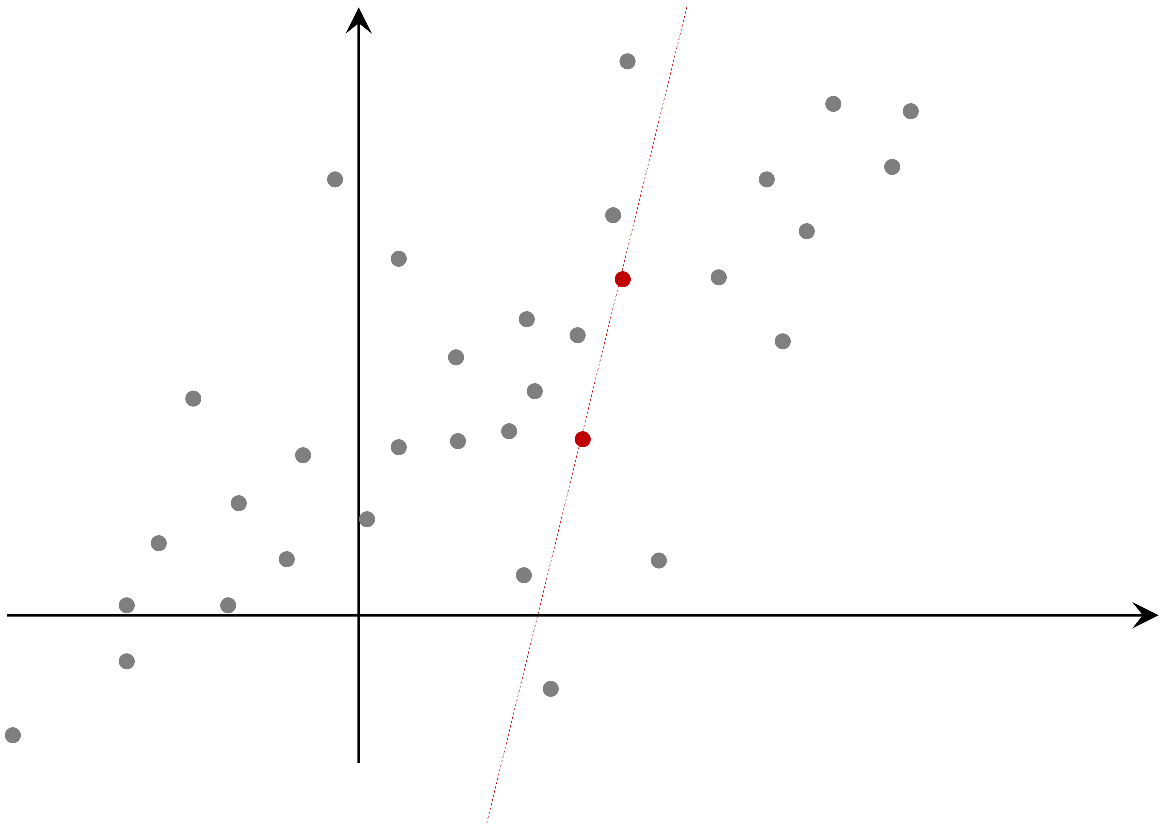


RANSAC: Random Sample Consensus



1. Random sampling

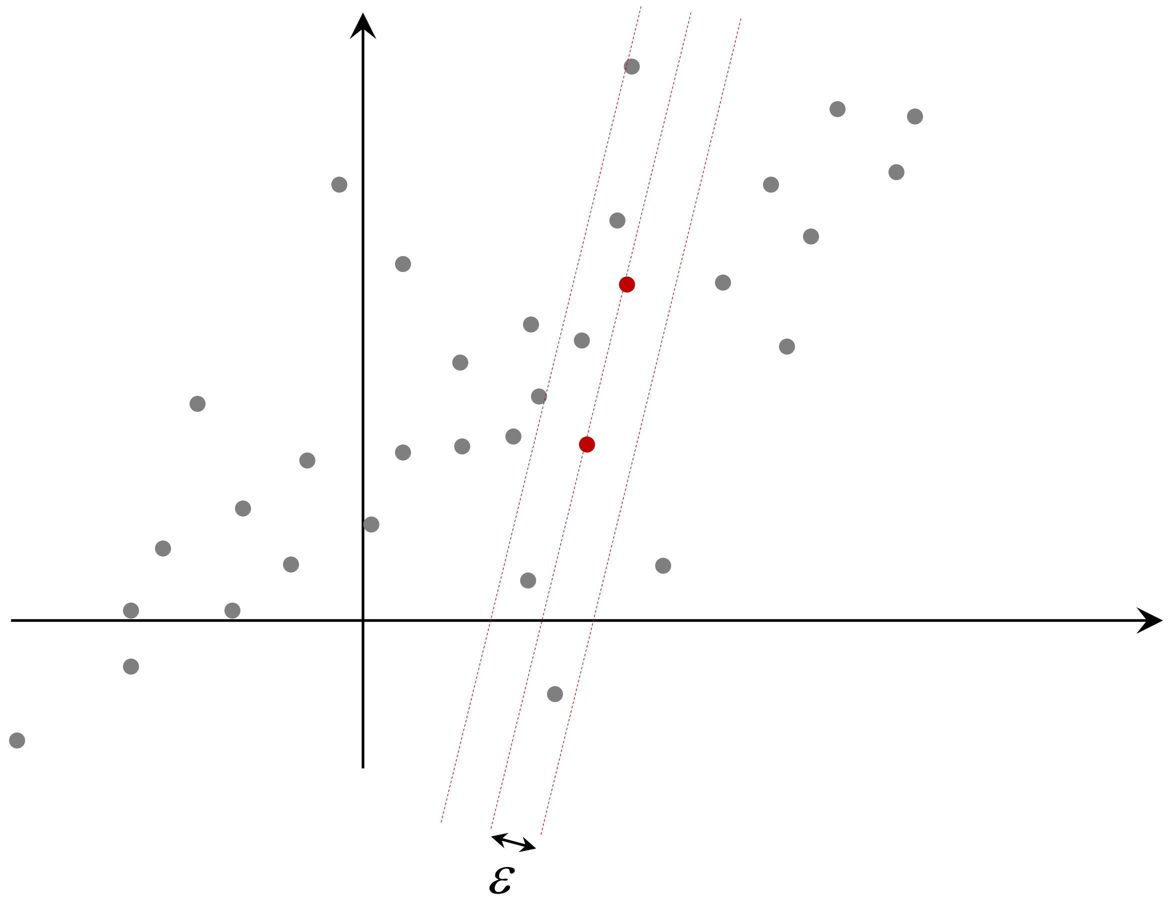
RANSAC: Random Sample Consensus



1. Random sampling

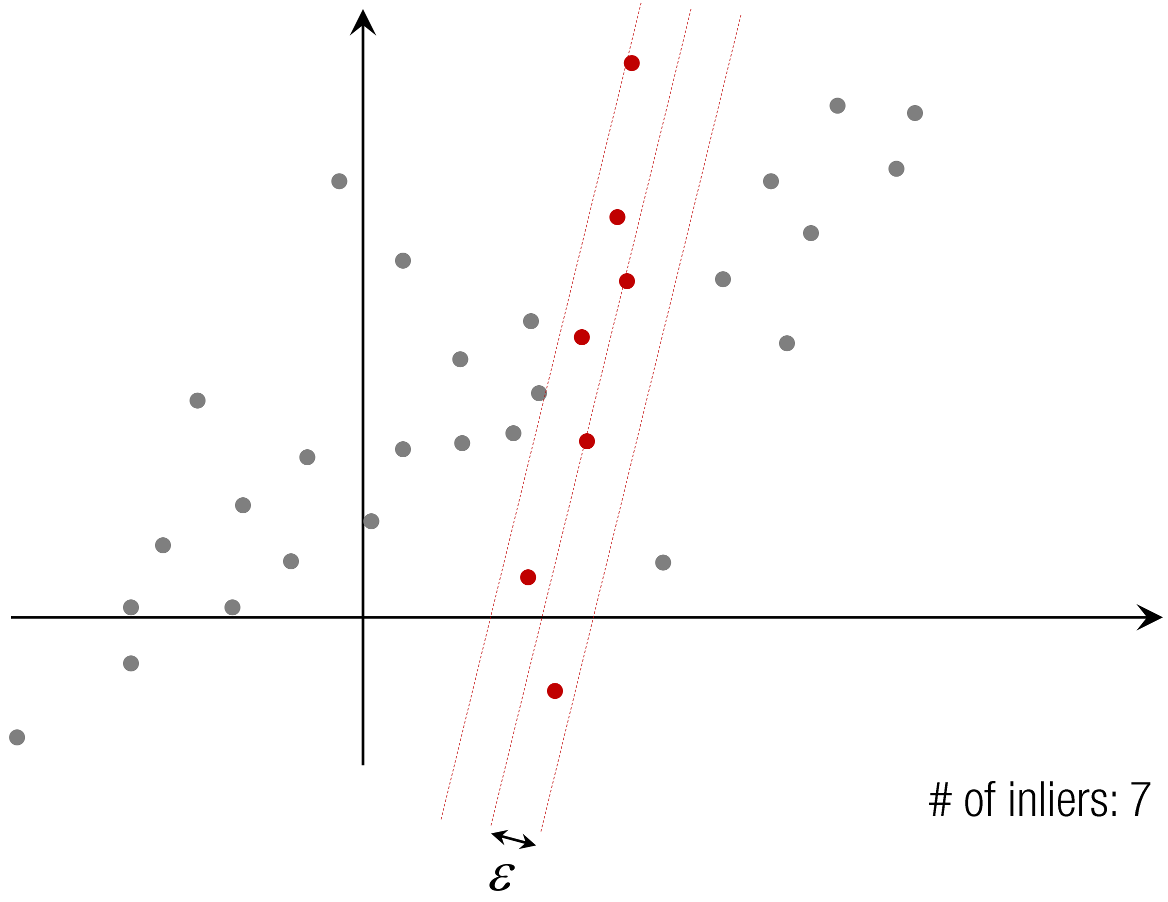
2. Model building

RANSAC: Random Sample Consensus



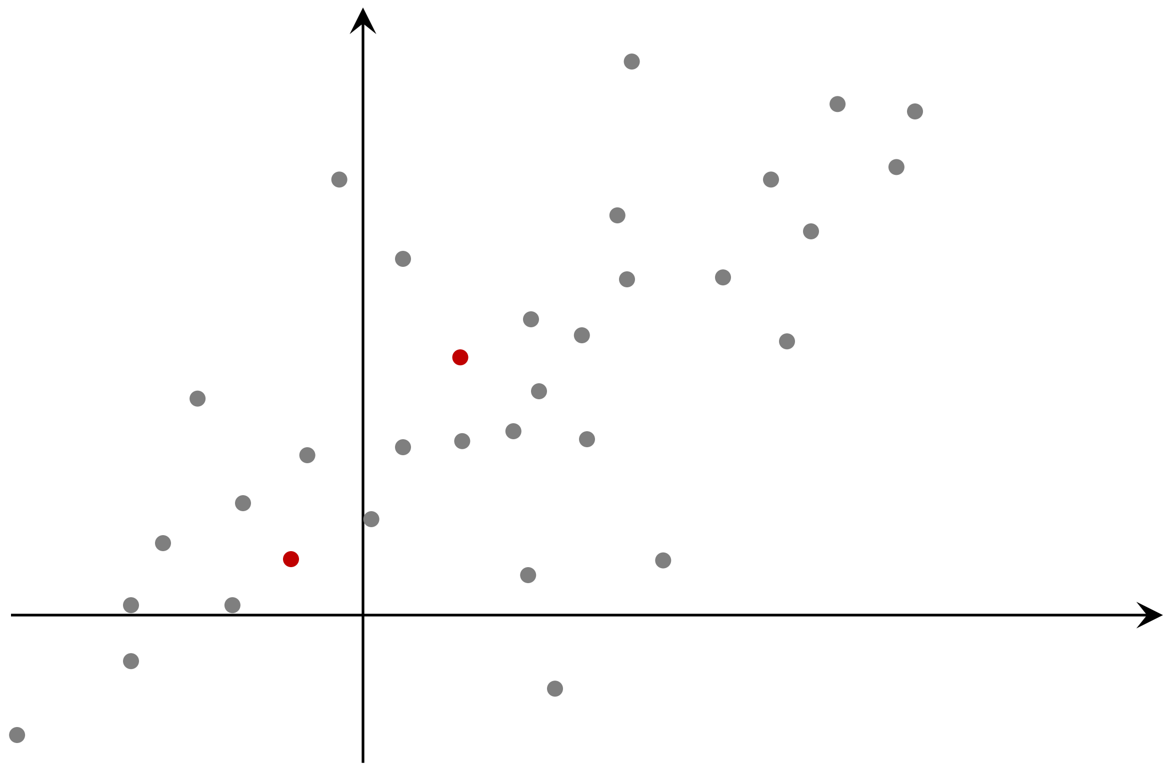
1. Random sampling
2. Model building
3. Thresholding

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
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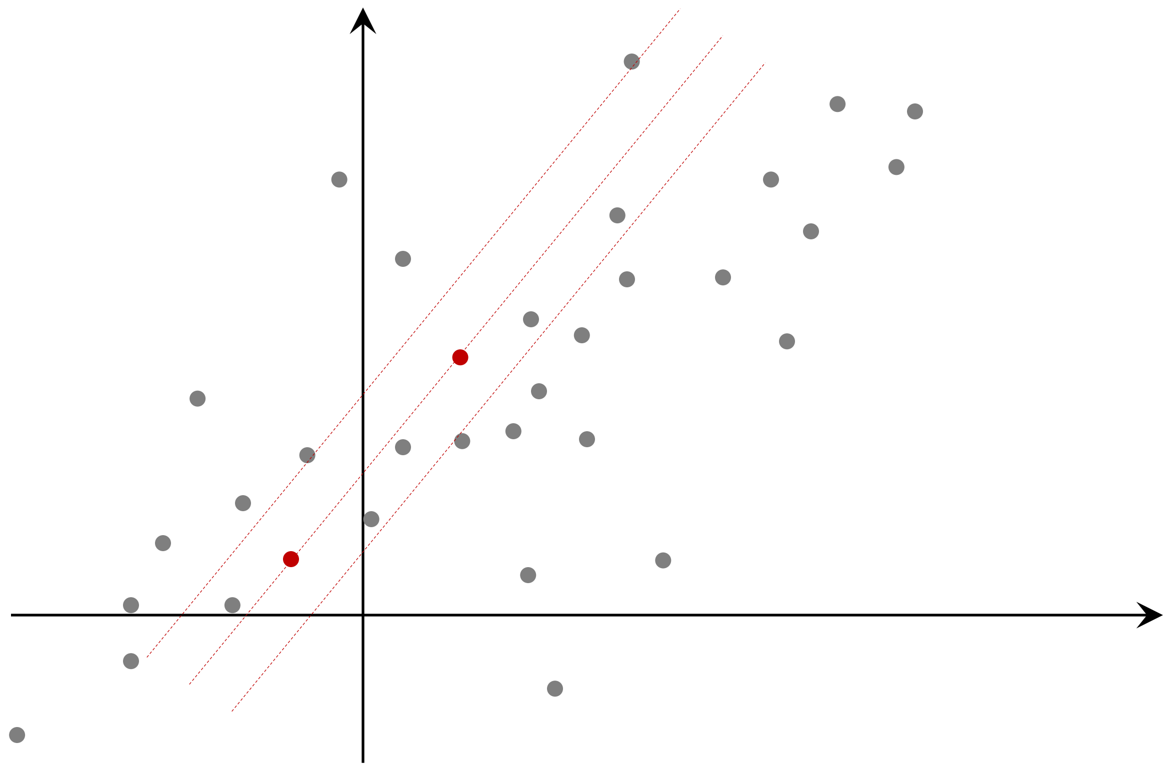
RANSAC: Random Sample Consensus



1. Random sampling
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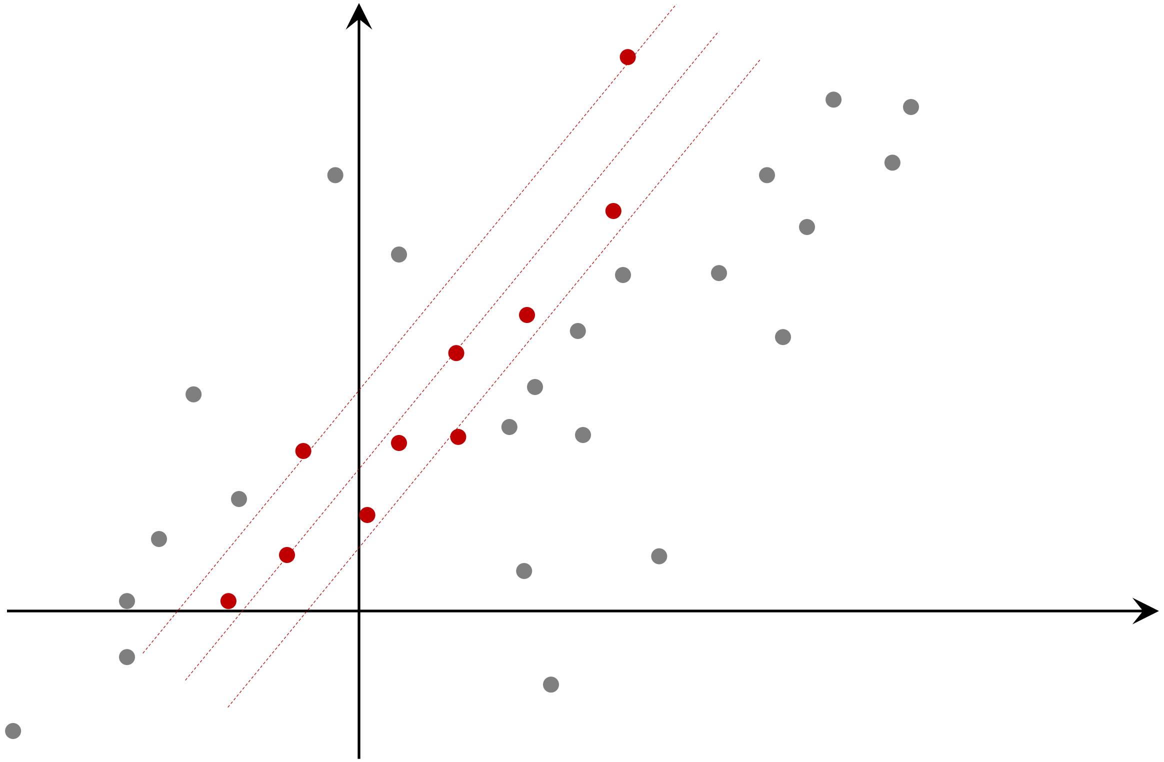
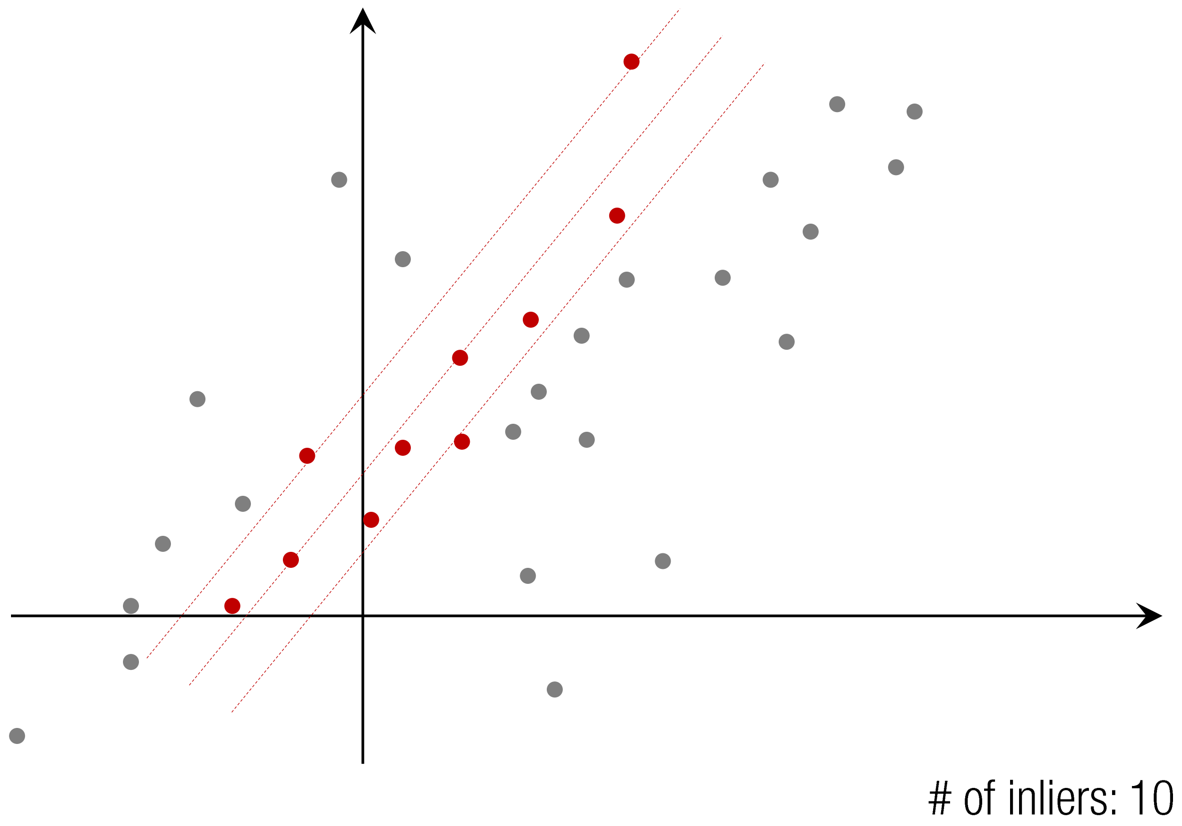
4. Inlier counting

RANSAC: Random Sample Consensus



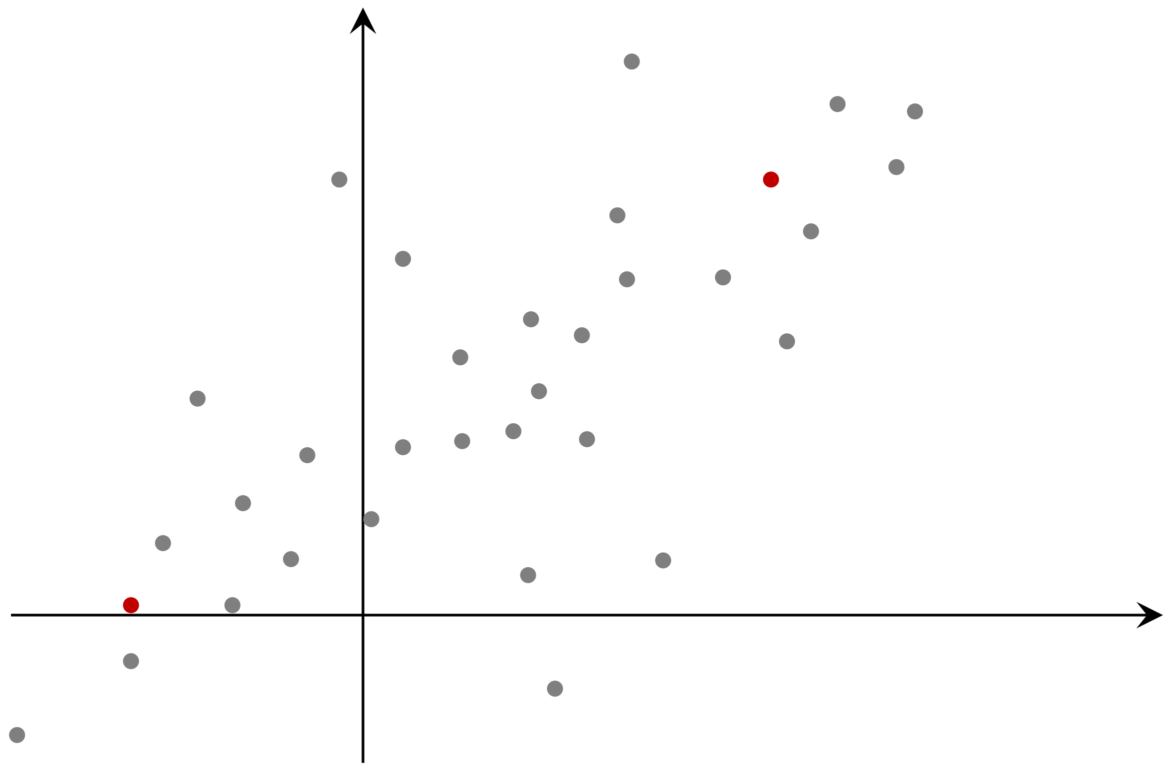
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
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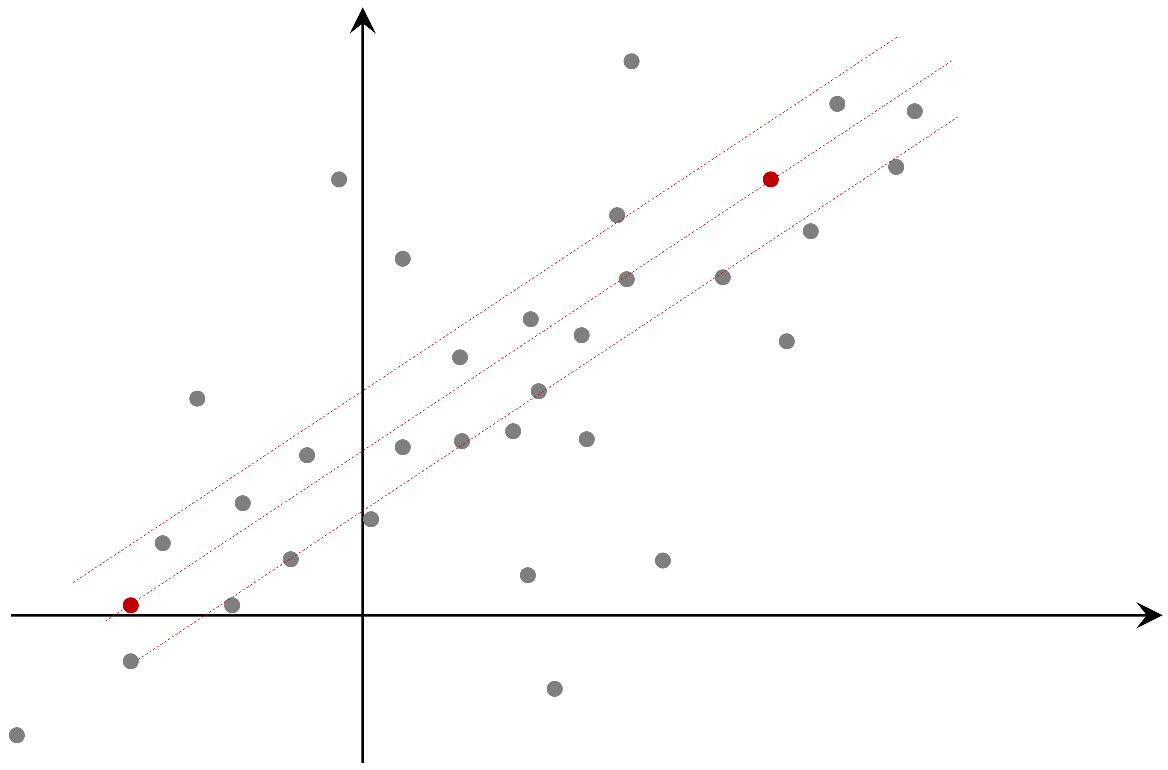
RANSAC: Random Sample Consensus



1. Random sampling
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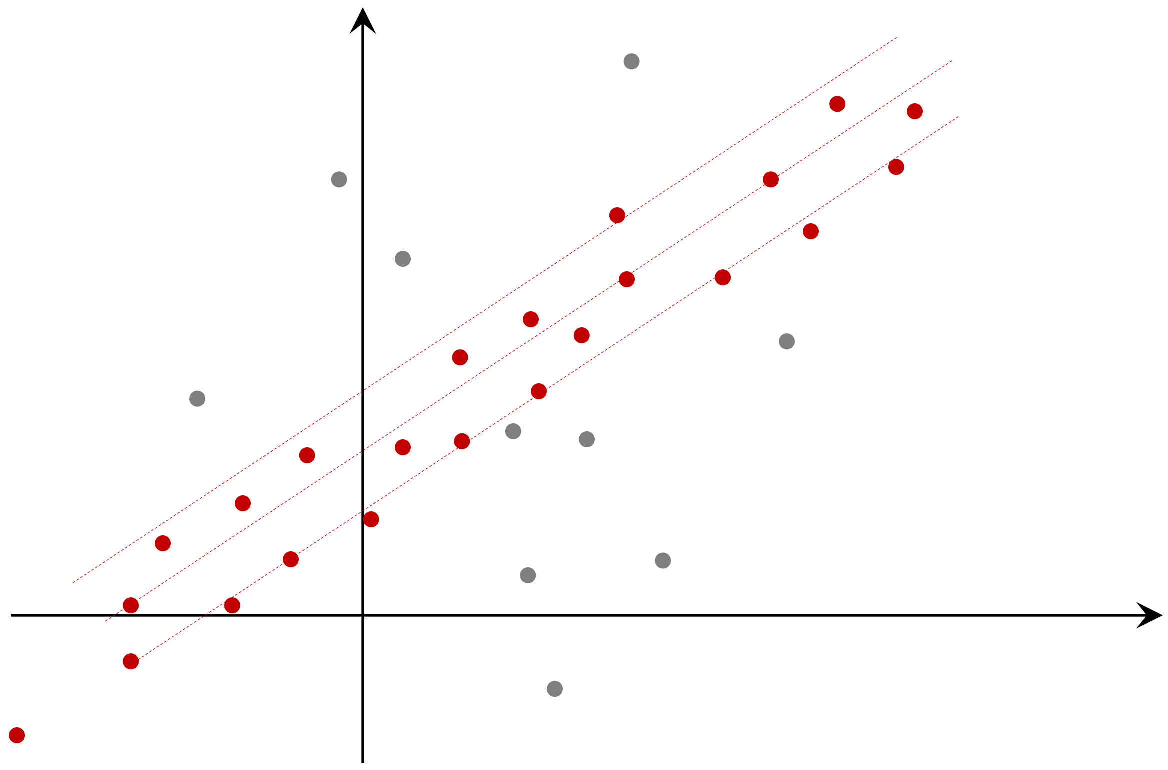
RANSAC: Random Sample Consensus



1. Random sampling
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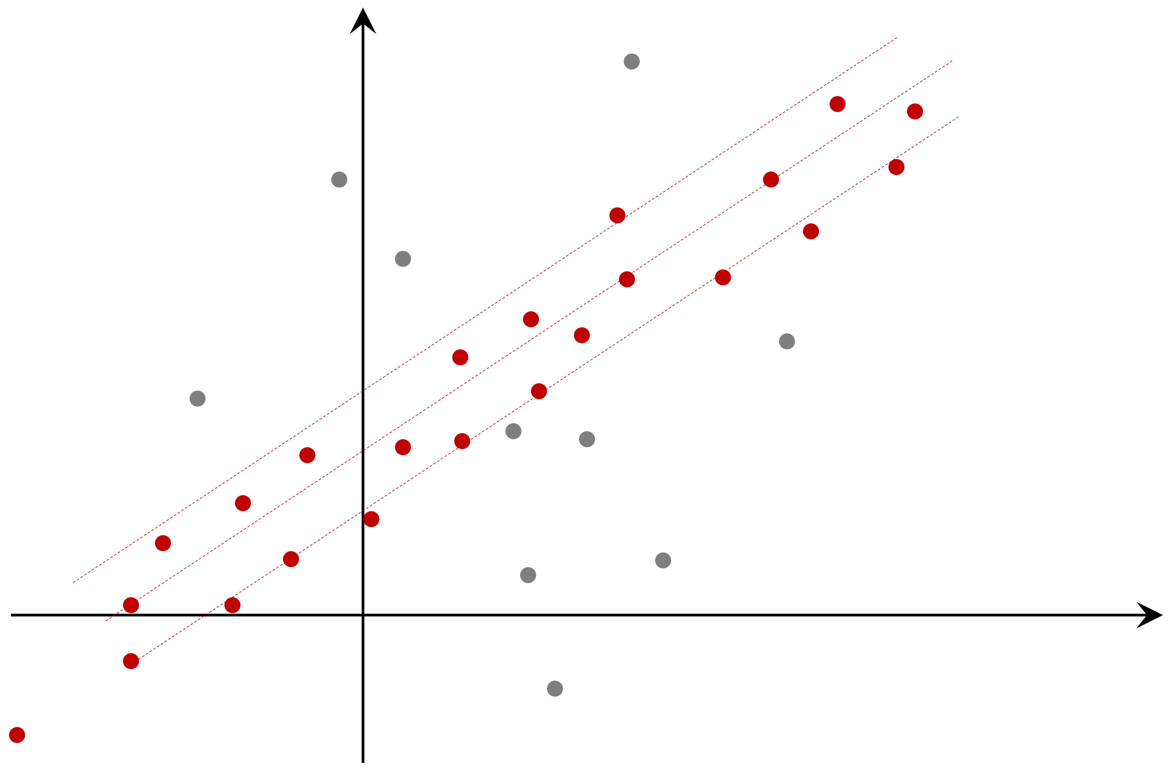
RANSAC: Random Sample Consensus

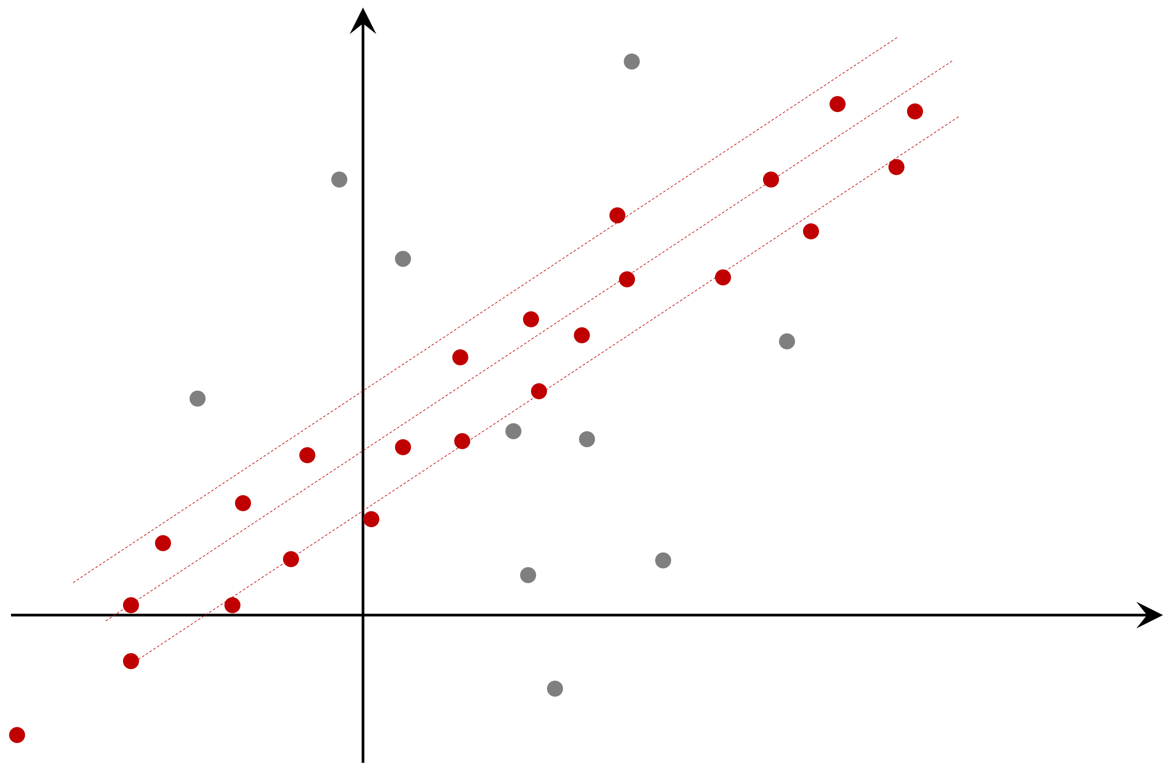


1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

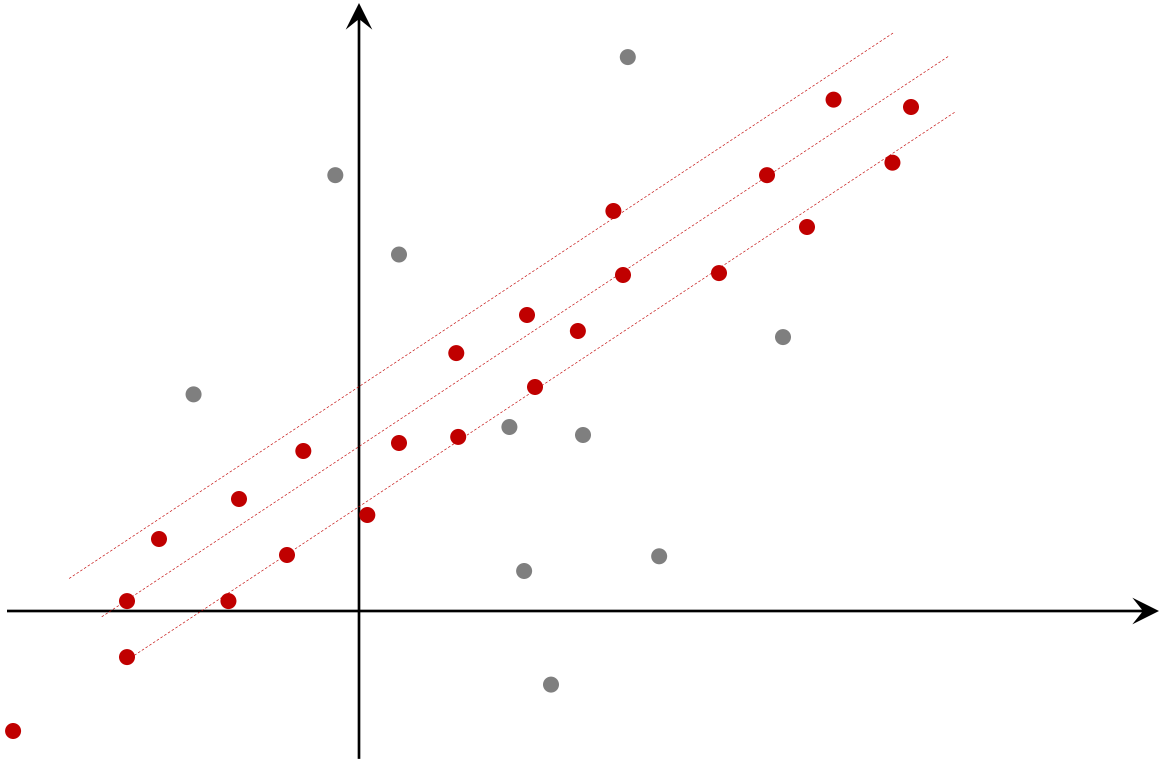
# of inliers: 23

Maximum number of inliers

RANSAC: Random Sample Consensus

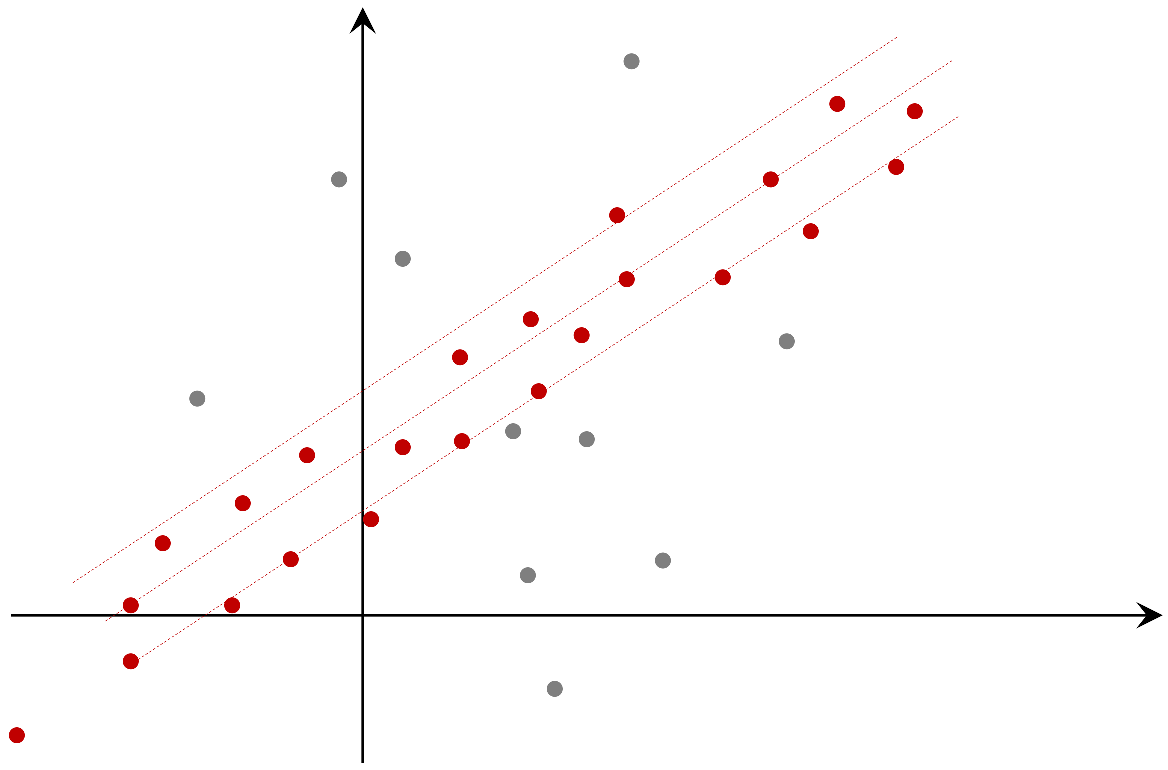


Required number of iterations with p success rate:



Required number of iterations with *p* success rate:

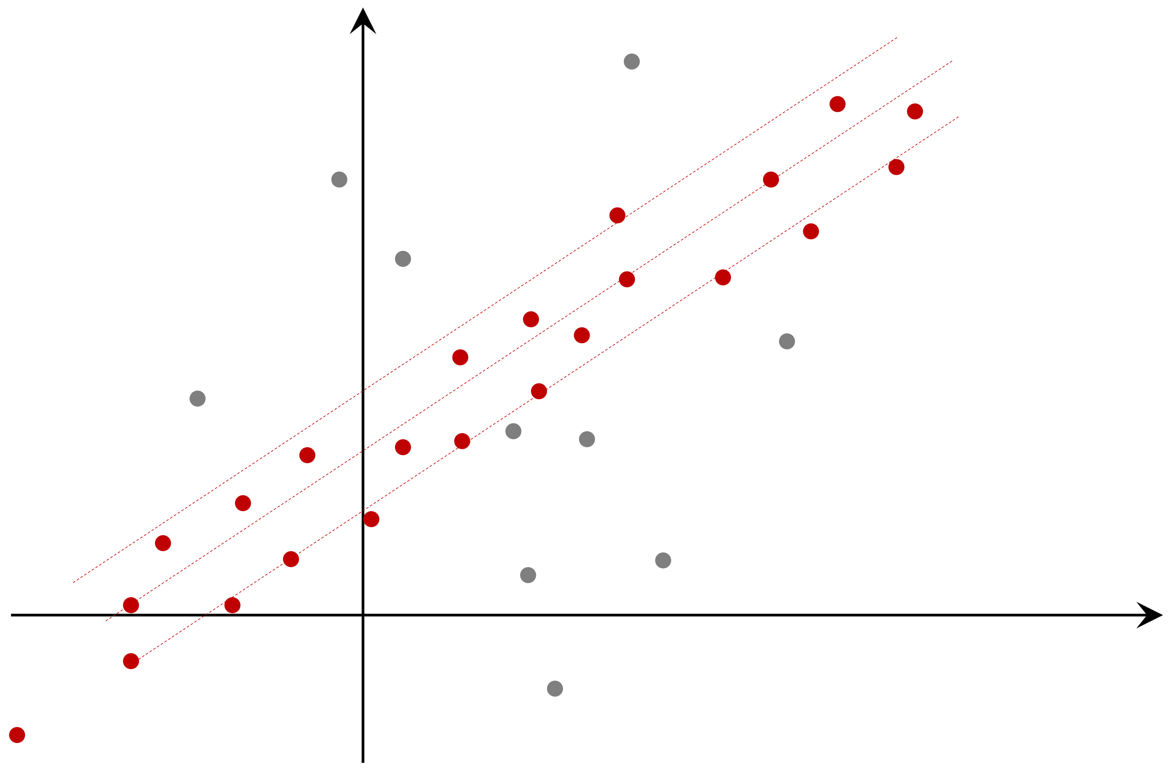
Probability of choosing an inlier: $w = \frac{\text{\# of inliers}}{\text{\# of samples}}$



Required number of iterations with *p* success rate:

Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.



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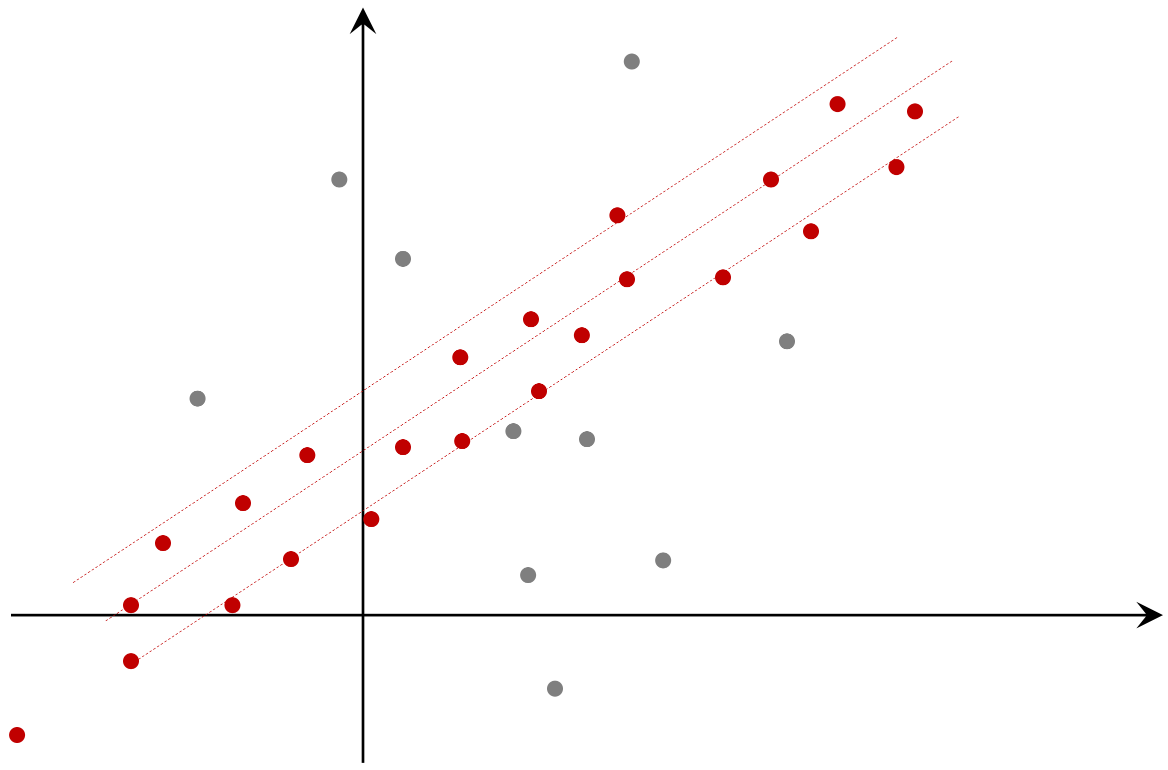
Probability of not building a correct model during *k* iterations: $\left(1 - w^n\right)^k$



Required number of iterations with p success rate:

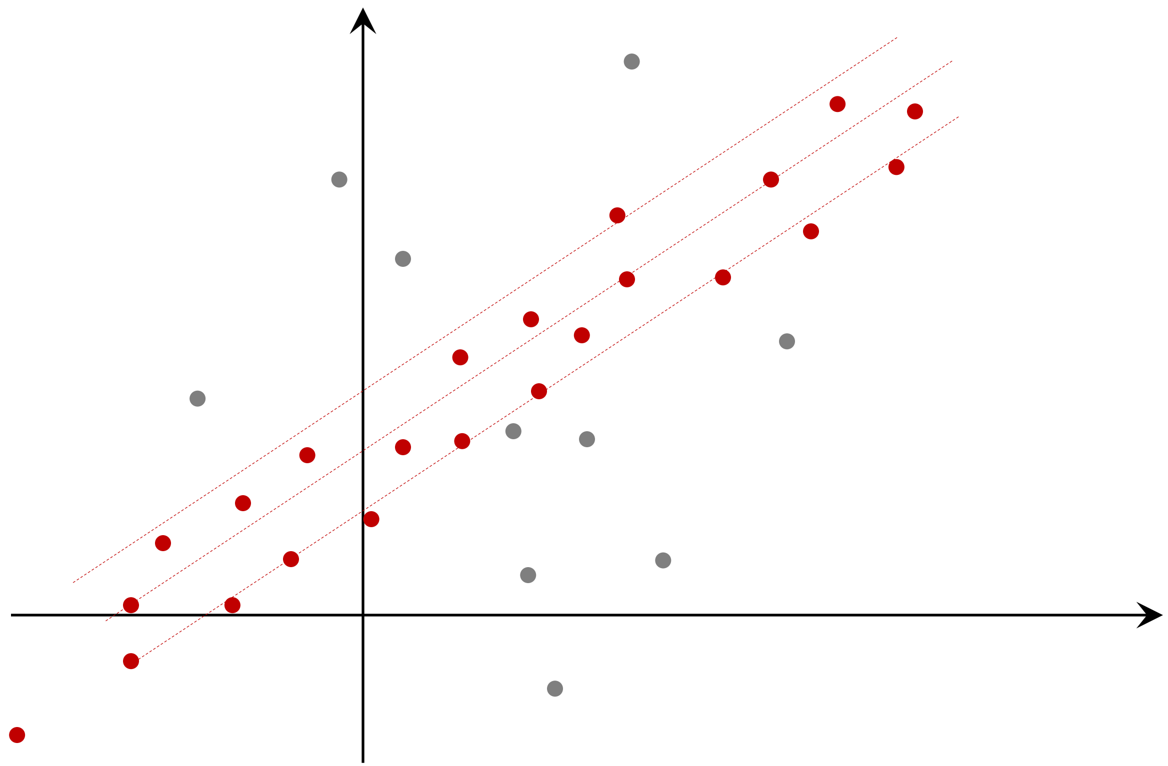
Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during k iterations: $\left(1-w^n\right)^k$

$\left(1-w^n\right)^k = 1-p$ where p is desired RANSAC success rate.

$$k = \frac{\log\left(1-p\right)}{\log\left(1-w^n\right)}$$



Required number of iterations with *p* success rate:

$$k = \frac{\log(1-p)}{\log(1-w^n)} \qquad \text{where} \quad w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$$

Probability of choosing an inlier: $$w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during *k* iterations: $(1-w^n)^k$

$(1-w^n)^k = 1-p$ where *p* is desired RANSAC success rate.

$$k = \frac{\log(1-p)}{\log(1-w^n)}$$

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{ \begin{matrix} \mathbf{v}_1 \leftrightarrow \mathbf{u}_1 \\ \mathbf{v}_2 \leftrightarrow \mathbf{u}_2 \\ \mathbf{v}_3 \leftrightarrow \mathbf{u}_3 \\ \mathbf{v}_4 \leftrightarrow \mathbf{u}_4 \end{matrix} \right\} \rightarrow \mathbf{H}$$

Homography computation

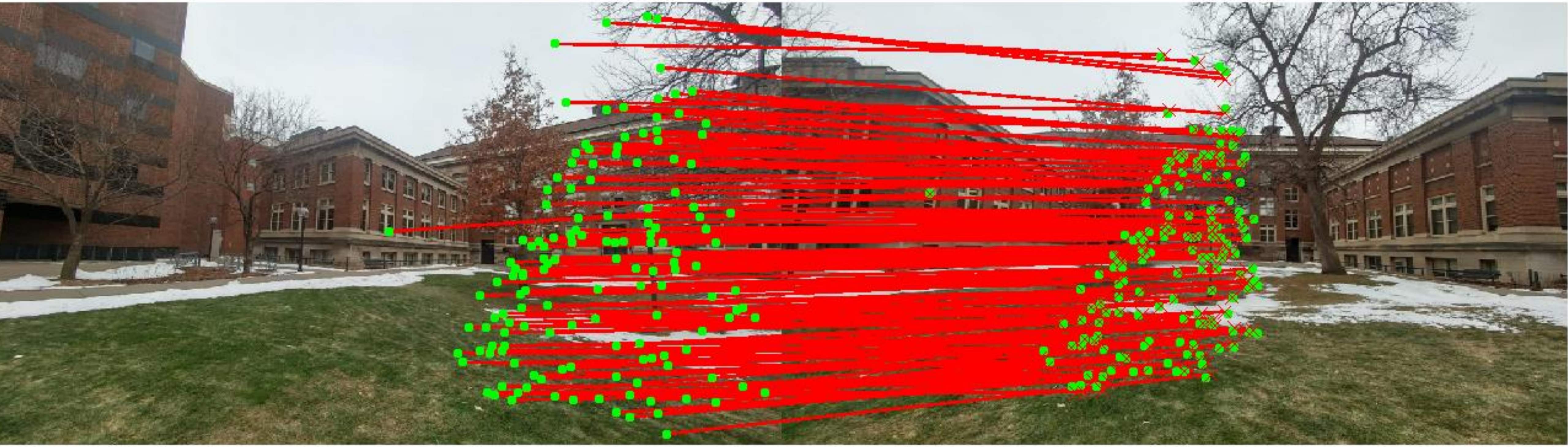
$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{\begin{matrix} \mathbf{v}_1 \leftrightarrow \mathbf{u}_1 \\ \mathbf{v}_2 \leftrightarrow \mathbf{u}_2 \\ \mathbf{v}_3 \leftrightarrow \mathbf{u}_3 \\ \mathbf{v}_4 \leftrightarrow \mathbf{u}_4 \end{matrix}\right\} \rightarrow \mathbf{H} \longrightarrow$$

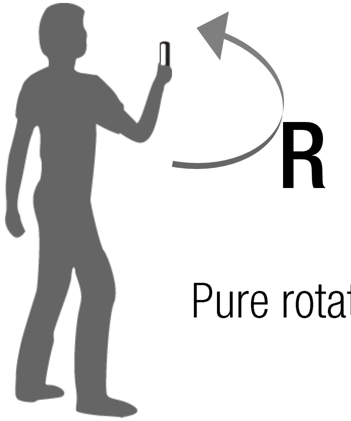
Homography computation

$$\longrightarrow \quad \mathbf{v} = \mathbf{H}\mathbf{u}$$

Inlier evaluation



$\mathcal{I}_1$ $\mathcal{I}_2$



Pure rotation

$$\mathcal{I}_2(\mathbf{v}) = \mathcal{I}_1(\mathbf{Hu})$$

where

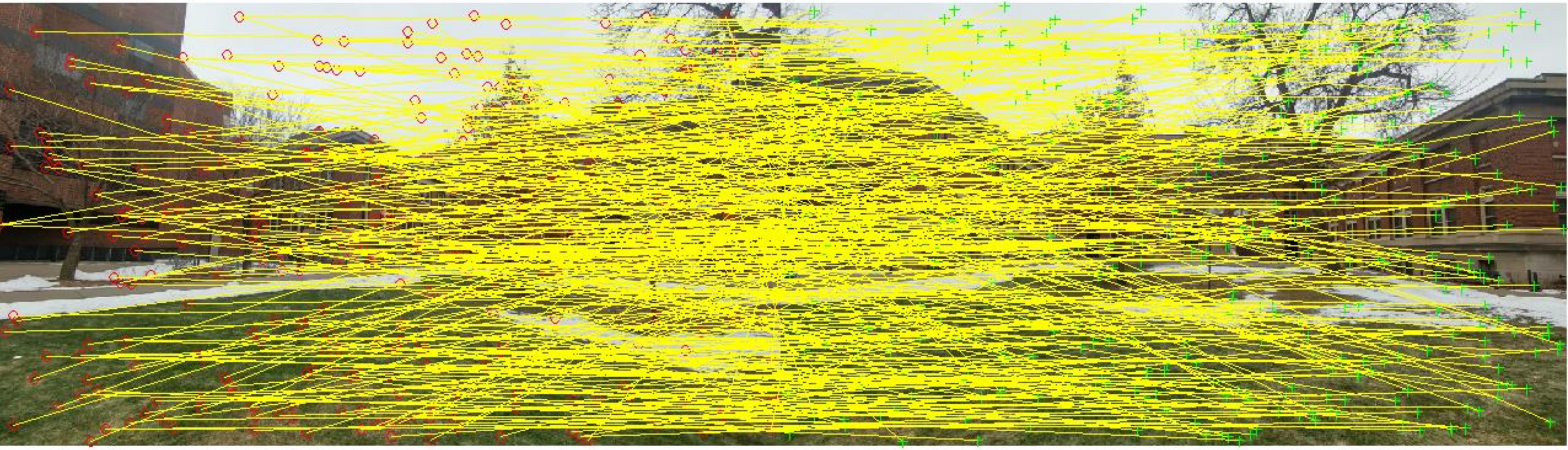
$$\mathbf{v} = \mathbf{Hu}$$

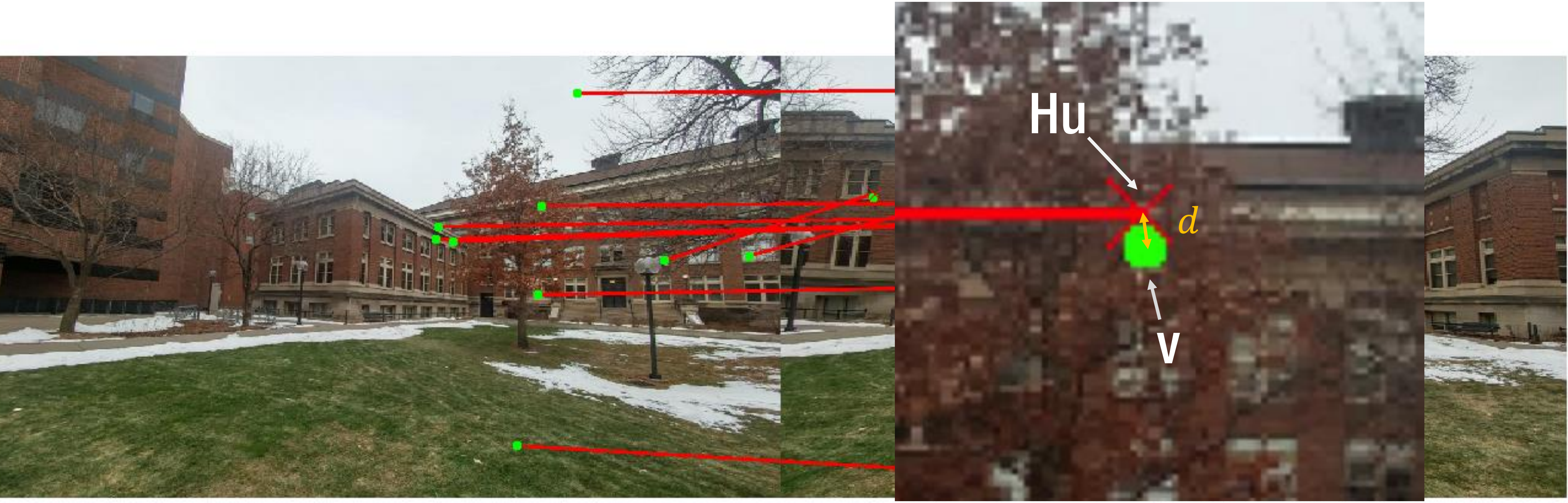
$\mathcal{I}_1$ $\mathcal{I}_2$

If the correspondence is bad, the computed homography will fit the four points still perfectly, but how do we know it is wrong?

$\mathcal{I}_1$ $\mathcal{I}_2$

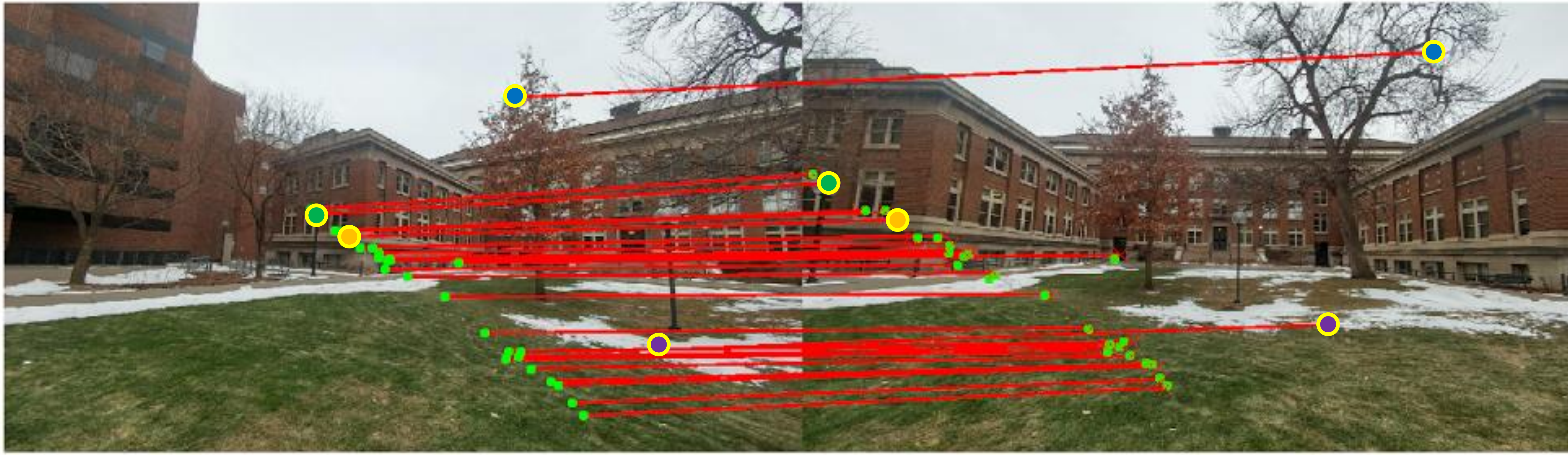
If the correspondence is bad, it has no prediction power!




Hu
d
V

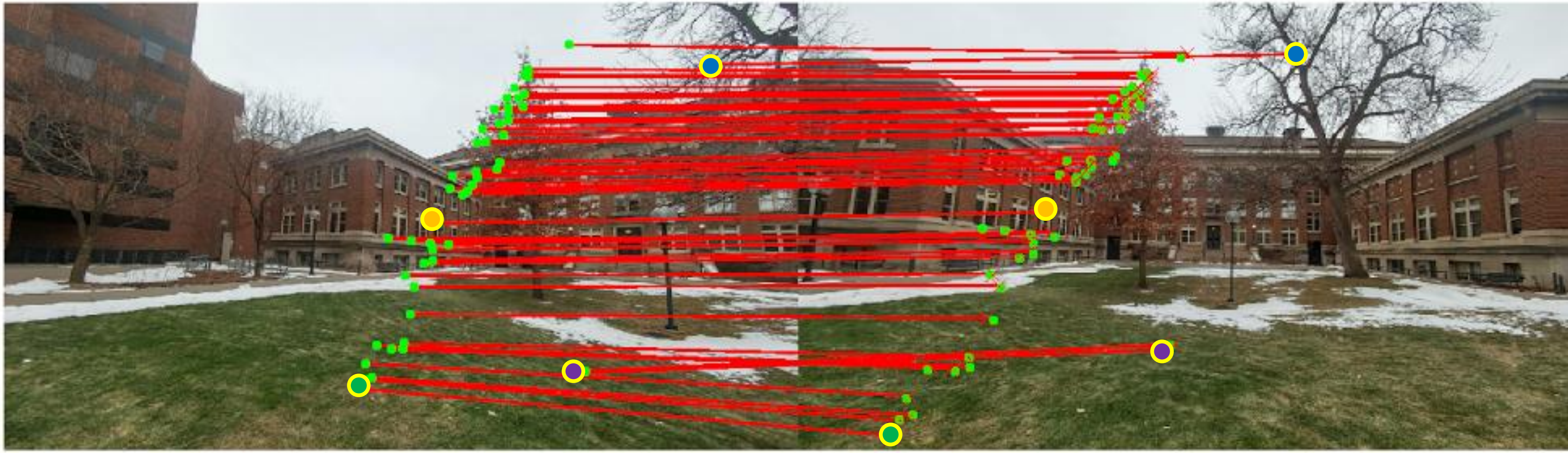
# of inliers: 16 out of 1865

# of inliers: 16 out of 1865



# of inliers: 36 out of 1865

# of inliers: 36 out of 1865



# of inliers: 57 out of 1865

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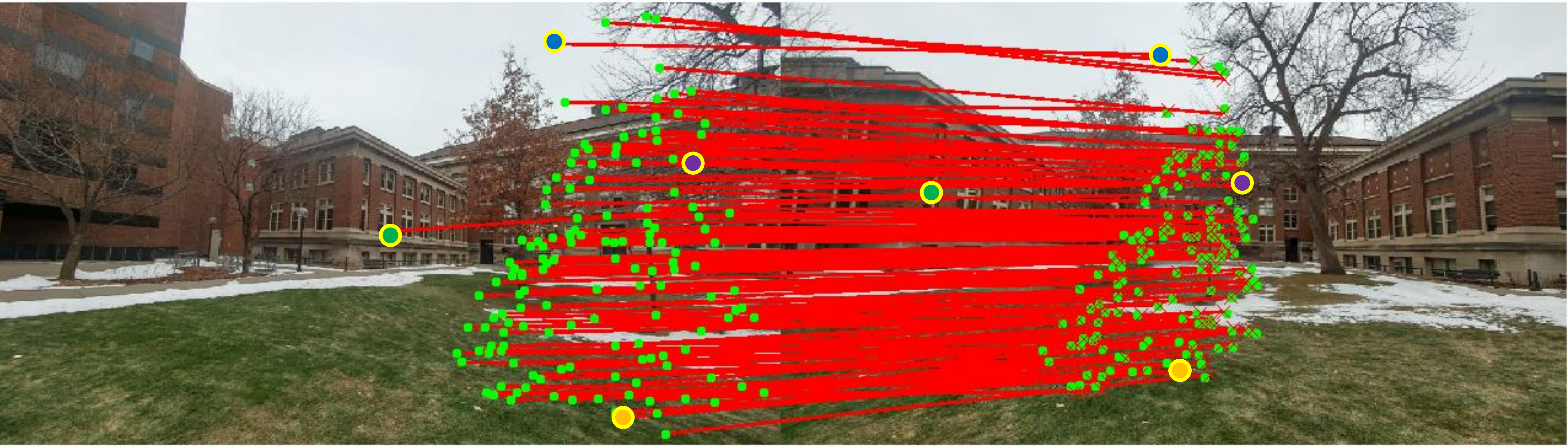


# of inliers: 216 out of 1865

# of inliers: 216 out of 1865

Euclidean Transform (Translation)

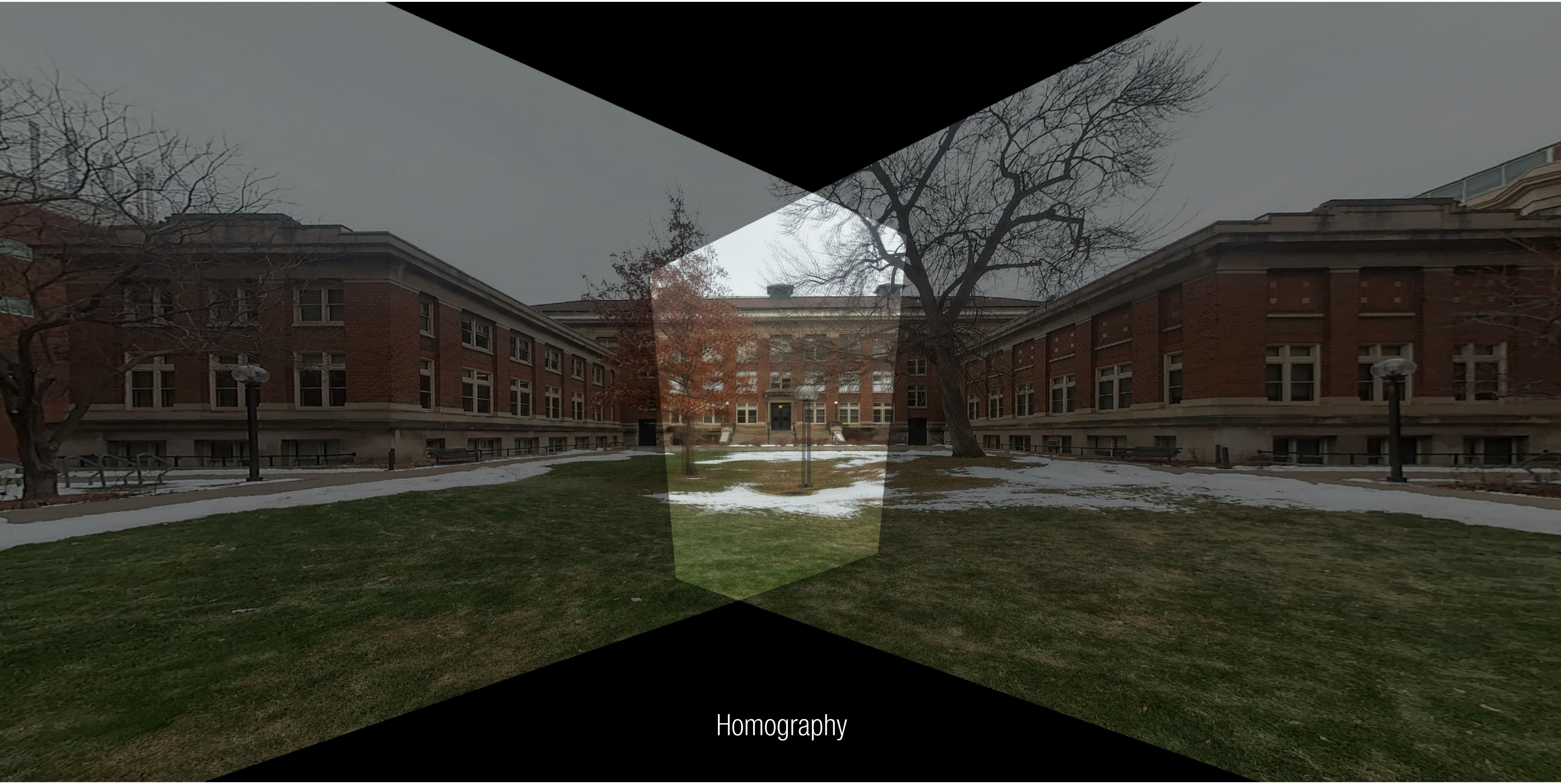
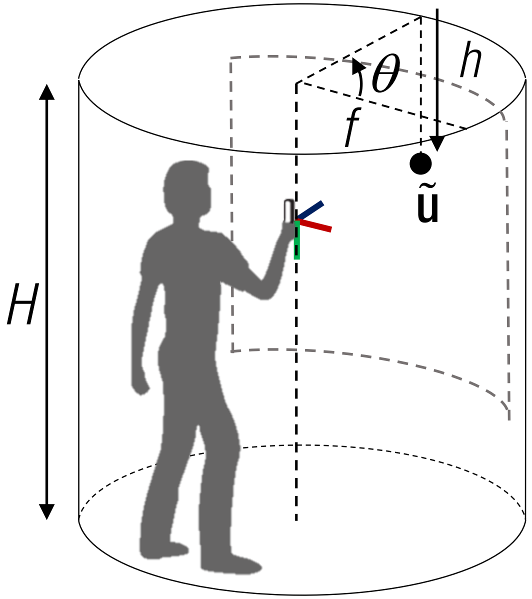

Homography

Image Panorama (Cylindrical Projection)



First camera:

Point on cylindrical surface: $[h, \theta]$

$\longleftrightarrow$ Point in 3D space: $[f\cos(\theta), h, f\sin(\theta)]$

$\longleftrightarrow$ Point in image coordinate: $\mathrm{K}[f\cos(\theta), h, f\sin(\theta)]^{\mathrm{T}}$

Image Panorama (Cylindrical Projection)

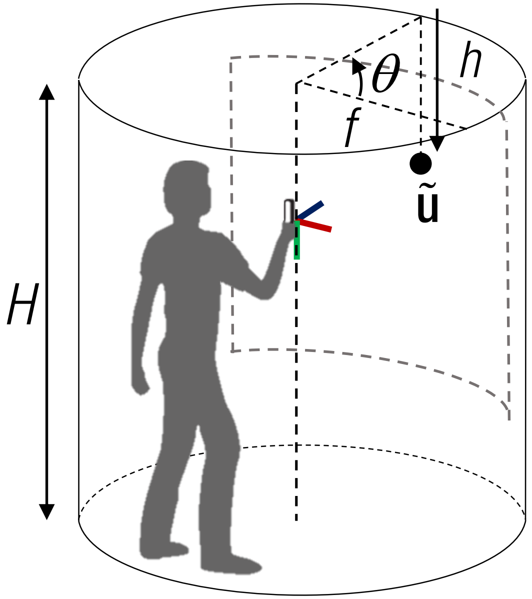
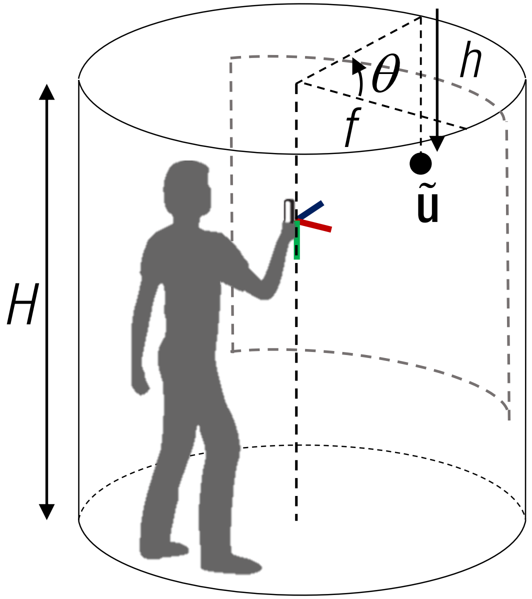


Image Panorama (Cylindrical Projection)



Second camera:

Point on cylindrical surface: $[h, \theta]$

$\leftrightarrow$ Point in 3D space: $[f\cos(\theta), h, f\sin(\theta)]$

$\leftrightarrow$ Point in image coordinate: $\mathrm{K}\mathrm{R}[f\cos(\theta), h, f\sin(\theta)]^{\mathrm{T}}$

where R is given by $\mathrm{R} = \mathrm{K}^{-1}\mathrm{HK}$

Image Panorama (Cylindrical Projection)

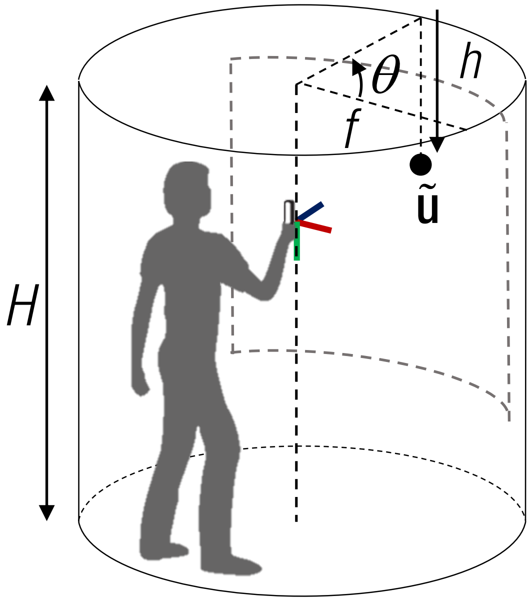


Image Panorama (Cylindrical Projection)