Morphing = Object Averaging

"an average" between two objects

Not an average of two *images of objects*…

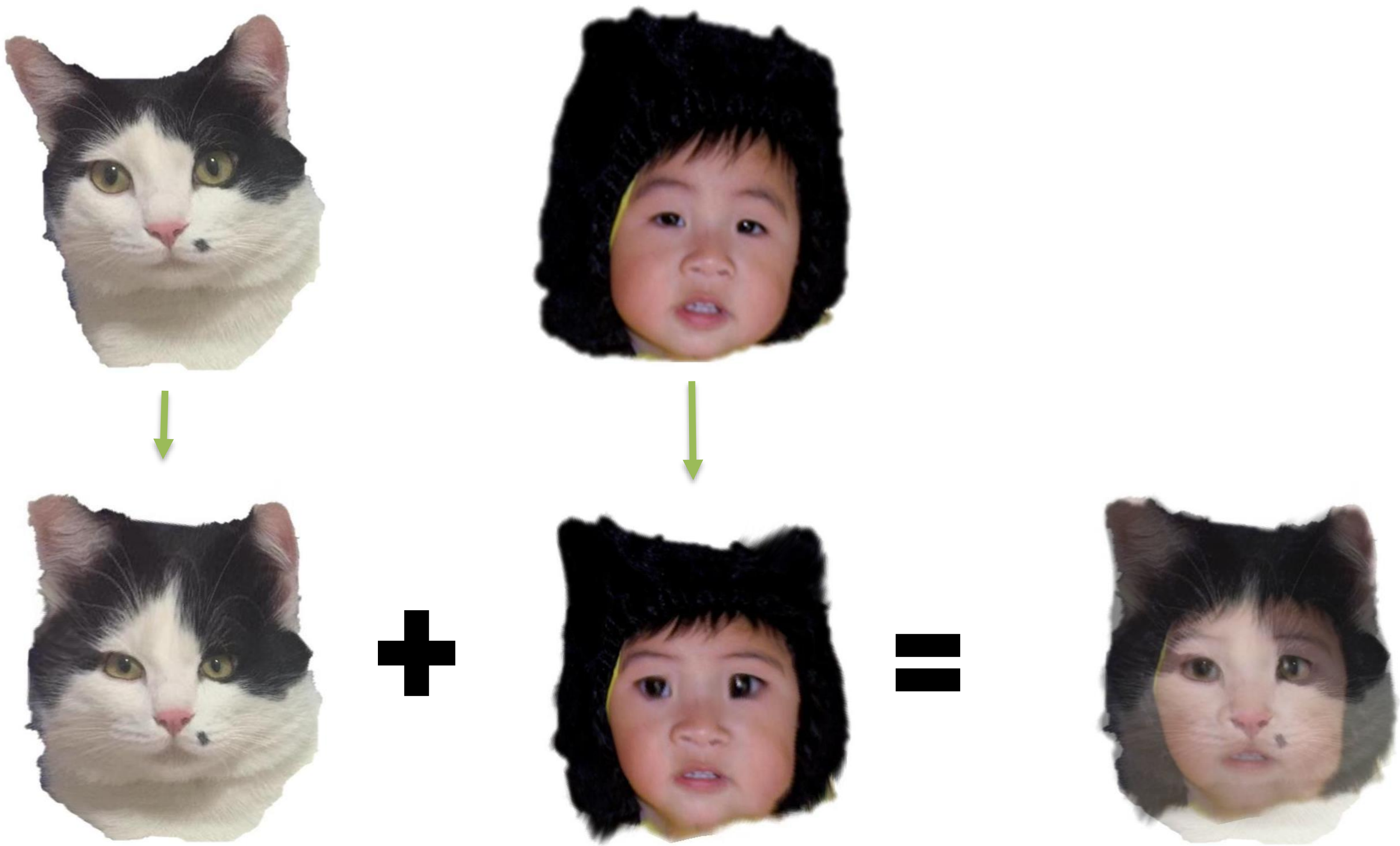
…but an image of the *average object*!

Morphing = Object Averaging

How do we know what the average object looks like?

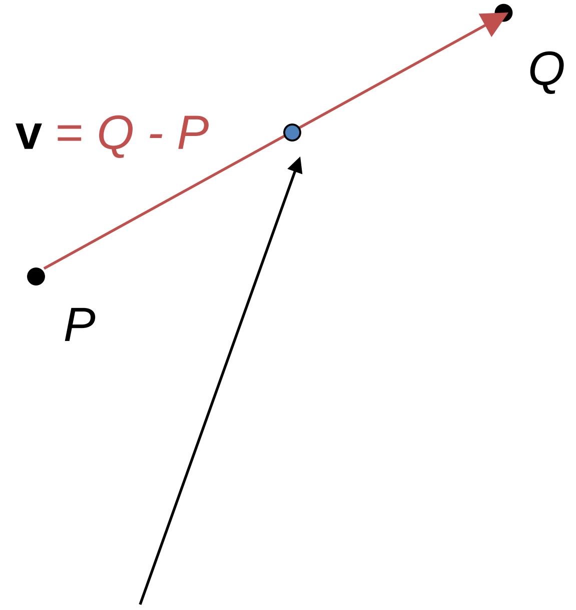
– We haven't a clue!
– But we can often fake something reasonable

Morphing = Warping + Cross Dissolving



Averaging Points

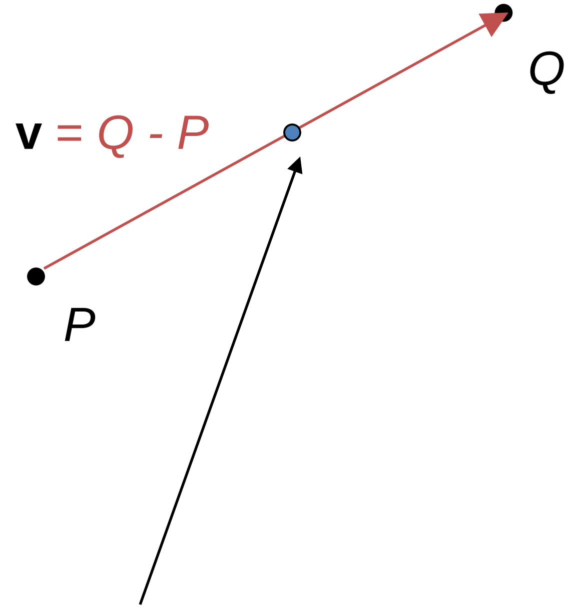
What's the average
of P and Q?



$$
\begin{aligned}
&P + 0.5\mathrm{v} \\
&= \; P + 0.5(Q - P) \\
&= \; 0.5P + 0.5\,Q
\end{aligned}
$$

Averaging Points

What’s the average of P and Q?



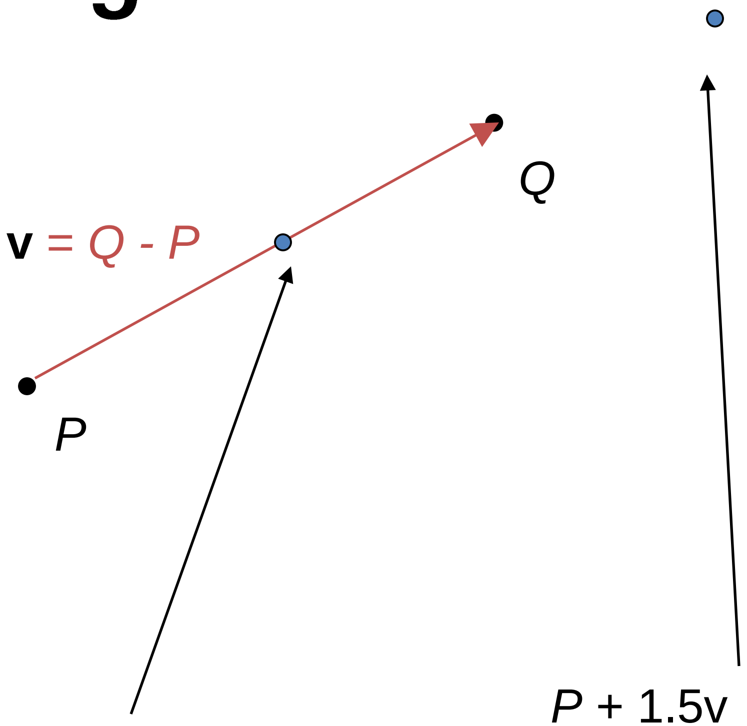
$P + 0.5\mathbf{v}$

$= P + 0.5(Q - P)$

$= 0.5P + 0.5\,Q$

Linear Interpolation
(Affine Combination):
New point $aP + bQ$,
defined only when $a+b = 1$
So $aP+bQ = aP+(1-a)Q$

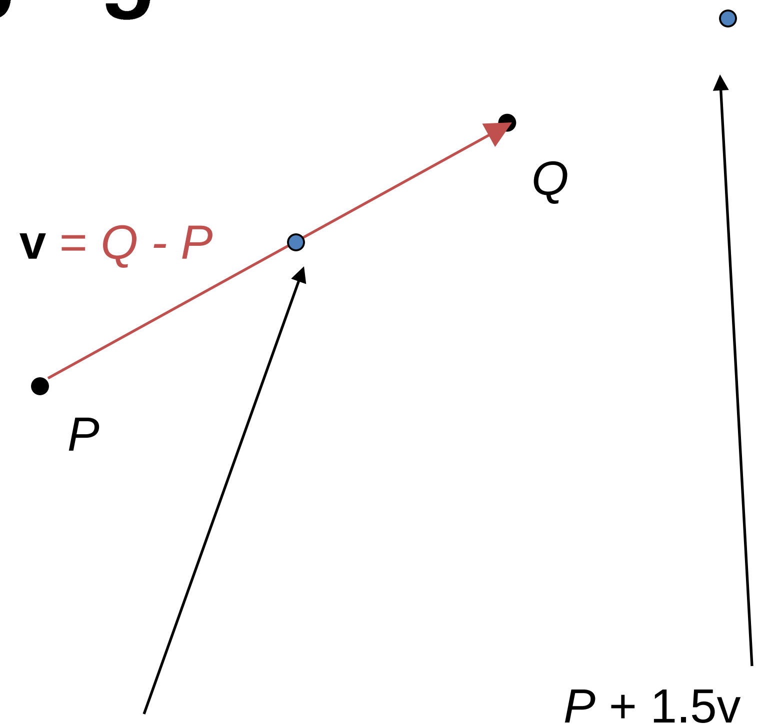
Averaging Points



$P + 0.5\mathrm{v}$
$= P + 0.5(Q - P)$
$= 0.5P + 0.5\,Q$

$P + 1.5\mathrm{v}$
$= P + 1.5(Q - P)$
$= -0.5P + 1.5\,Q$
(extrapolation)

Averaging Points



$P + 0.5\mathbf{v}$
$= P + 0.5(Q - P)$
$= 0.5P + 0.5\,Q$

$P + 1.5\mathbf{v}$
$= P + 1.5(Q - P)$
$= -0.5P + 1.5\,Q$
(extrapolation)

- P and Q can be anything:
  - points on a plane (2D) or in space (3D)
  - Colors in RGB or HSV (3D)
  - Whole images (m-by-n D)… etc.

Averaging Images: Cross-Dissolve

Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1\text{-}t)*\text{Image}_1 + t*\text{image}_2$$

This is called **cross-dissolve** in film industry

Averaging Images: Cross-Dissolve

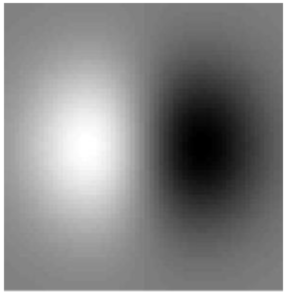
Interpolate whole images:

$$Image_{halfway} = (1\text{-}t)*Image_1 + t*image_2$$

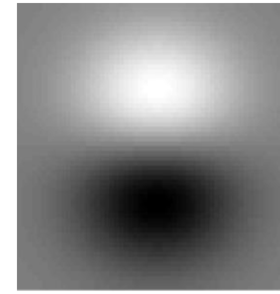
This is called **cross-dissolve** in film industry

Averaging Images

Averaging Images = Rotating Objects

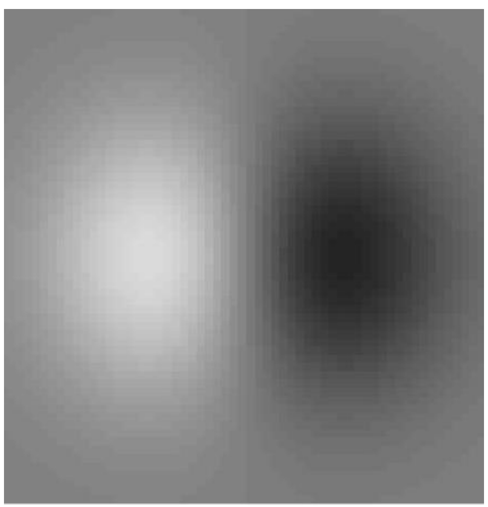


$Image_1$

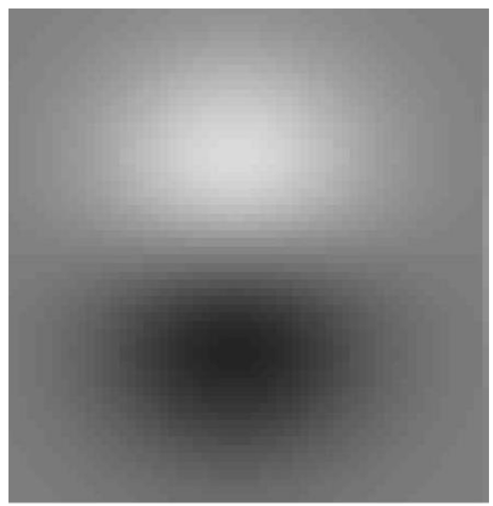


$Image_2$

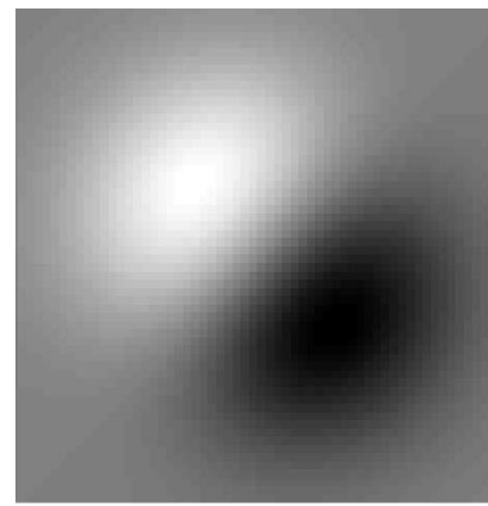
t = 0 t = 0.5 t = 1



+



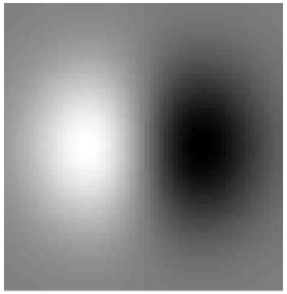
=



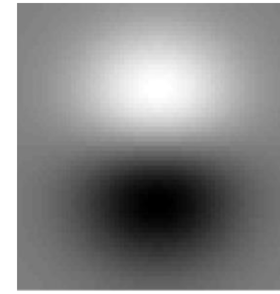
$(1-t)*Image_1 + t*Image_2 = Image_{halfway}$

Averaging Images

Averaging Images = Rotating Objects

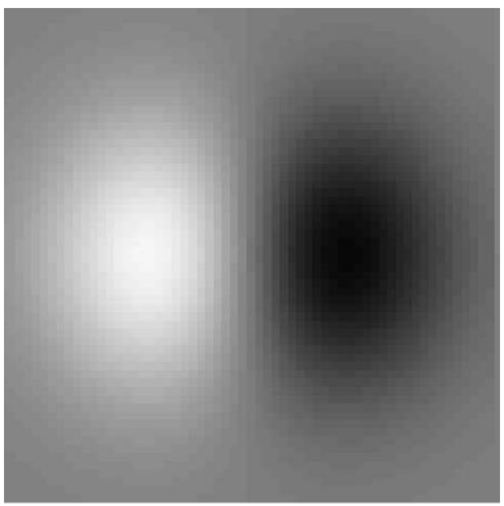


$Image_1$



$Image_2$

t = 0 | t = 0.3 | t = 1



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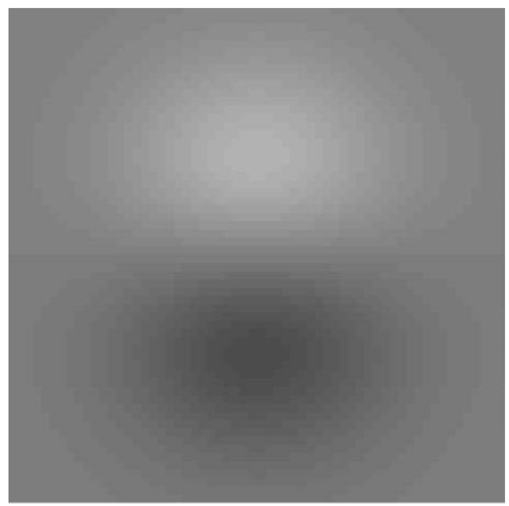

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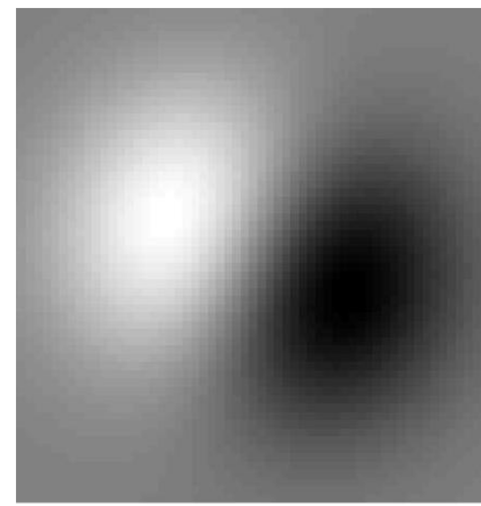
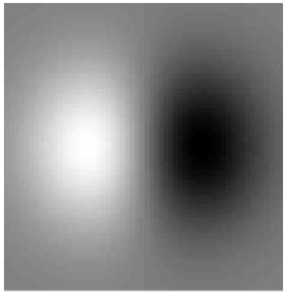
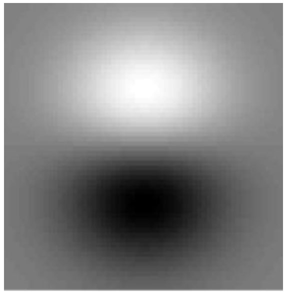


$(1\text{-}t)*Image_1$ + $t*Image_2$ = $Image_{halfway}$

Averaging Images



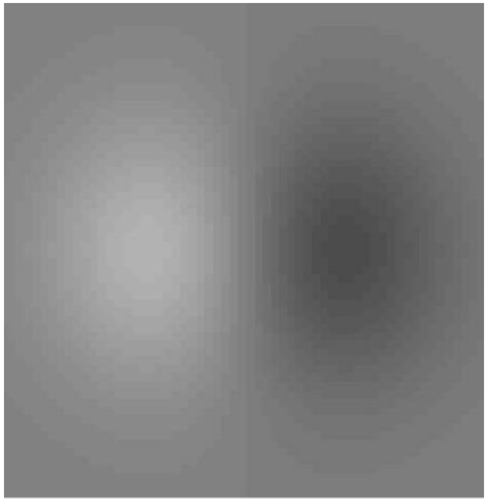
Averaging Images = Rotating Objects

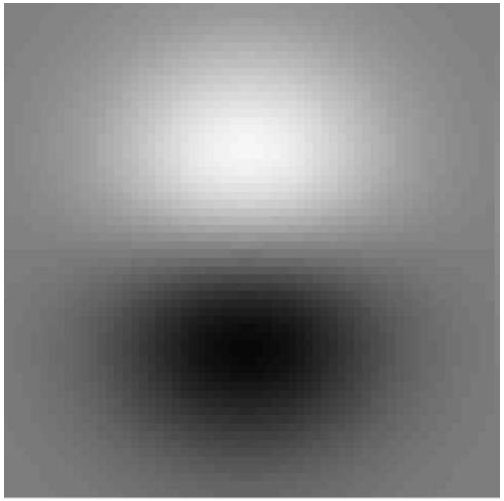


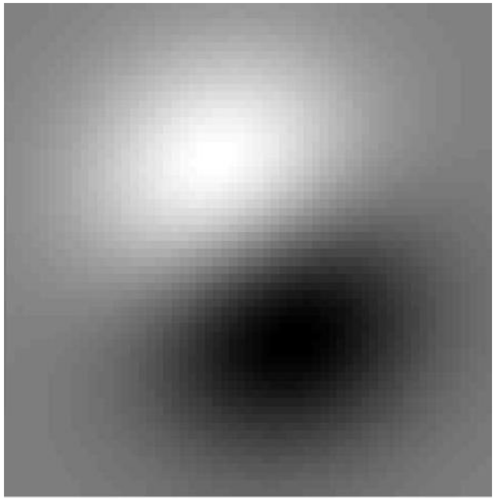
$Image_1$

$Image_2$

$t = 0$ $t = 0.7$ $t = 1$







$(1-t)*Image_1 + t*Image_2 = Image_{halfway}$

Averaging Images

Image_1 Image_2

+

= ?

Averaging Images

$Image_1$ $Image_2$

+

? =

Averaging Images

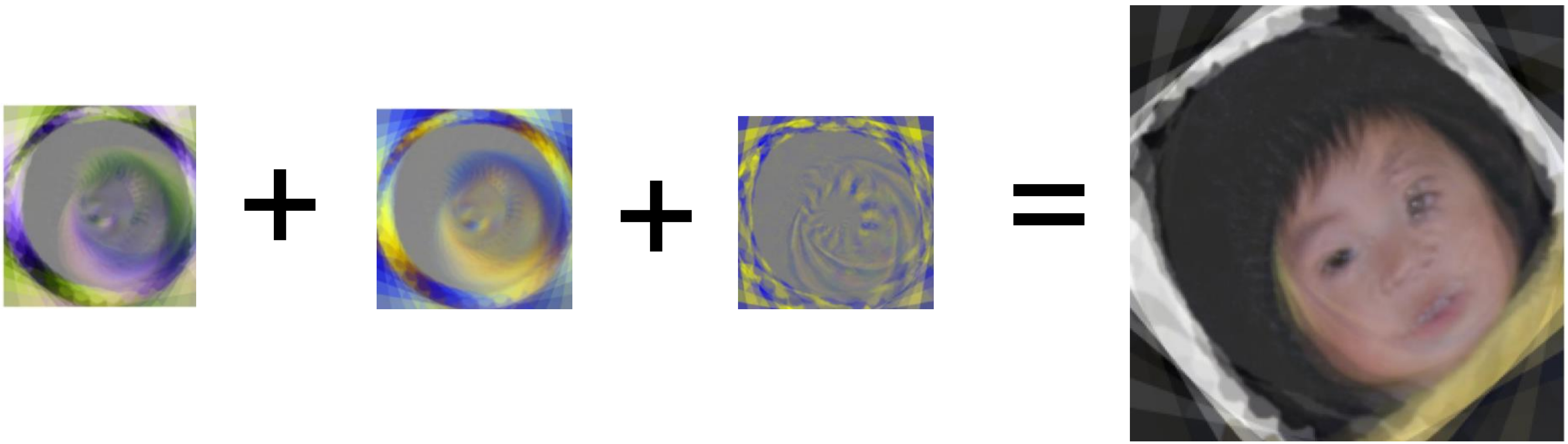
Image_1 Image_2

+

=

Averaging Images != Rotating Complex Objects

Averaging Images



Averaging **‘Eigen’** Images = Rotating Objects

Cat-Baby Averaging

Object Averaging with feature matching!

- Nose to nose, eye to eye, mouth to mouth, etc.
- This is a non-parametric **warp**

Cat-Baby Averaging

Object Averaging with feature matching (warping)!

- Nose to nose, eye to eye, mouth to mouth, etc.
- This is a non-parametric warp

Warping, then cross-dissolve

Morphing procedure:

1. Find the average shape
2. Non-parametric warping
3. Find the average color
   - Cross-dissolve the warped images

Image warping – non-parametric

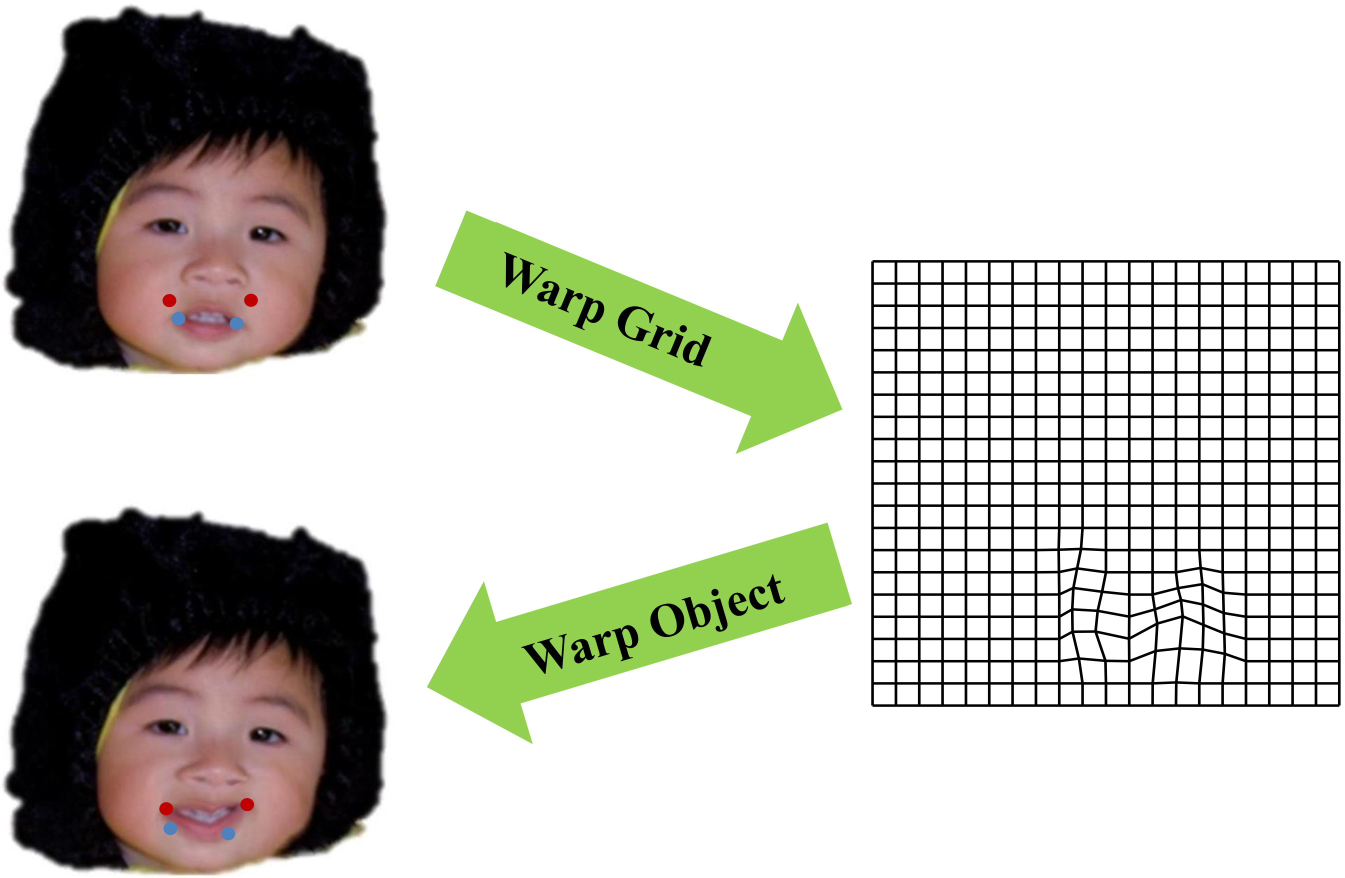
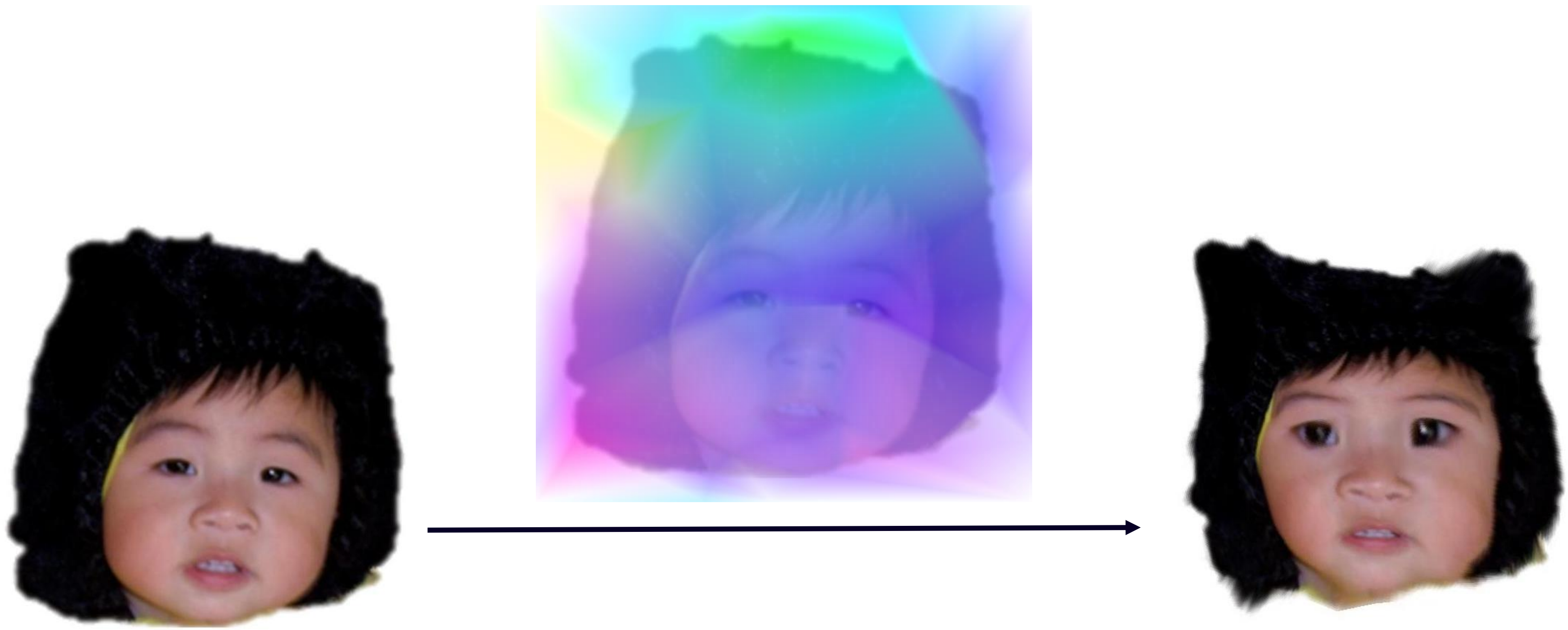


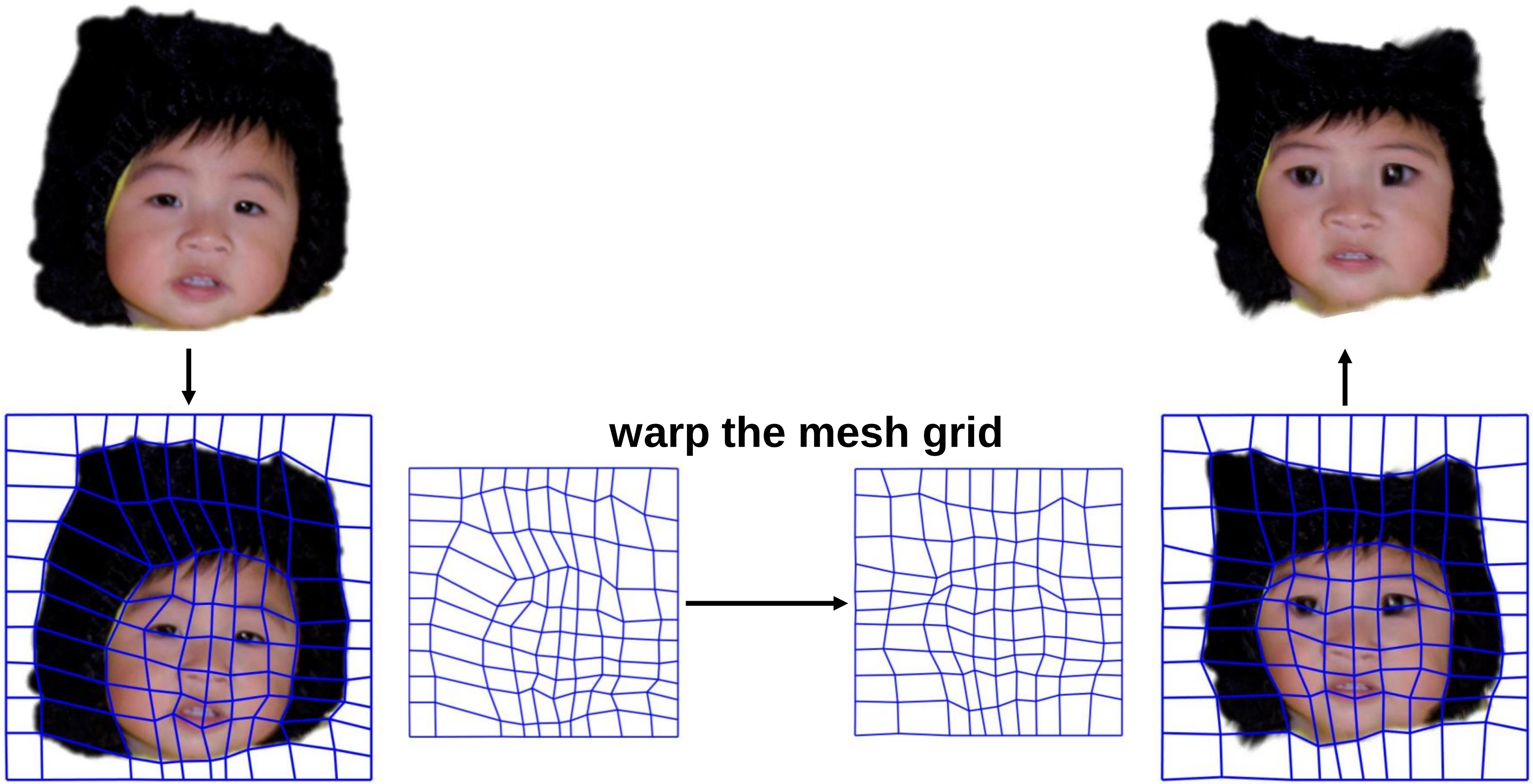
Image warping idea 1: dense flow



Displacement vector (u,v) for each pixel.

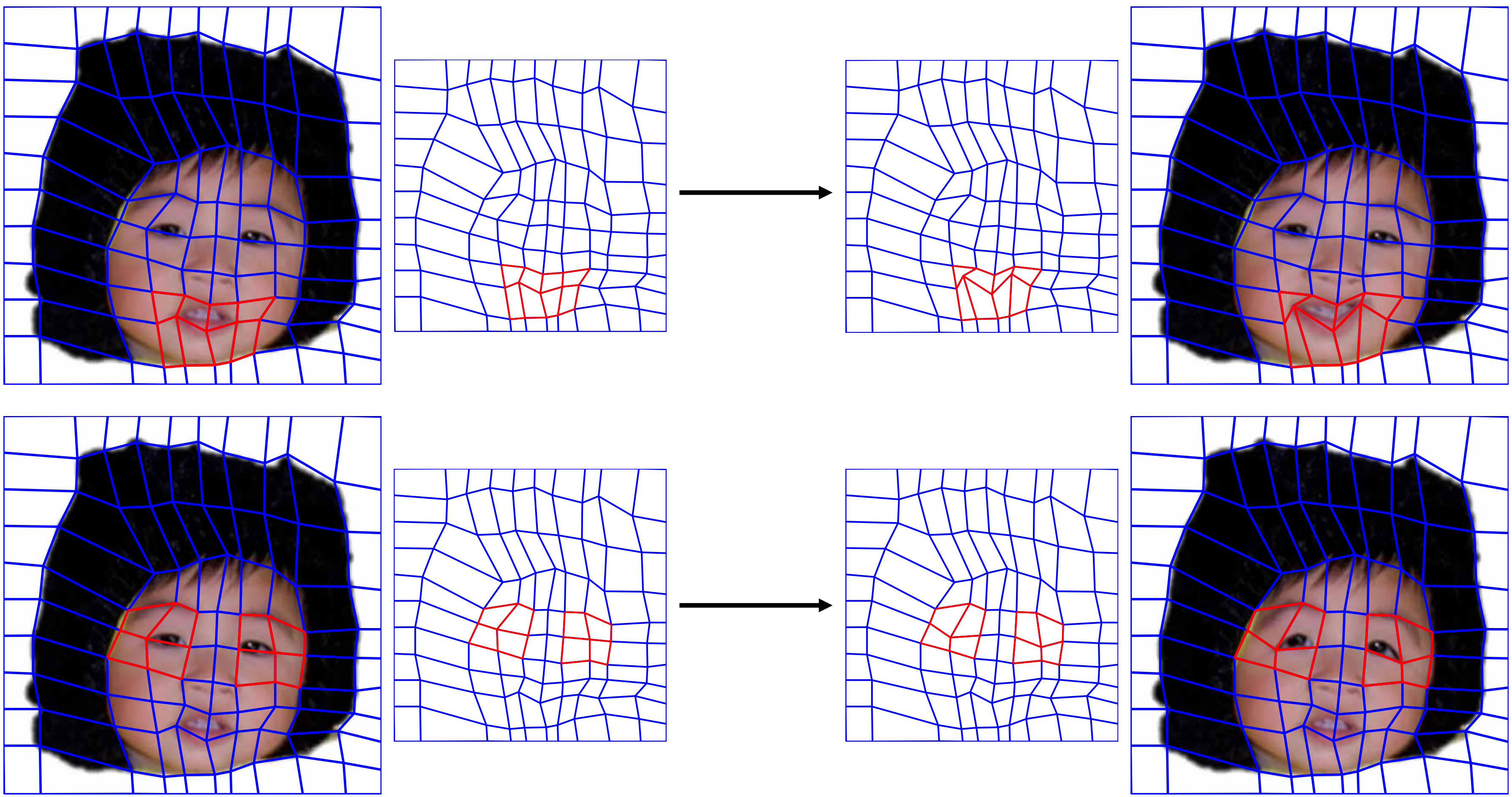
Great details… but too much work, let's simply it to mesh grid

Image warping idea 2 : dense grid



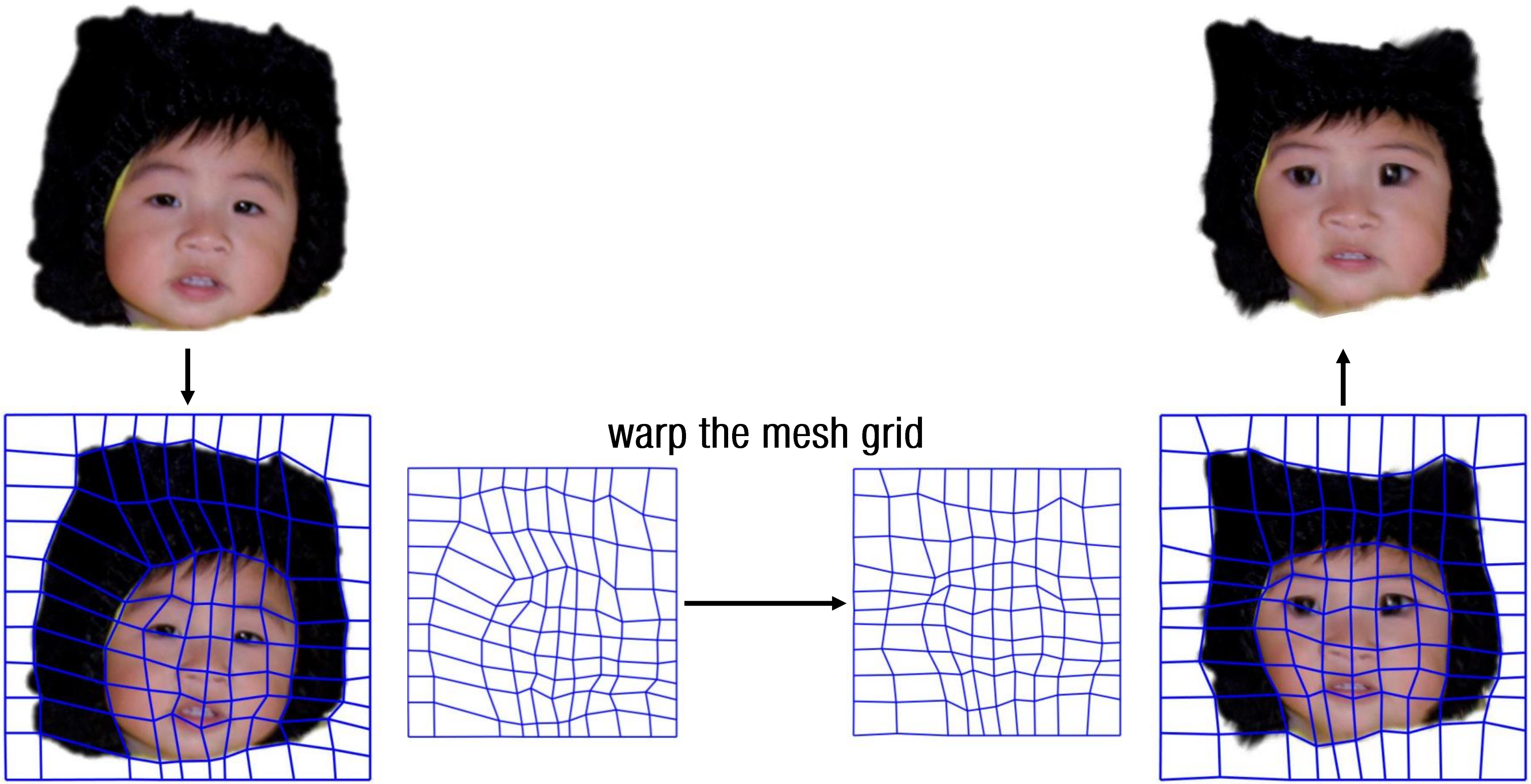
Define and manipulate the mesh grid

Image warping idea 2 : dense grid



Grid deformation generates expression change

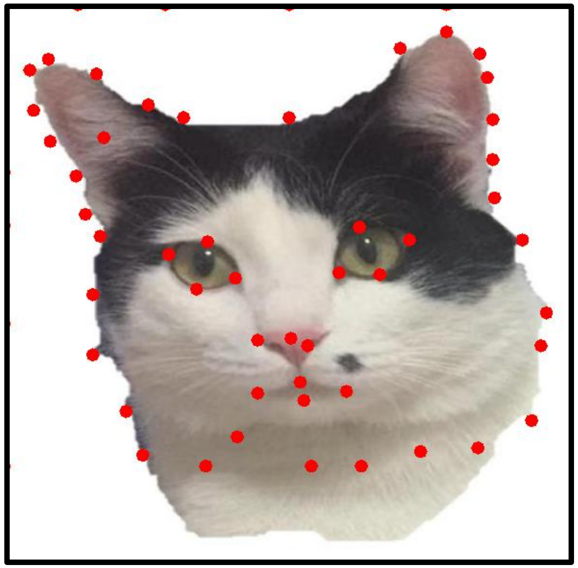
Image warping idea 2 : dense grid

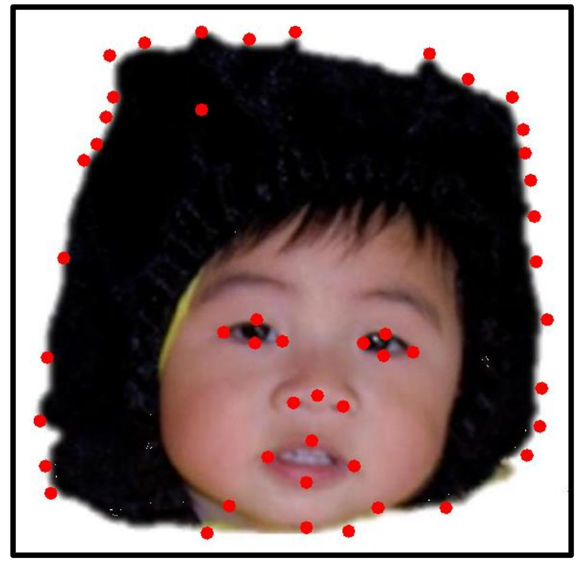


Still too much work…

simplify it to sparse control points and triangles

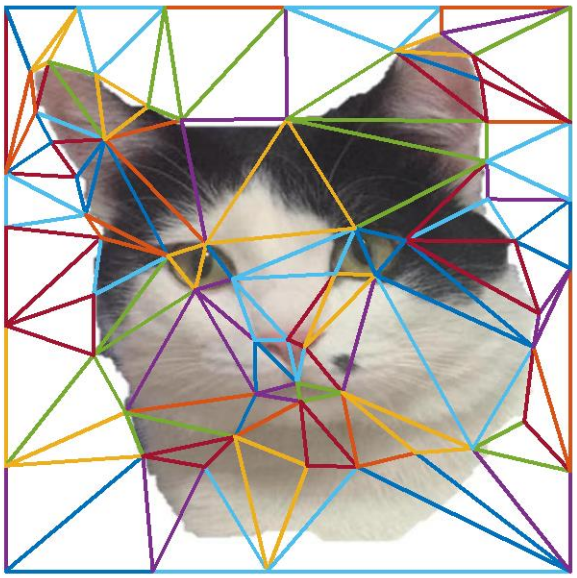
Image warping idea 3 : sparse points

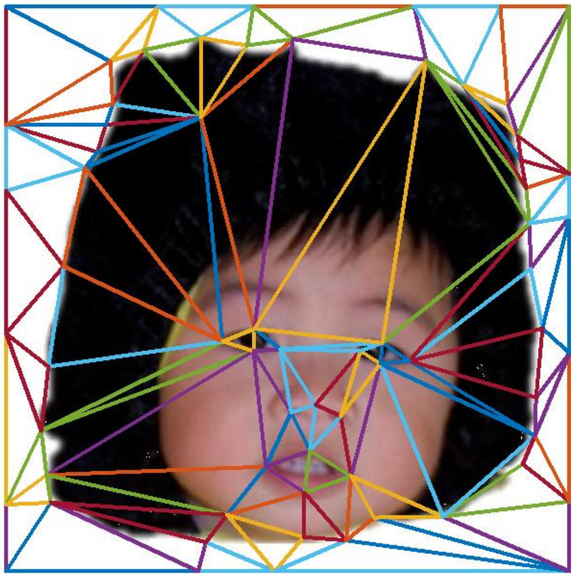




Specify sparse points and their correspondence

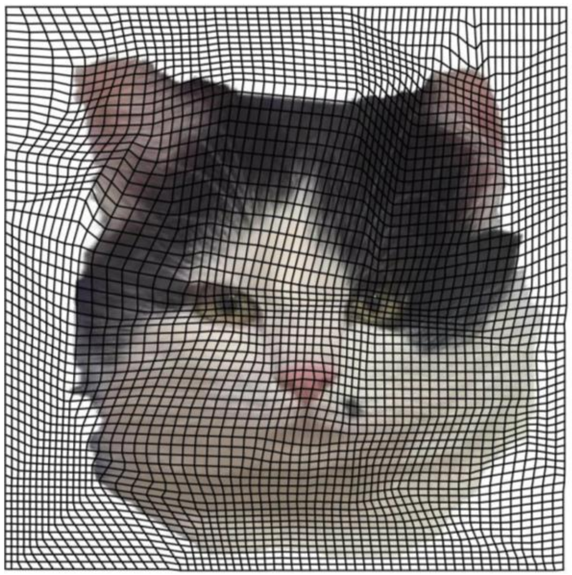
Image warping idea 3 : sparse points

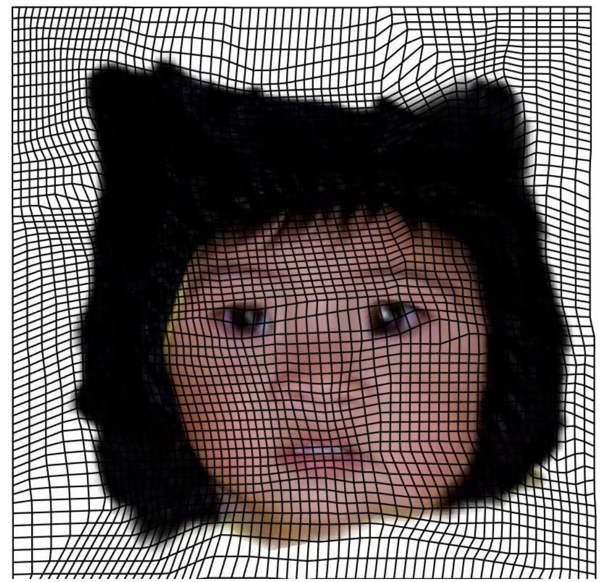




- Define a triangular mesh over the feature points
- Triangle-to-triangle correspondences
- Warp each triangle separately from source to destination

From sparse points to dense grid

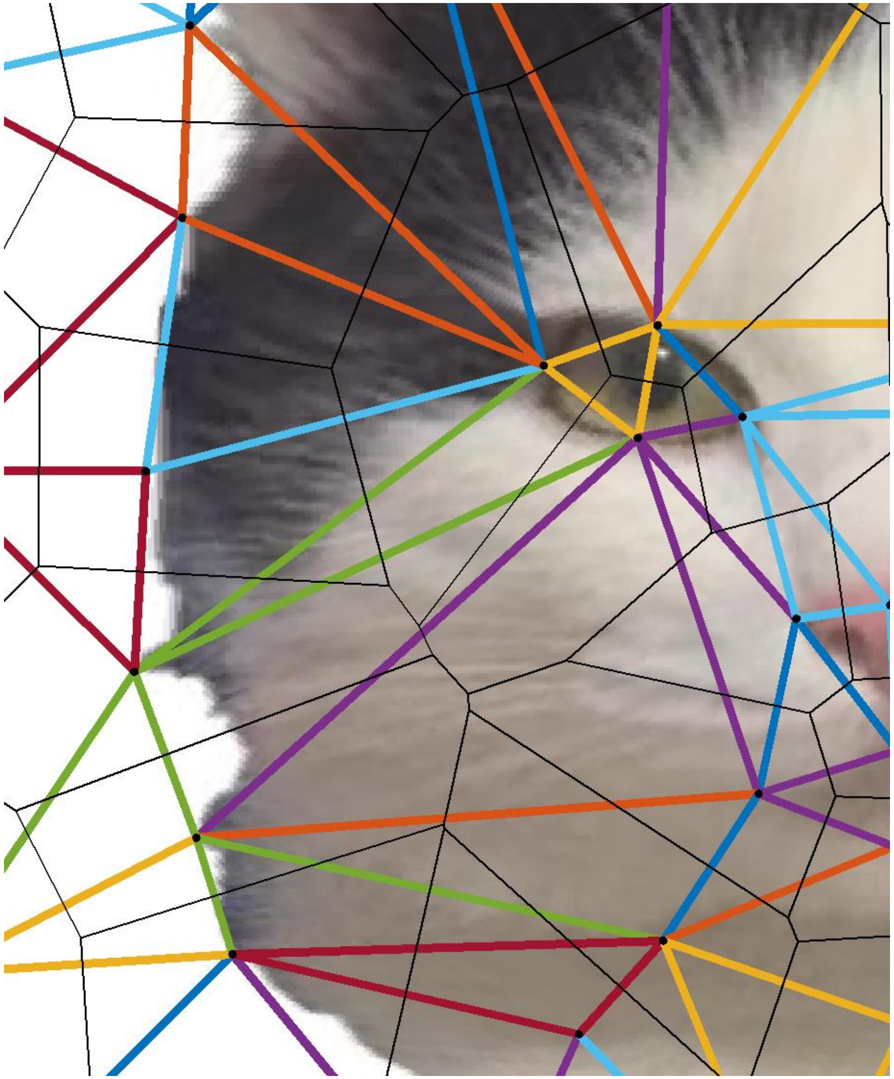




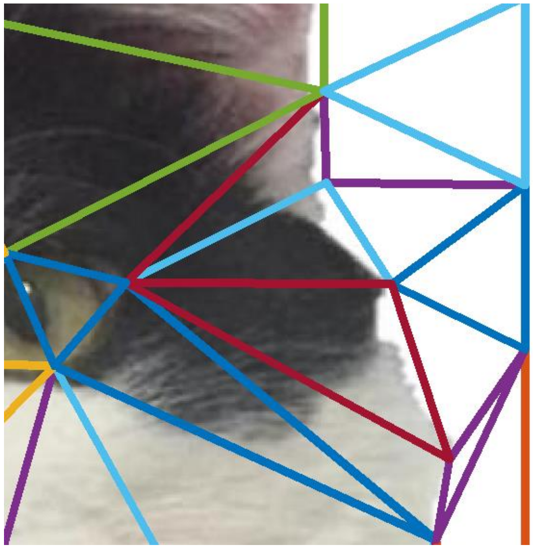
- Warping on triangulation corresponds to warping on dense grid, and dense pixel flow

Delaunay Triangulation

- Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.
- The DT may be constructed in O(nlogn) time.
- This is what Matlab's delaunay function uses.



What is good feature points

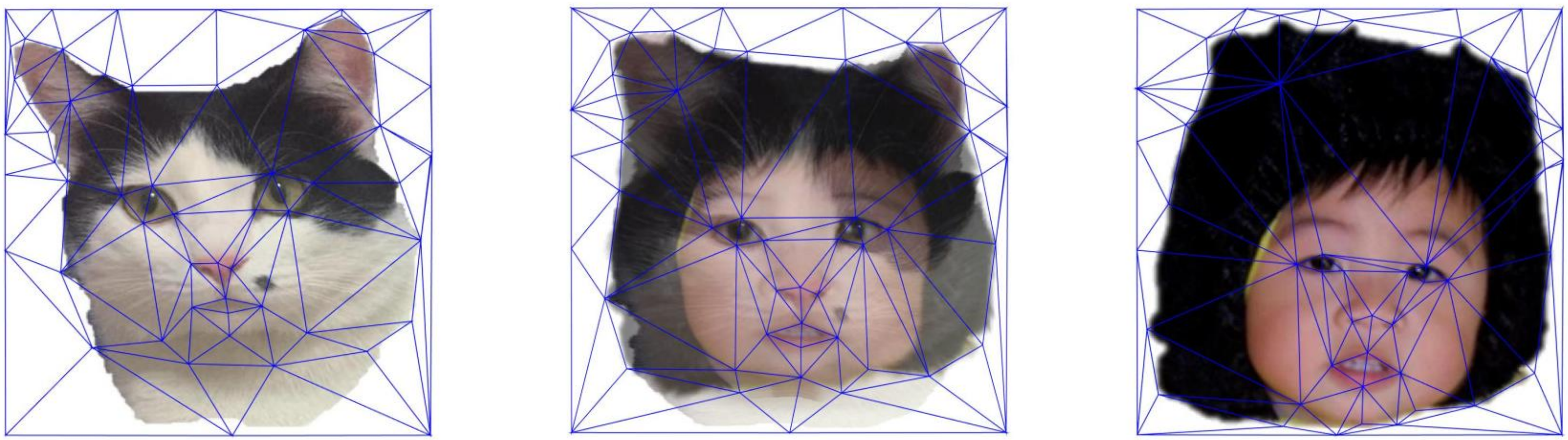


Good

Bad

- The triangulation is consistent with image boundary
  - Texture regions won't fade into the background when morphing
- Maintain the relationship between parts

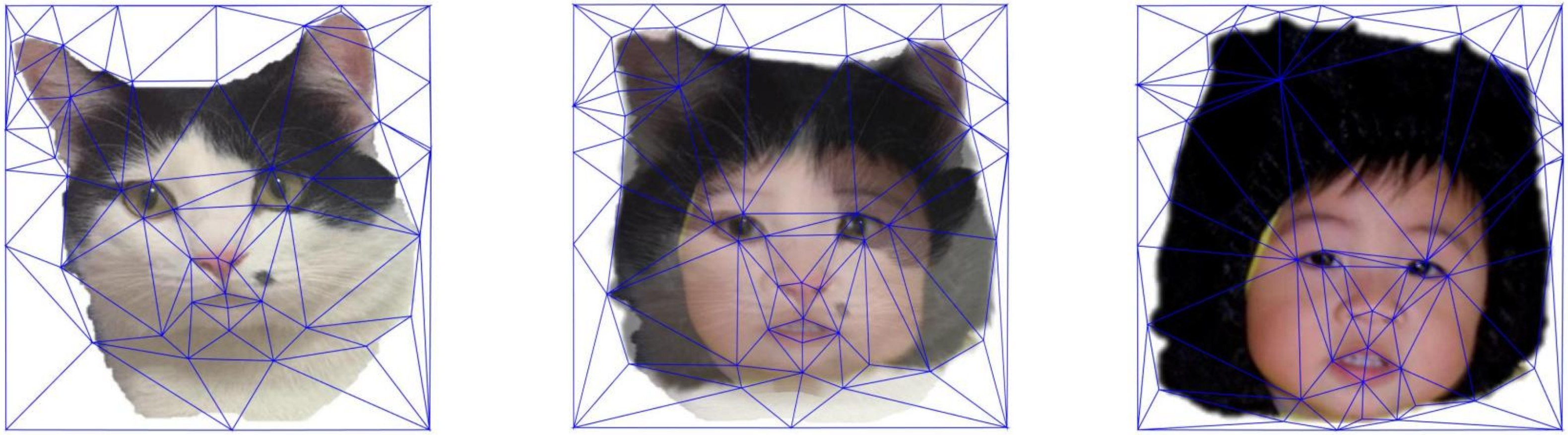
Triangular Mesh



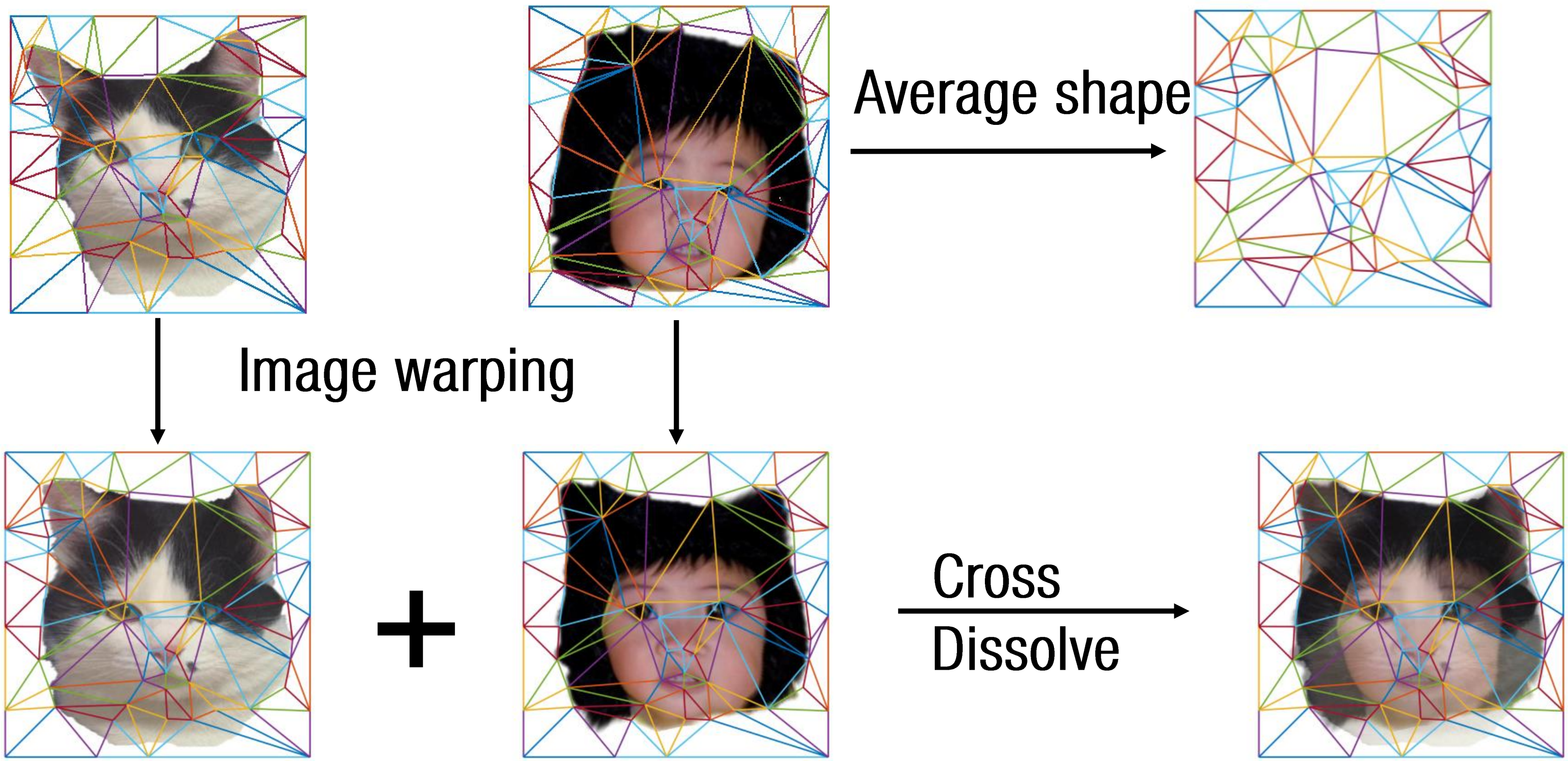
1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences

Warp interpolation

- How do we create an intermediate warp at time t?
  - Assume t = [0,1]
  - Simple linear interpolation of each feature pair
  - (1-t)*p1+t*p0 for corresponding features p0 and p1

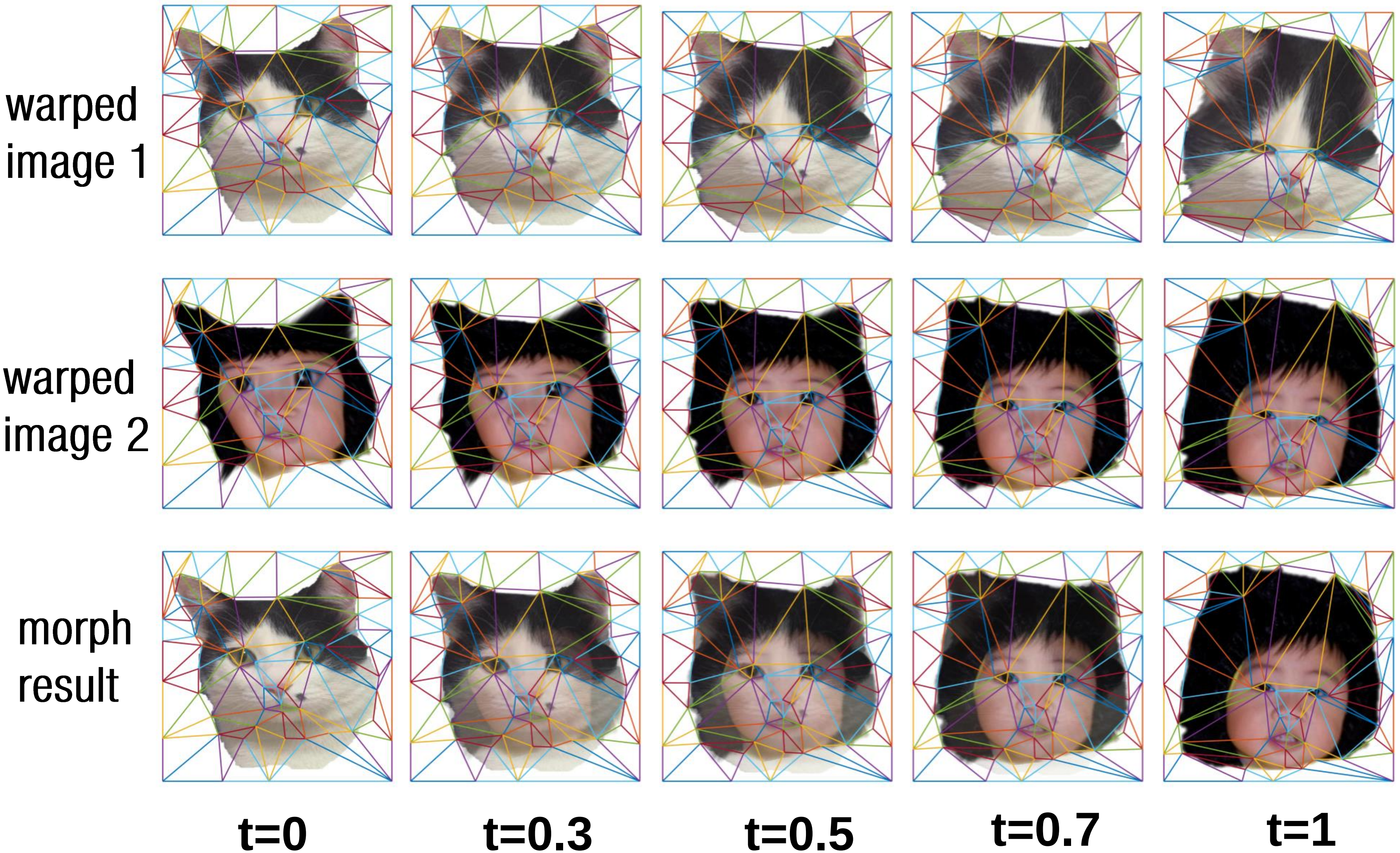


Morphing = Warping + Cross-dissolve

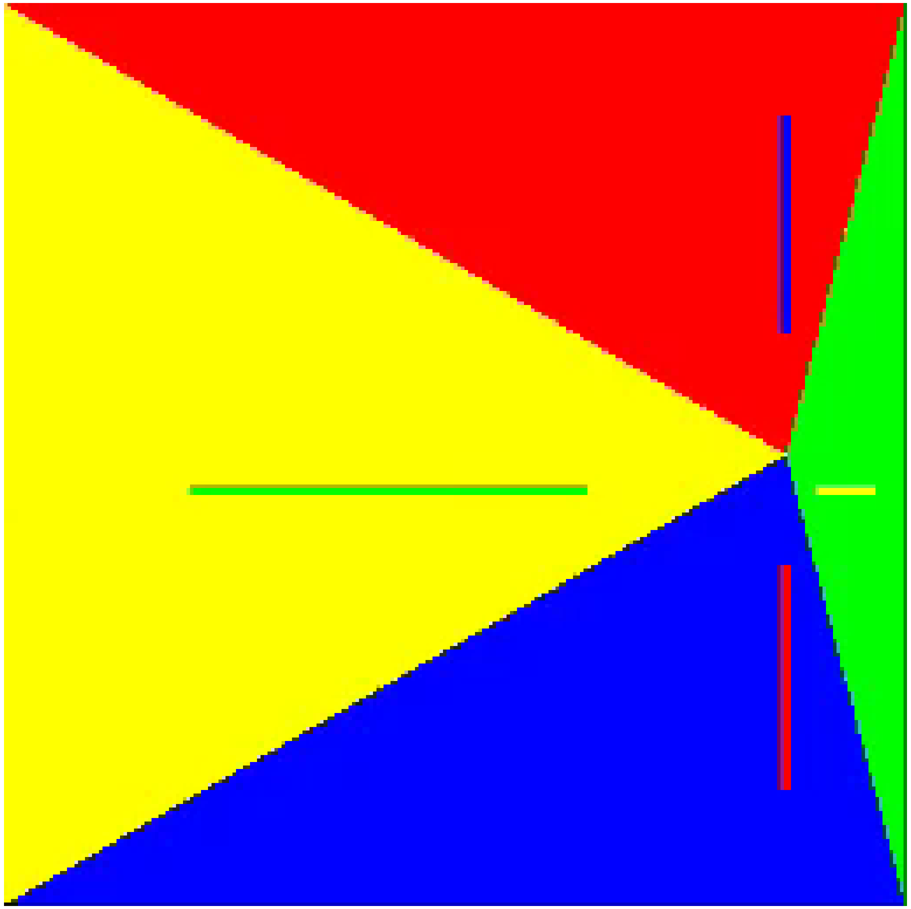


- For each time t, define the intermediate shape
  - $p_t = (1 - t) \times p_1 + t \times p_2$
  - triangulation doesn't change
- Warp both image to the intermediate shape
- Dissolve image = $(1\text{-}t)\times\text{image}_1 + t\times\text{Image}_2$

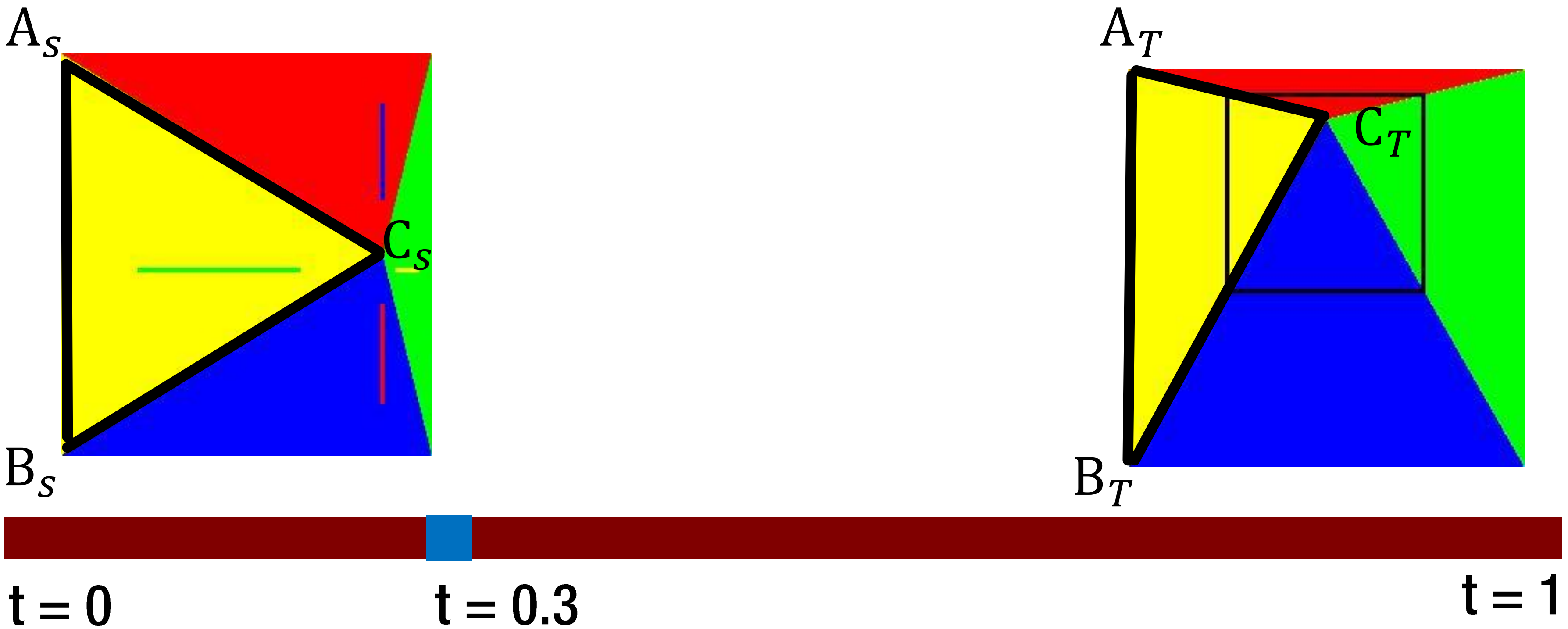
Morphing Sequence



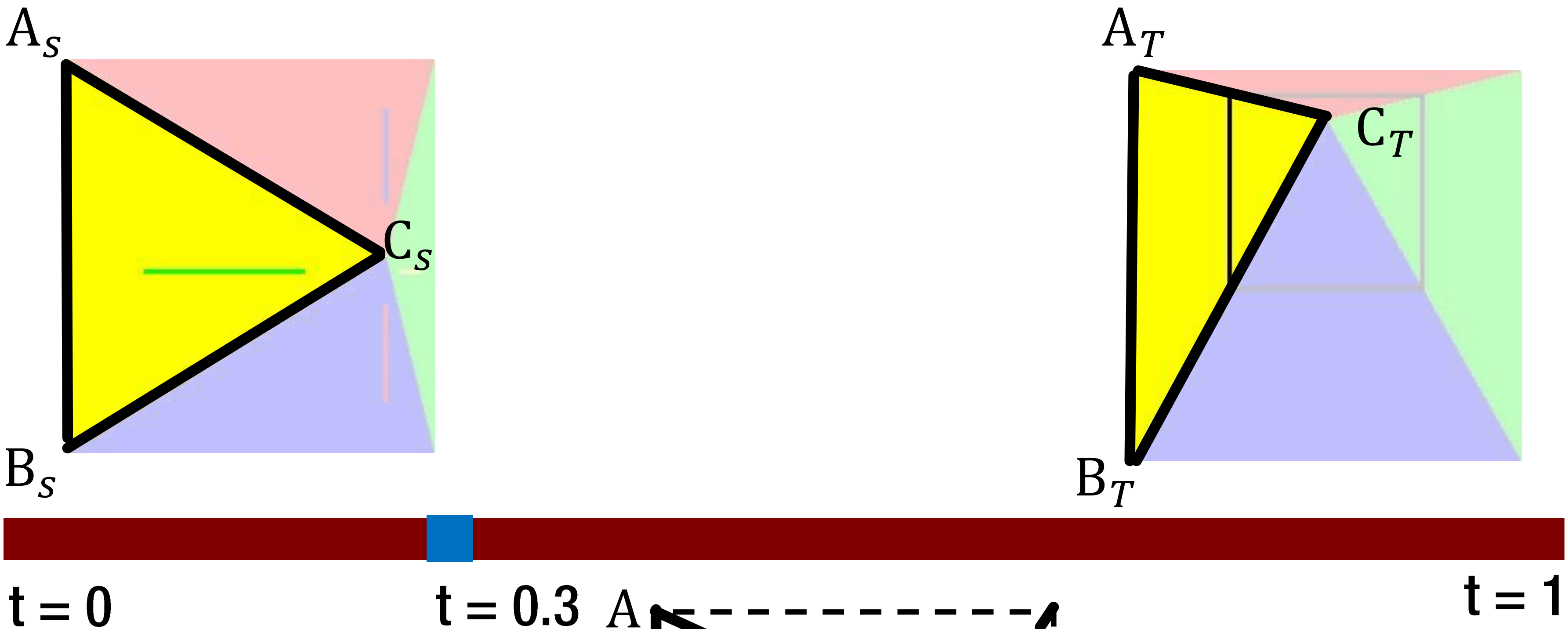
An Example

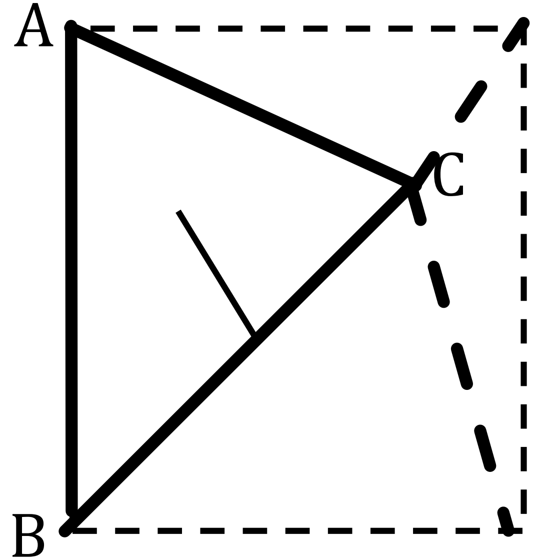


Morphing



Step 1: Triangle interpolation



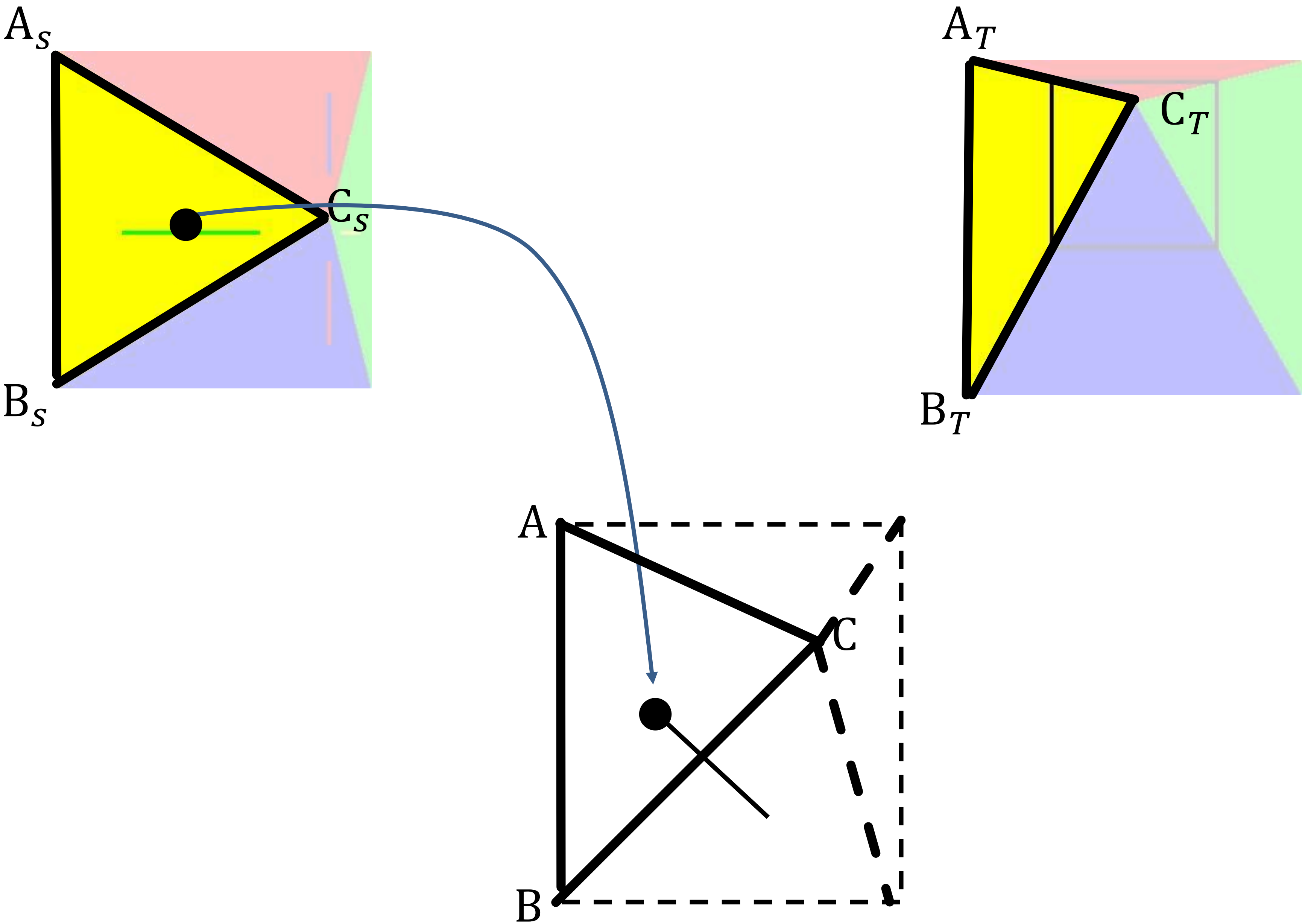


$$\mathbf{A}_t = (1-t)\mathbf{A}_S + t\mathbf{A}_T$$

$$\mathbf{B}_t = (1-t)\mathbf{B}_S + t\mathbf{B}_T$$

$$\mathbf{C}_t = (1-t)\mathbf{C}_S + t\mathbf{C}_T$$

Step 2: Warping



Step 2: Warping

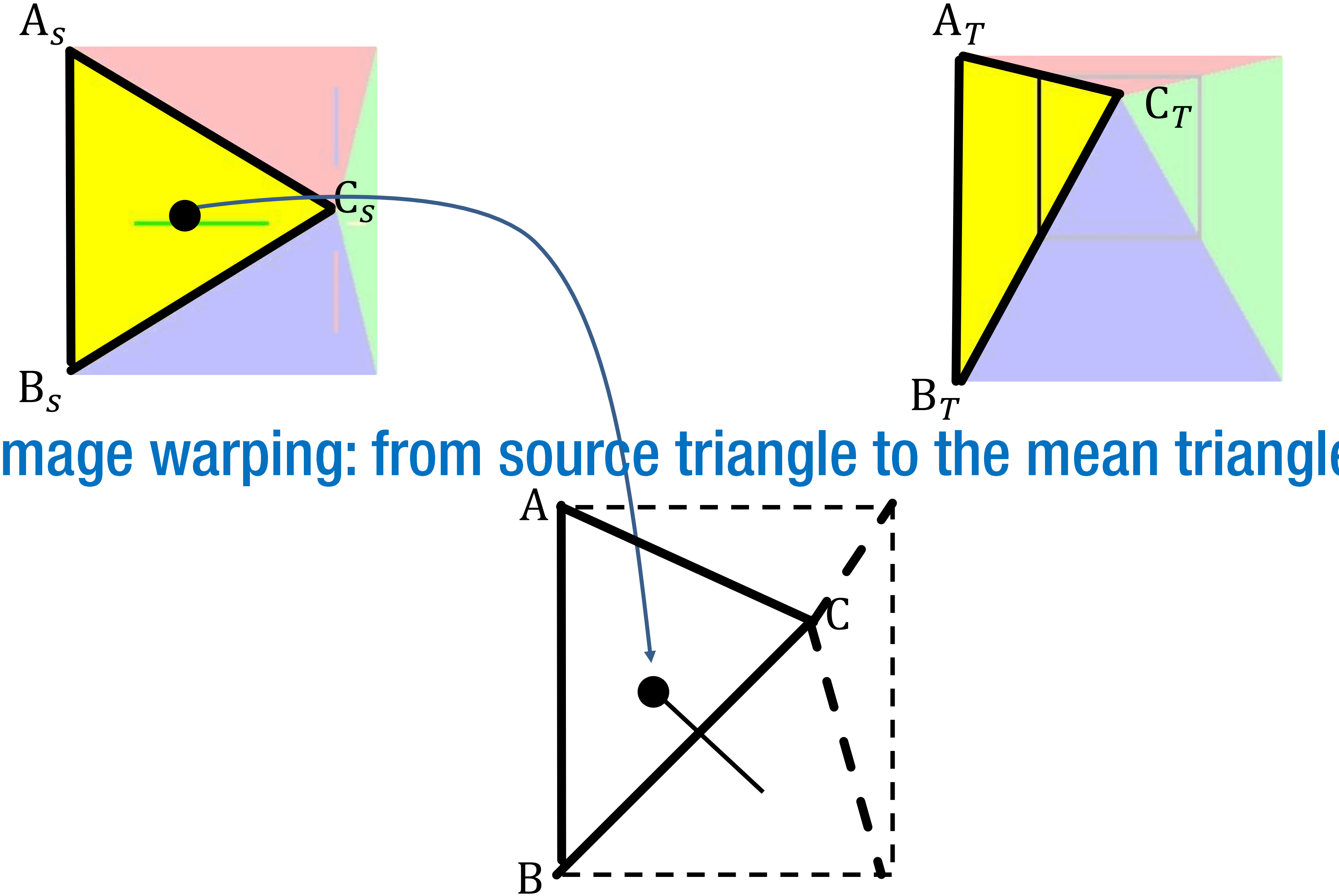
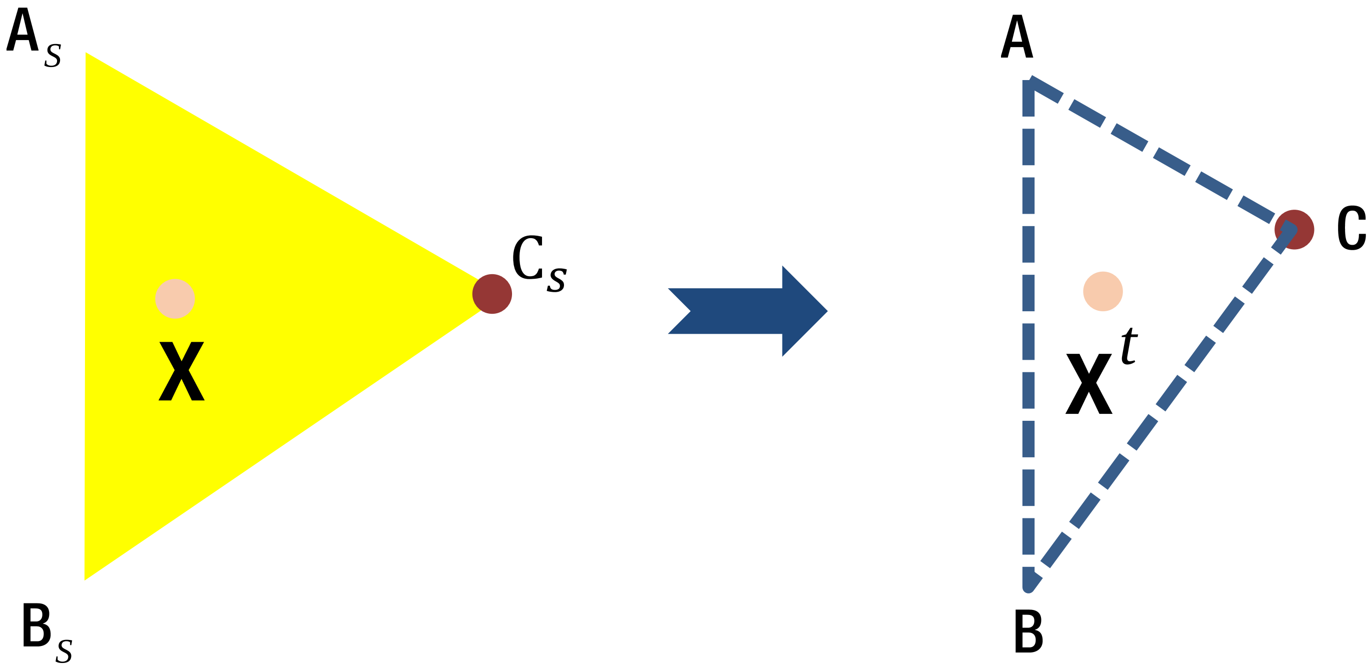


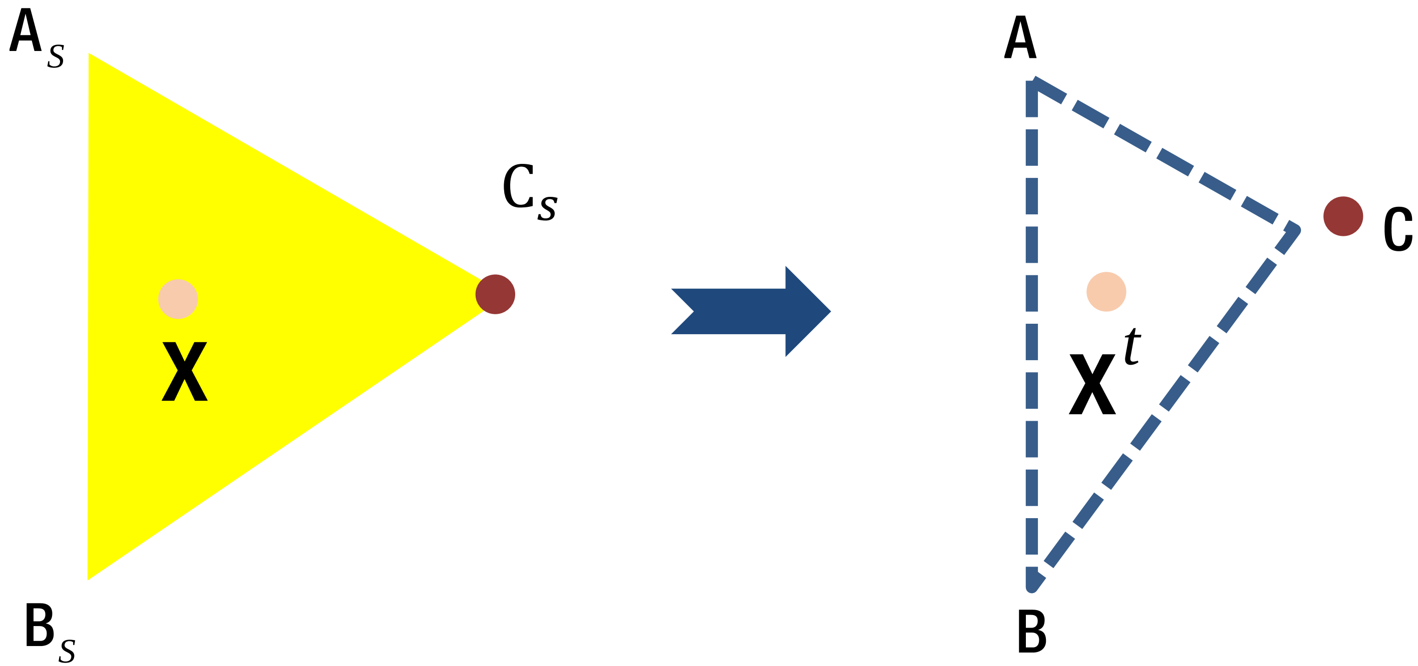
Image warping: from source triangle to the mean triangle

Triangle warping = Affine transform



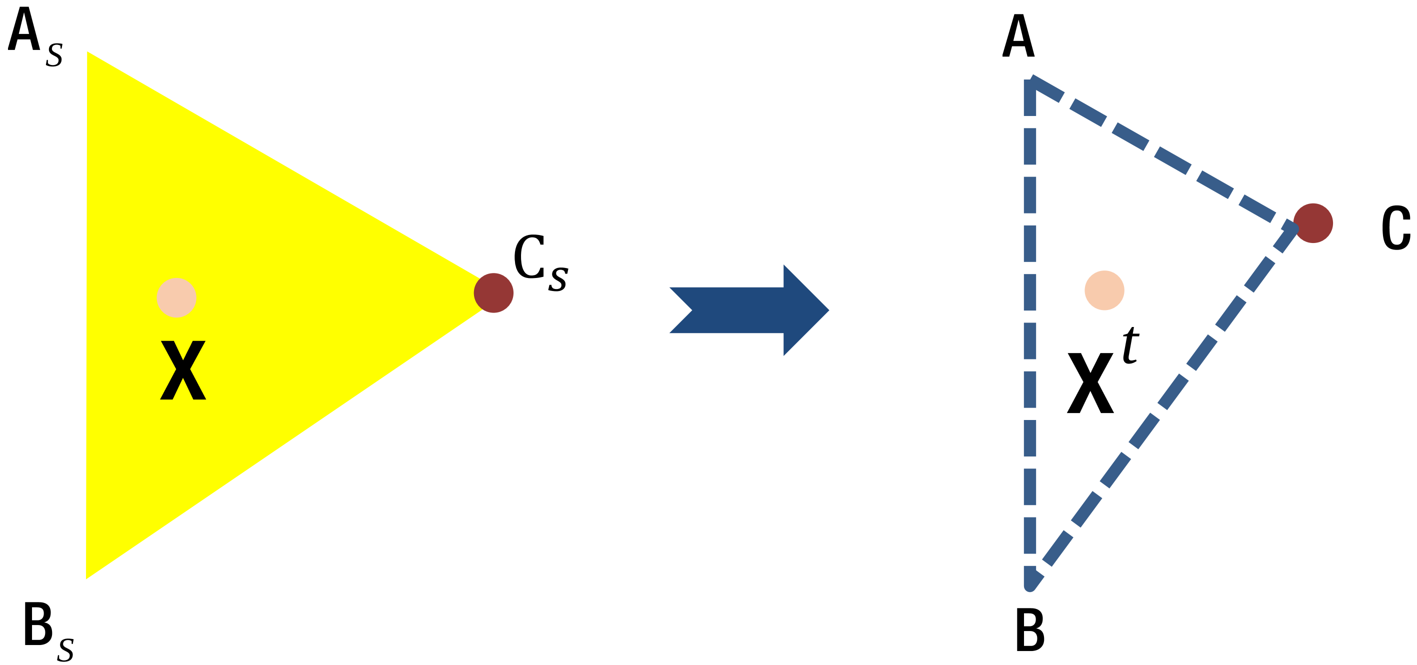
Affine transform is a pixel transportation $X \rightarrow X^t$
It is controlled by the movement of the three vertices of the triangle

Barycentric Coordinates



Each point **X** has an invariant representation with respect to the three vertices.

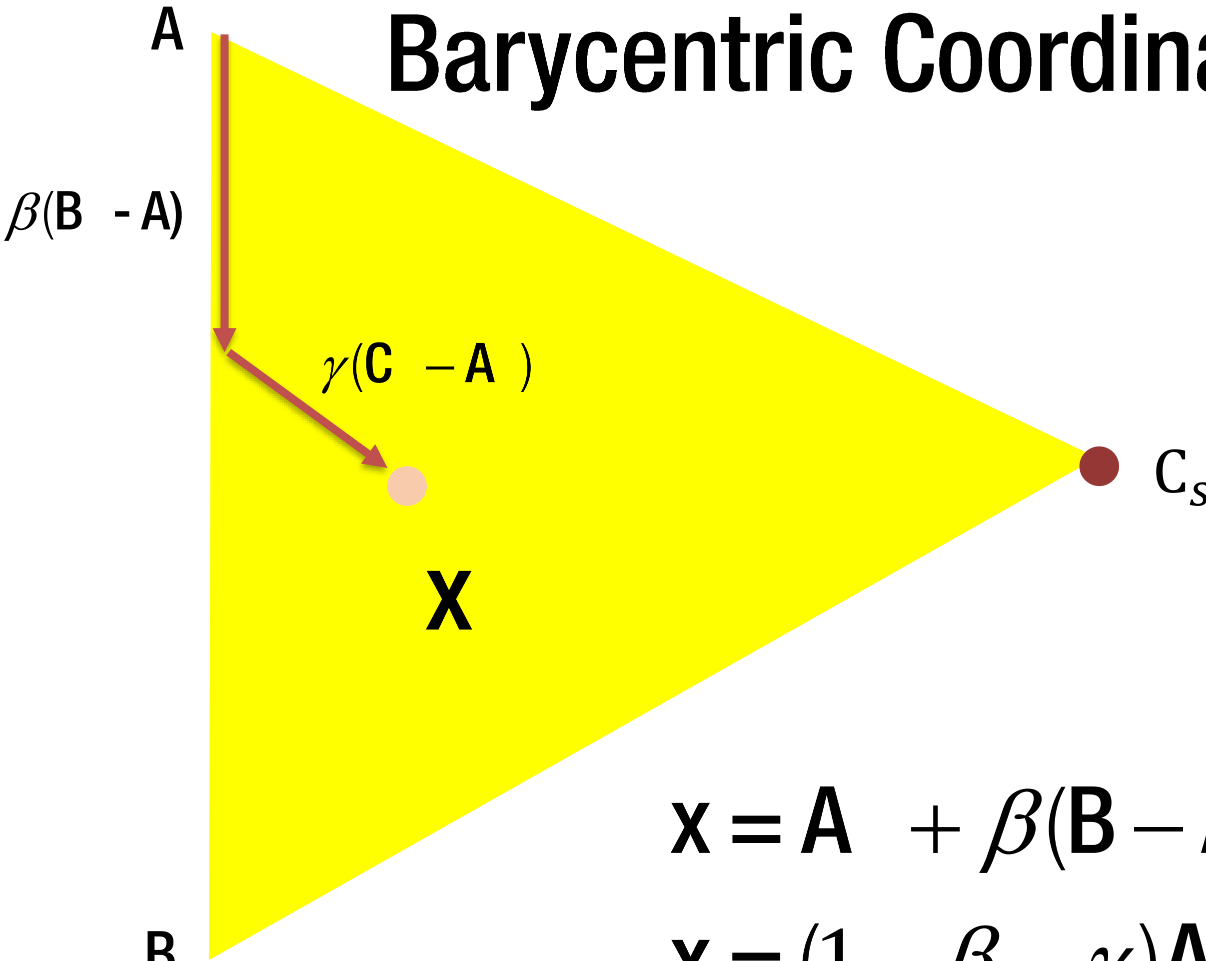
Barycentric Coordinates



$$\mathbf{X} = \alpha \mathbf{A}_s + \beta \mathbf{B}_s + \gamma \mathbf{C}_s \qquad \mathbf{X}^t = \alpha \mathbf{A}_t + \beta \mathbf{B}_t + \gamma \mathbf{C}_t$$

$$\alpha + \beta + \gamma = 1$$

Barycentric Coordinates

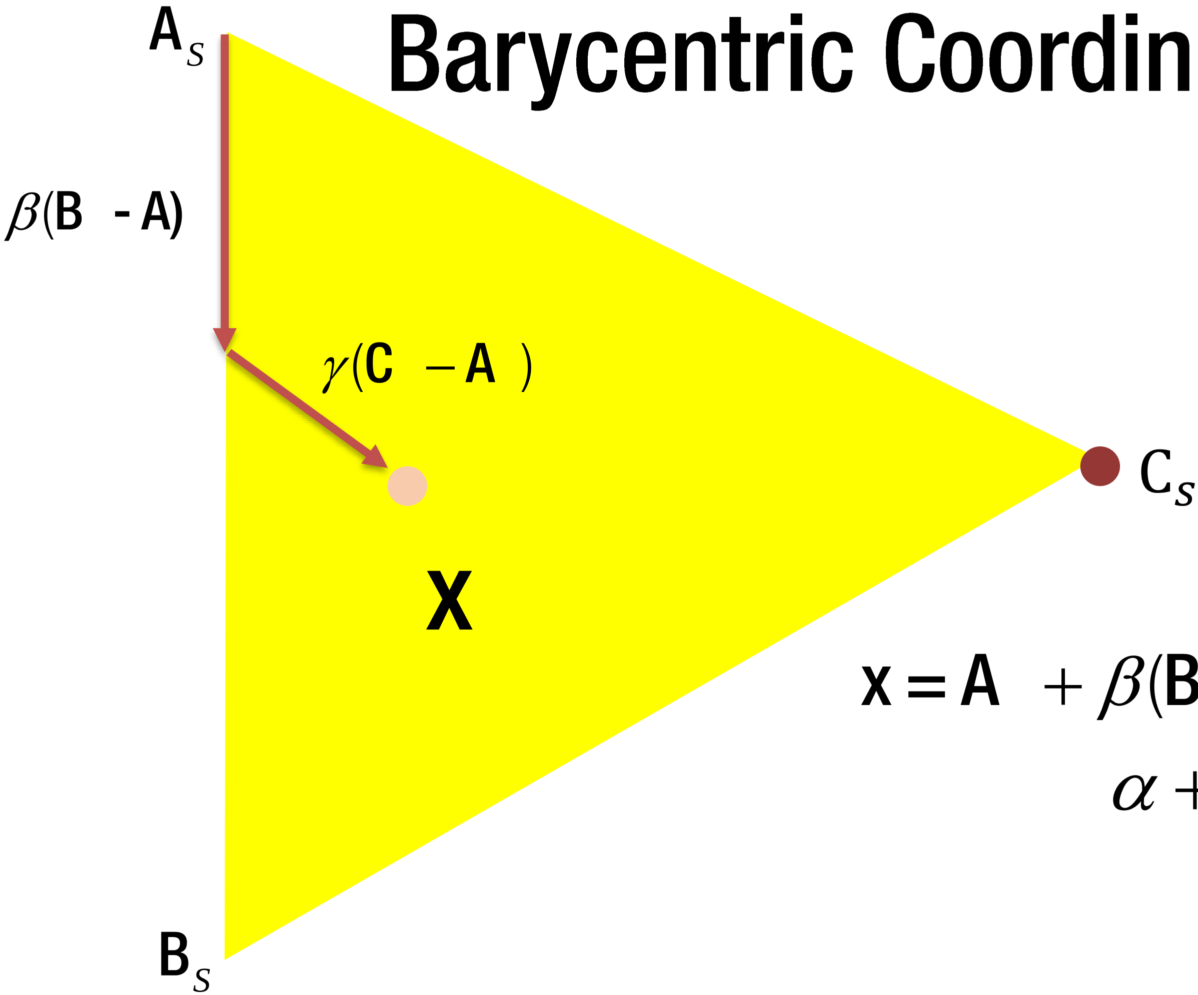


$$\mathbf{X} = \mathbf{A} + \beta(\mathbf{B} - \mathbf{A}) + \gamma(\mathbf{C} - \mathbf{A})$$

$$\mathbf{X} = (1 - \beta - \gamma)\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$$

$$\mathbf{X} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C} \qquad \alpha + \beta + \gamma = 1$$

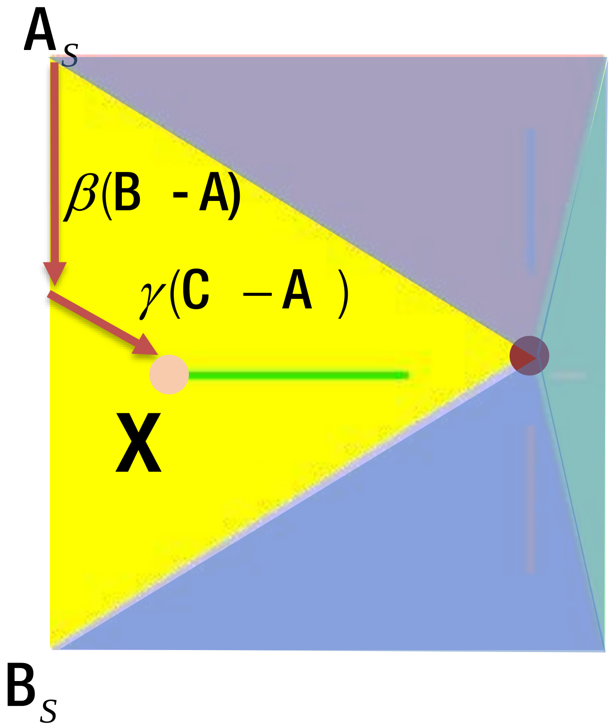
Barycentric Coordinates



$$\mathbf{x} = \mathbf{A} + \beta(\mathbf{B} - \mathbf{A}) + \gamma(\mathbf{C} - \mathbf{A})$$

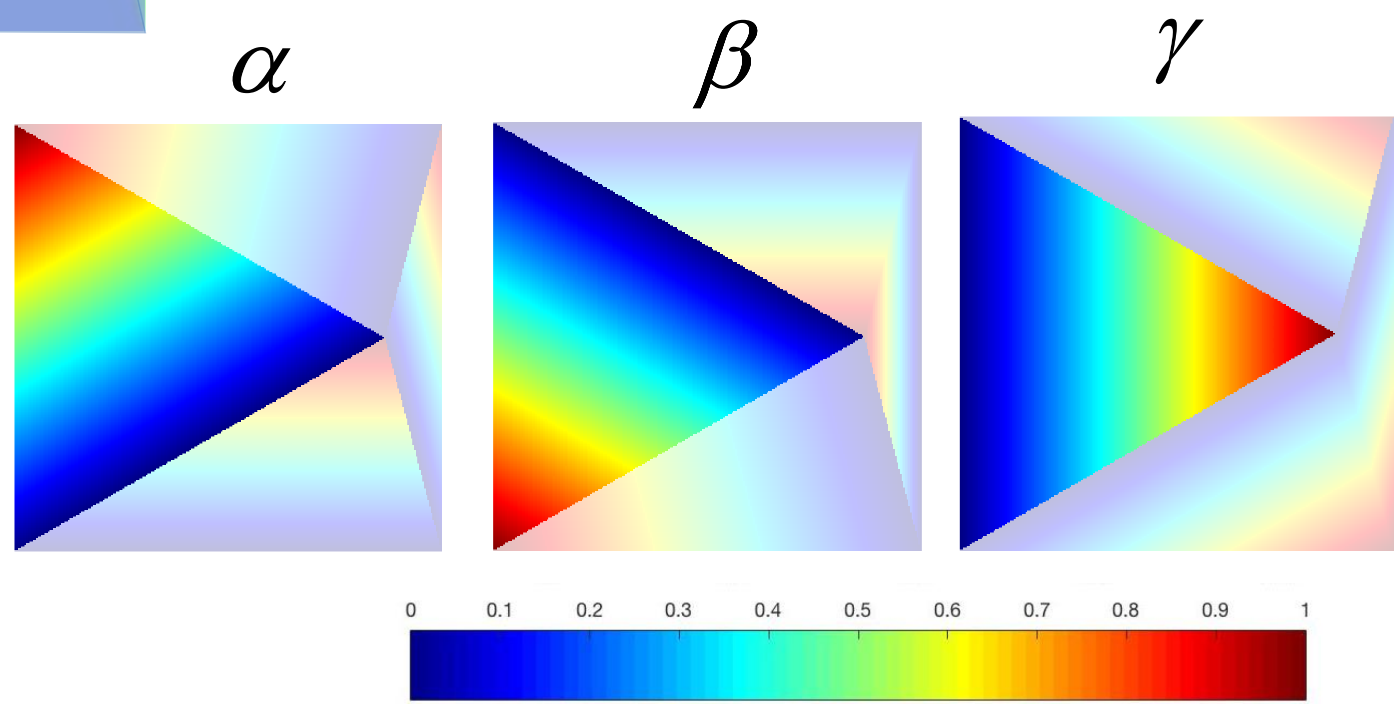
$$\alpha + \beta + \gamma = 1$$

$$\begin{bmatrix} \mathbf{A}_x & \mathbf{B}_x & \mathbf{C}_x \\ \mathbf{A}_y & \mathbf{B}_y & \mathbf{C}_y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ linear equations in 3 unknowns



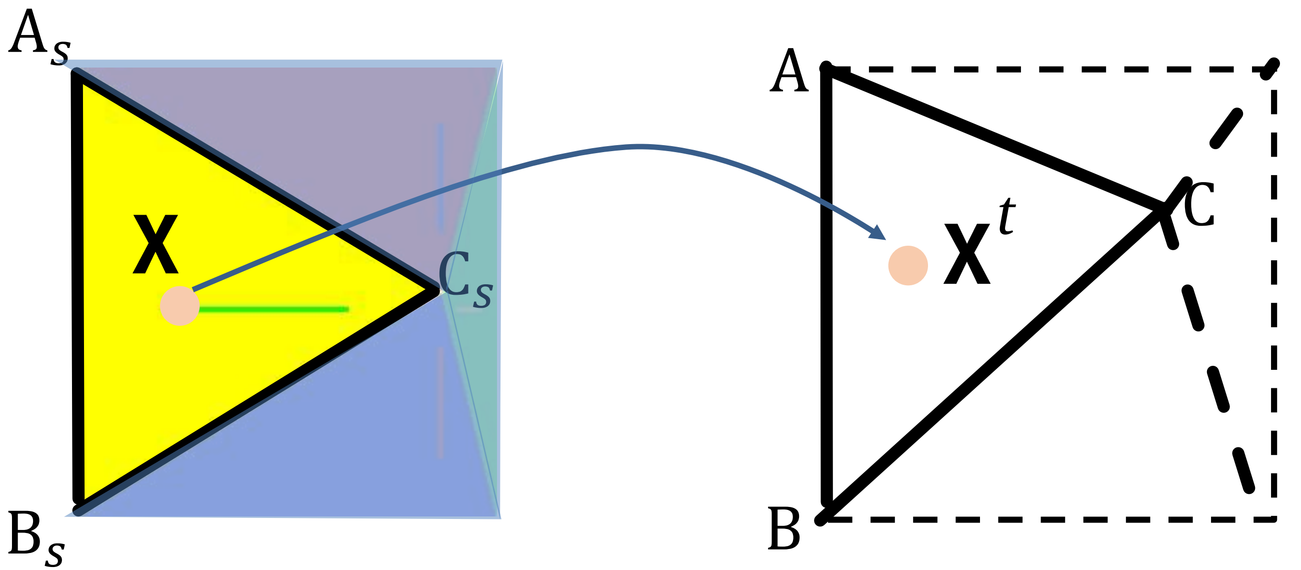
Barycentric coordinate

$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

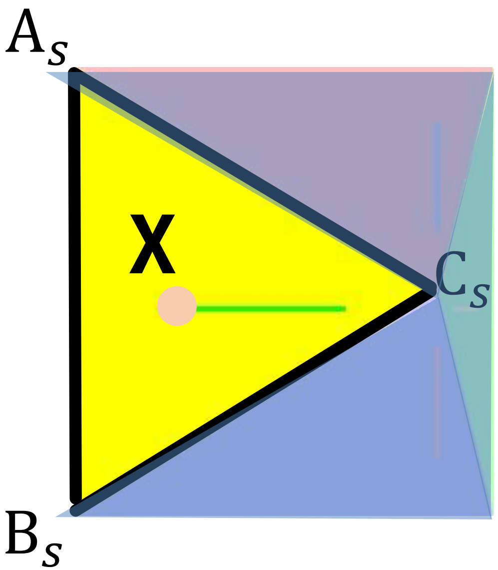


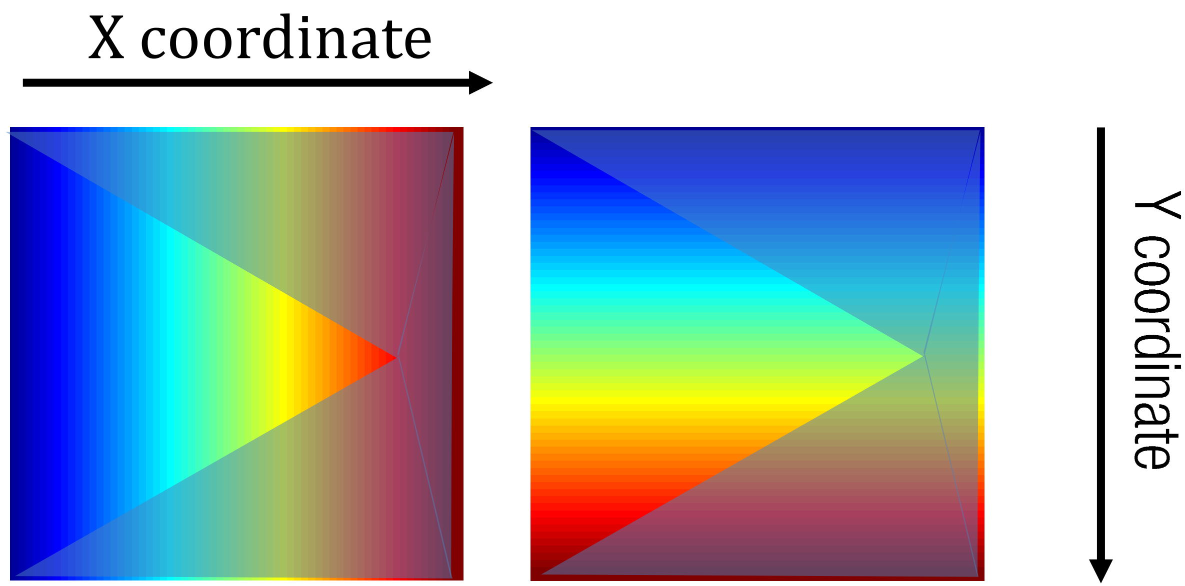
Warping with Barycentric Coordinate

$$\mathbf{X} = \alpha \mathbf{A}_S + \beta \mathbf{B}_S + \gamma \boldsymbol{C}_S \qquad \mathbf{X}^t = \alpha \mathbf{A} + \beta \mathbf{B} + \gamma \mathbf{C}$$

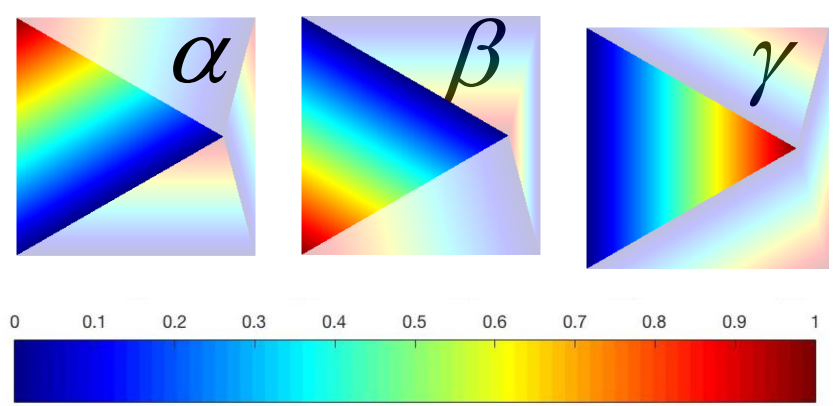


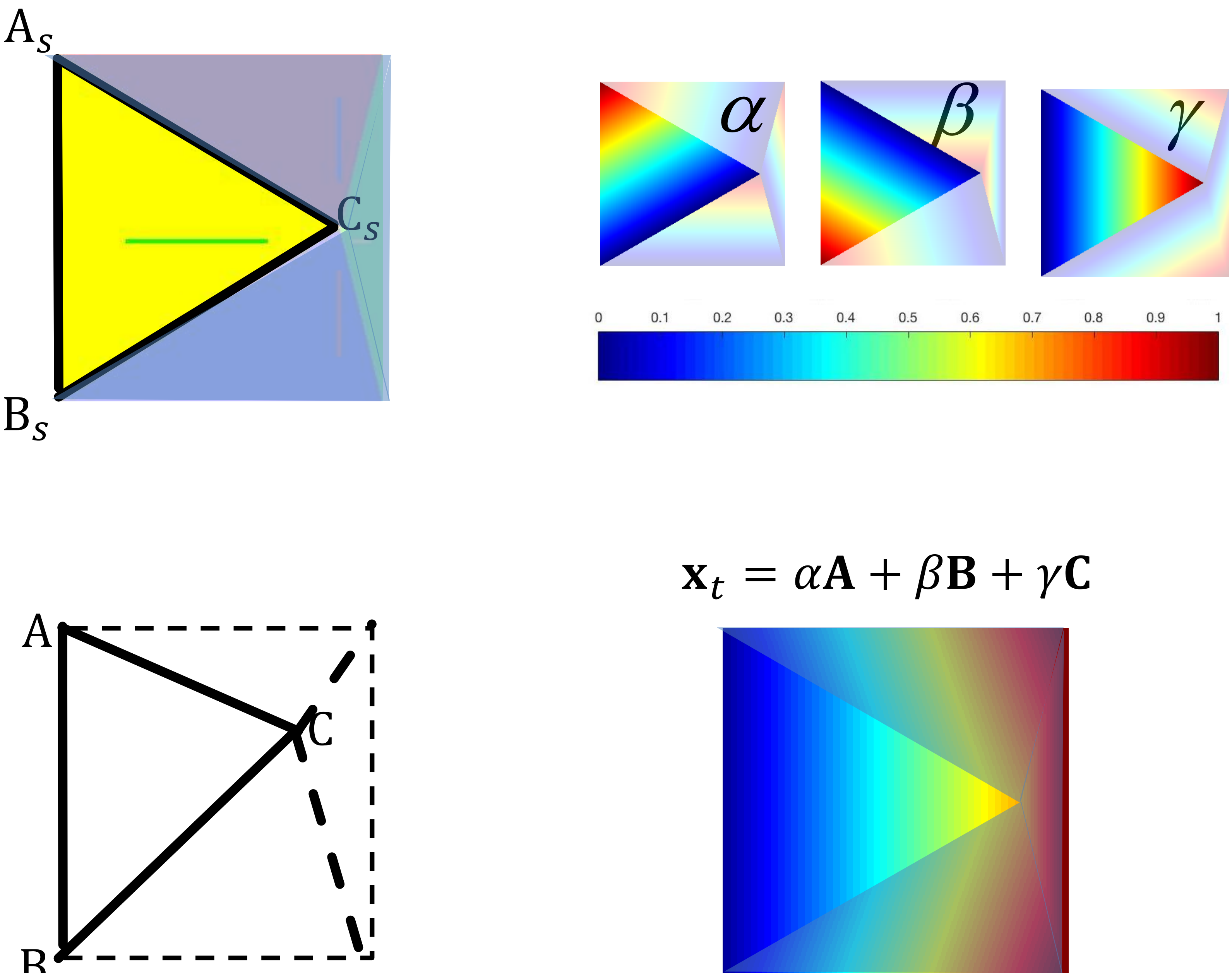
Warping with Barycentric Coordinate





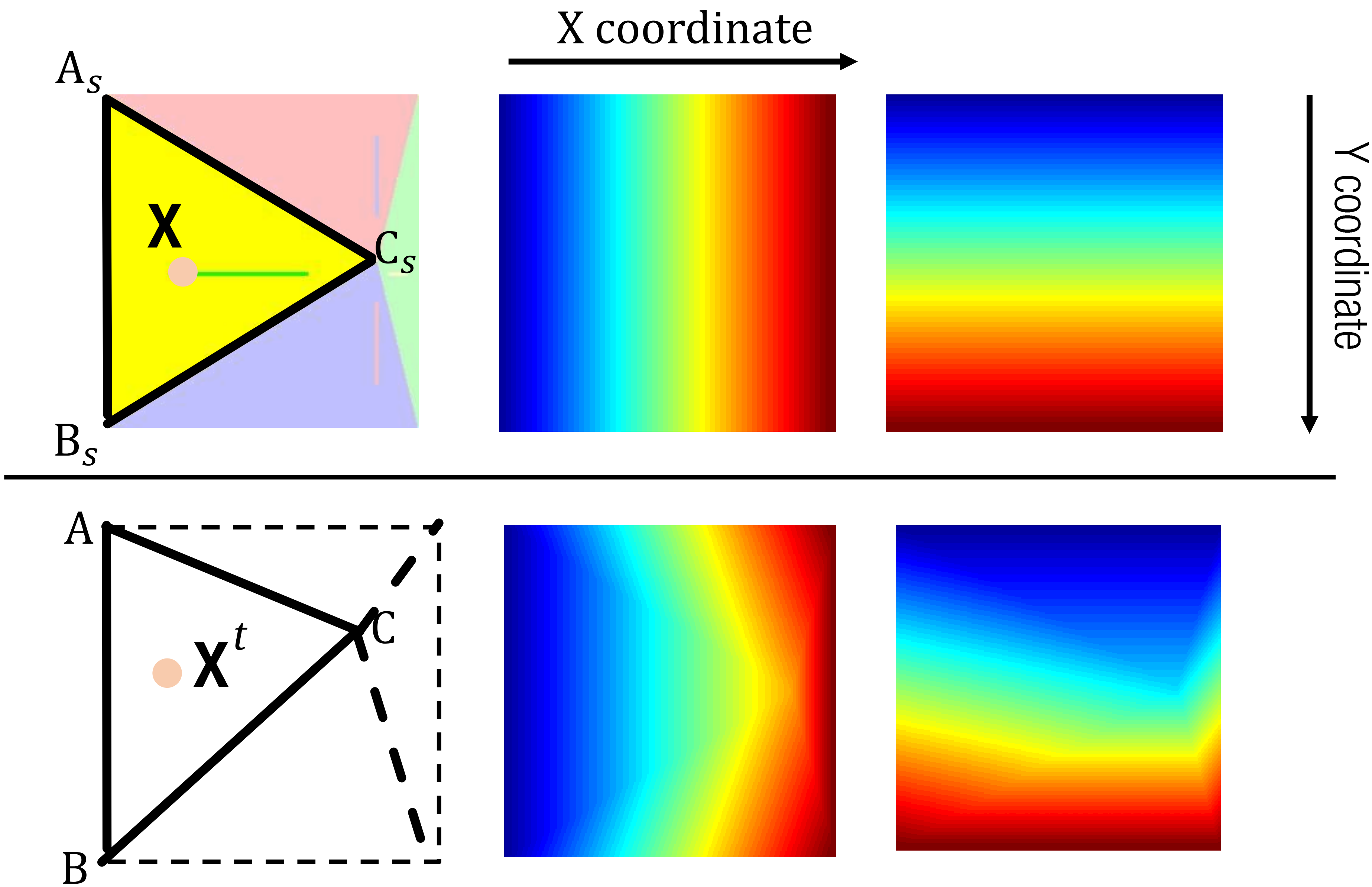
$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \lambda \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



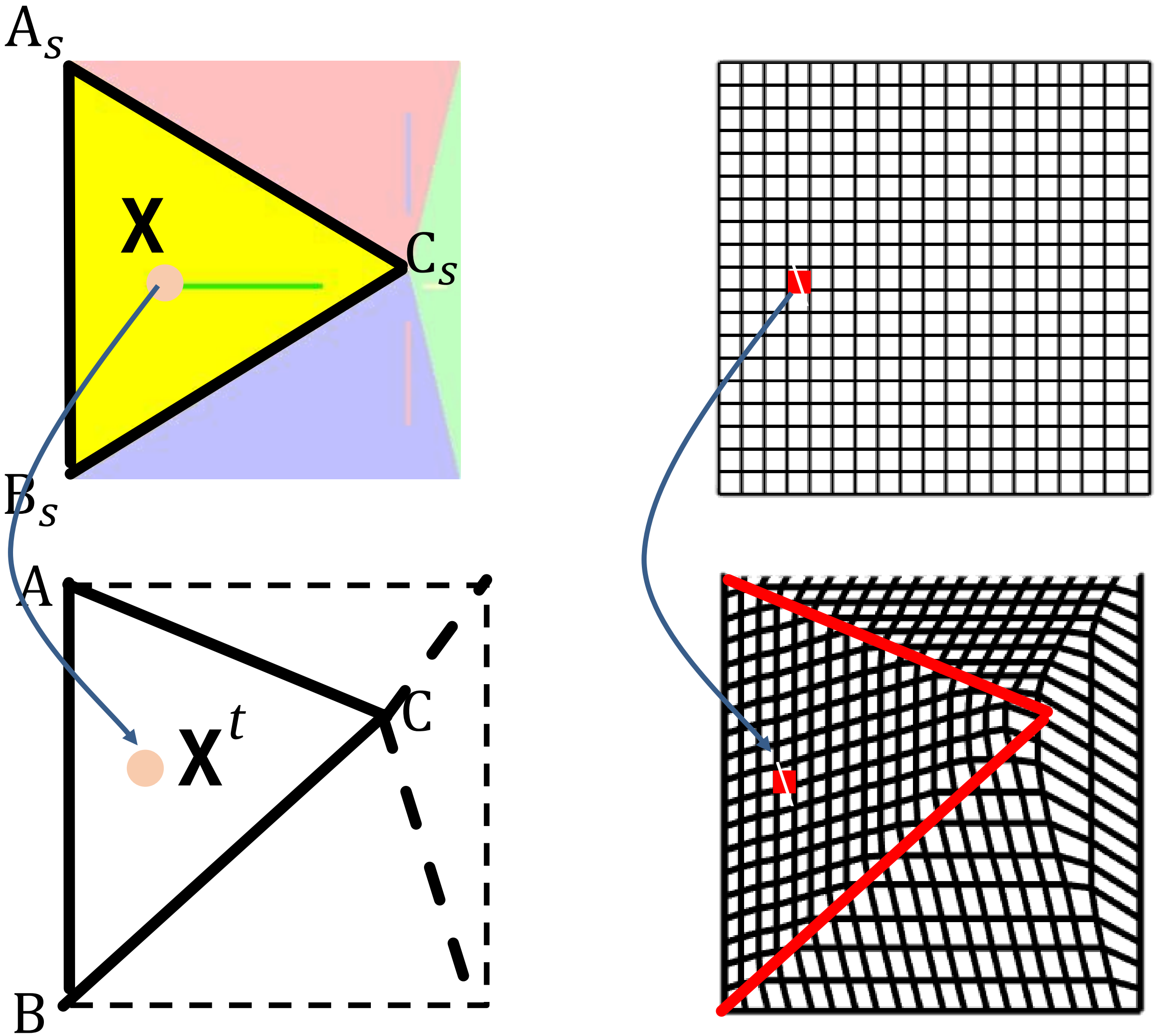


A_s
B_s
C_s
α
β
γ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
A
B
C
$\mathbf{x}_t = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma\mathbf{C}$

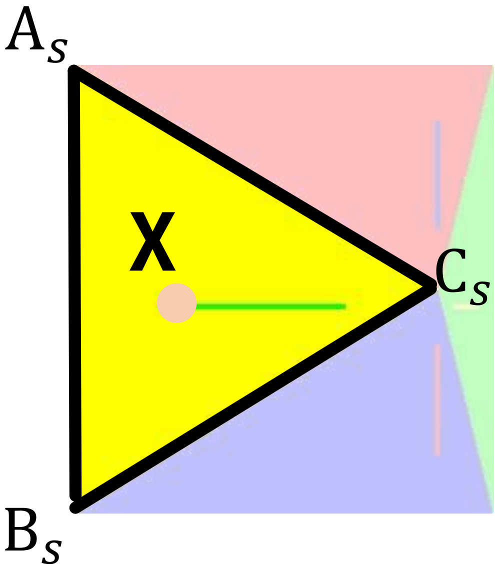
Warping with Barycentric Coordinate

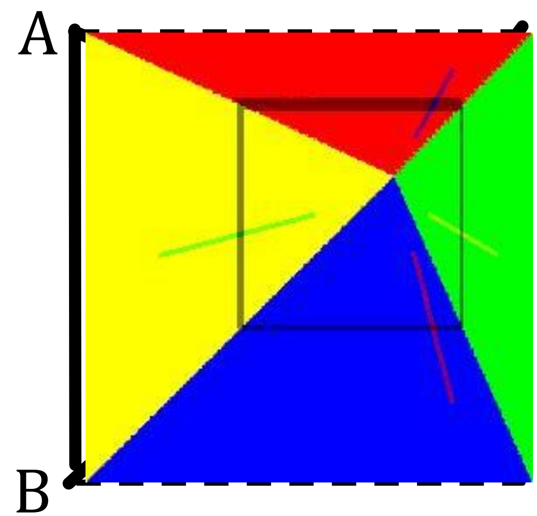


Grids before and after warping

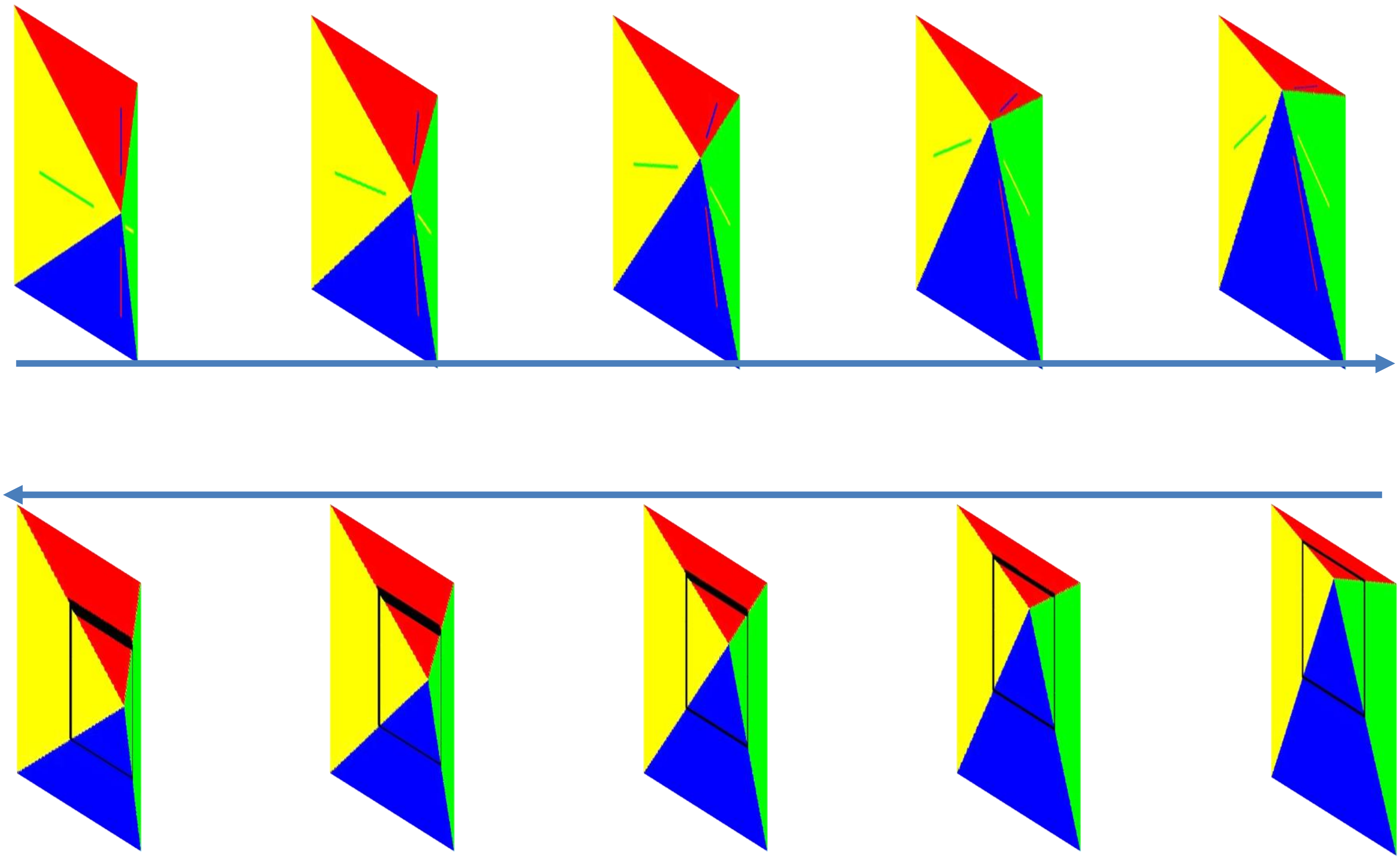


Step 3: Average warped image

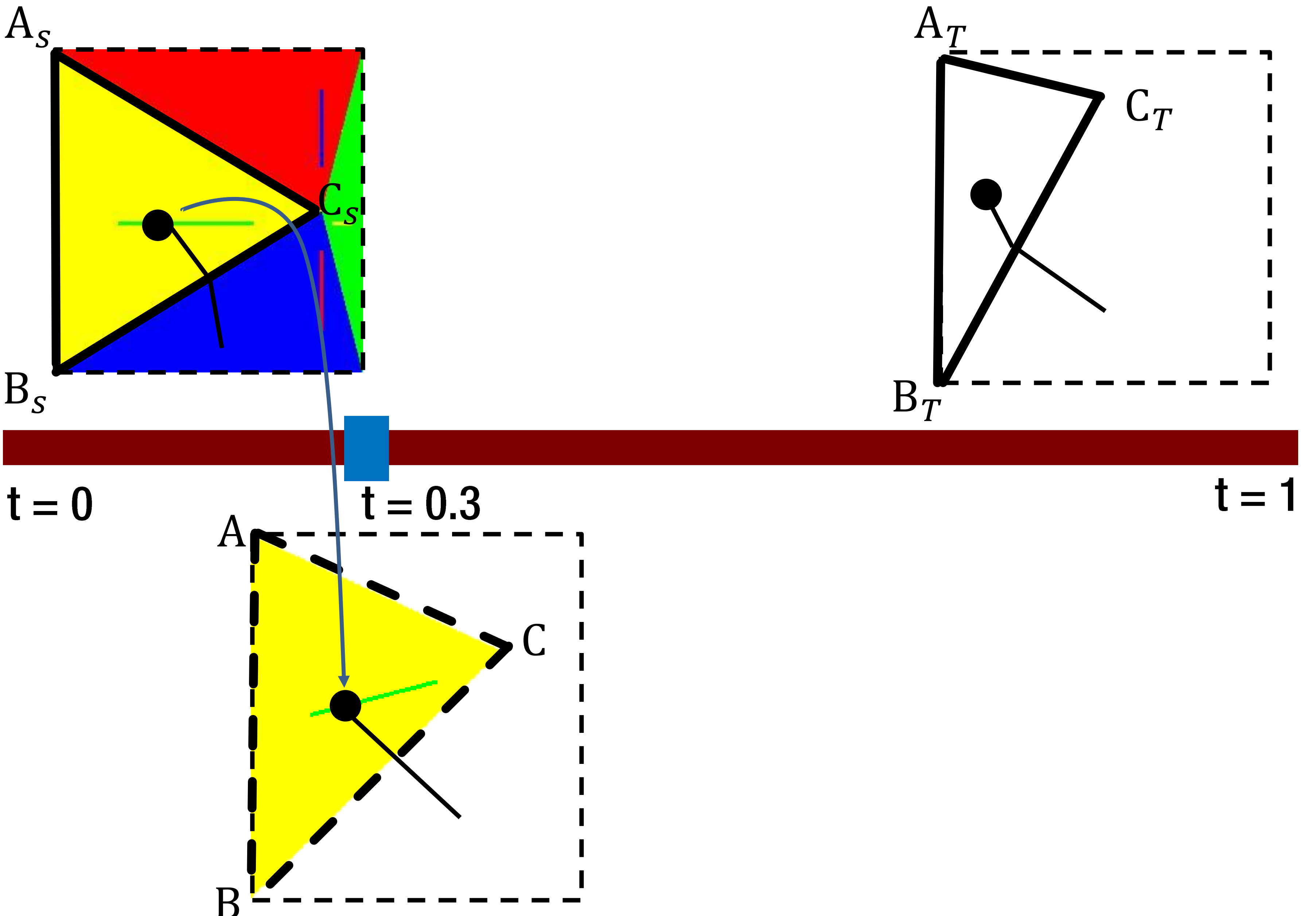




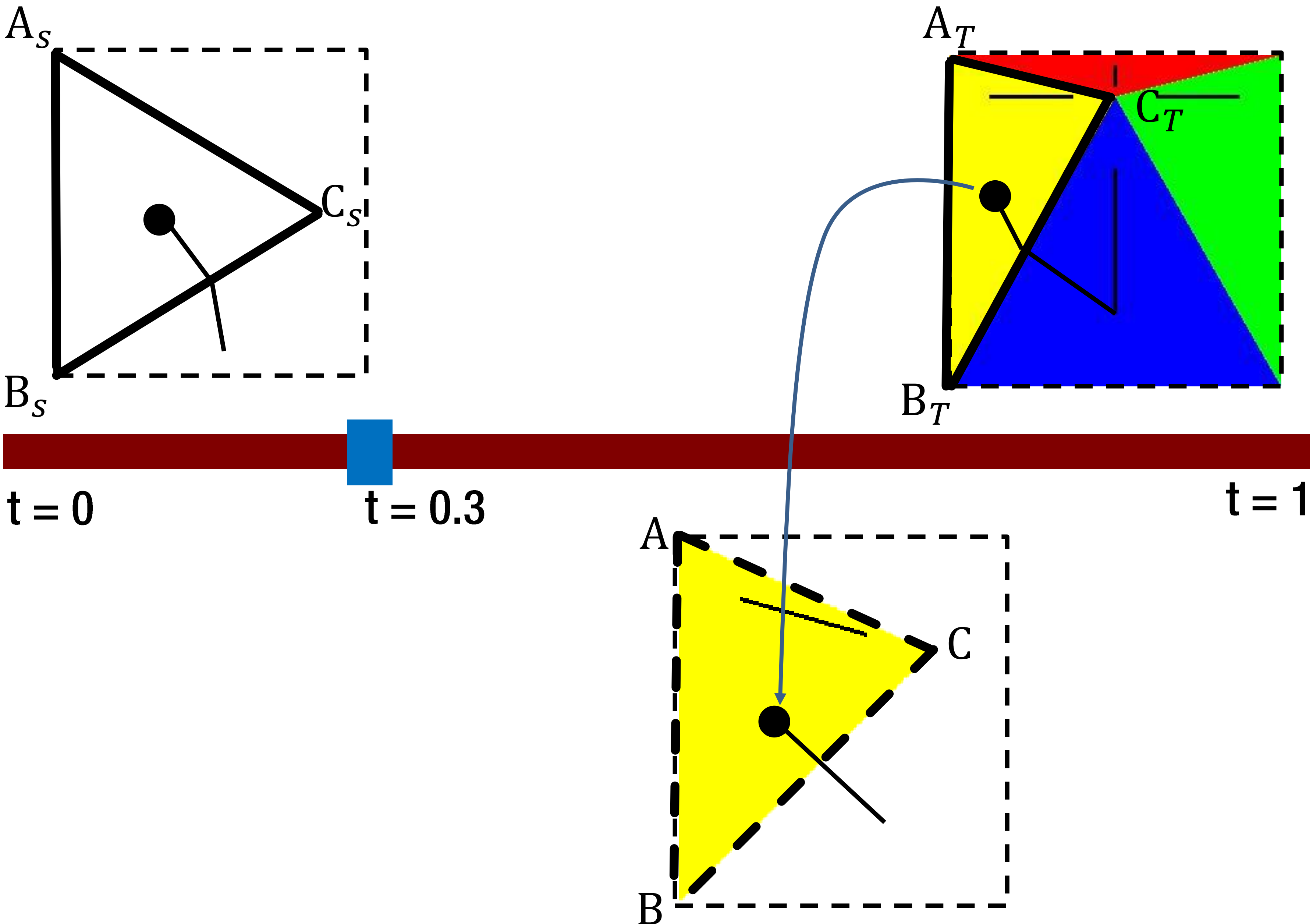
Warping and Cross Dissolve



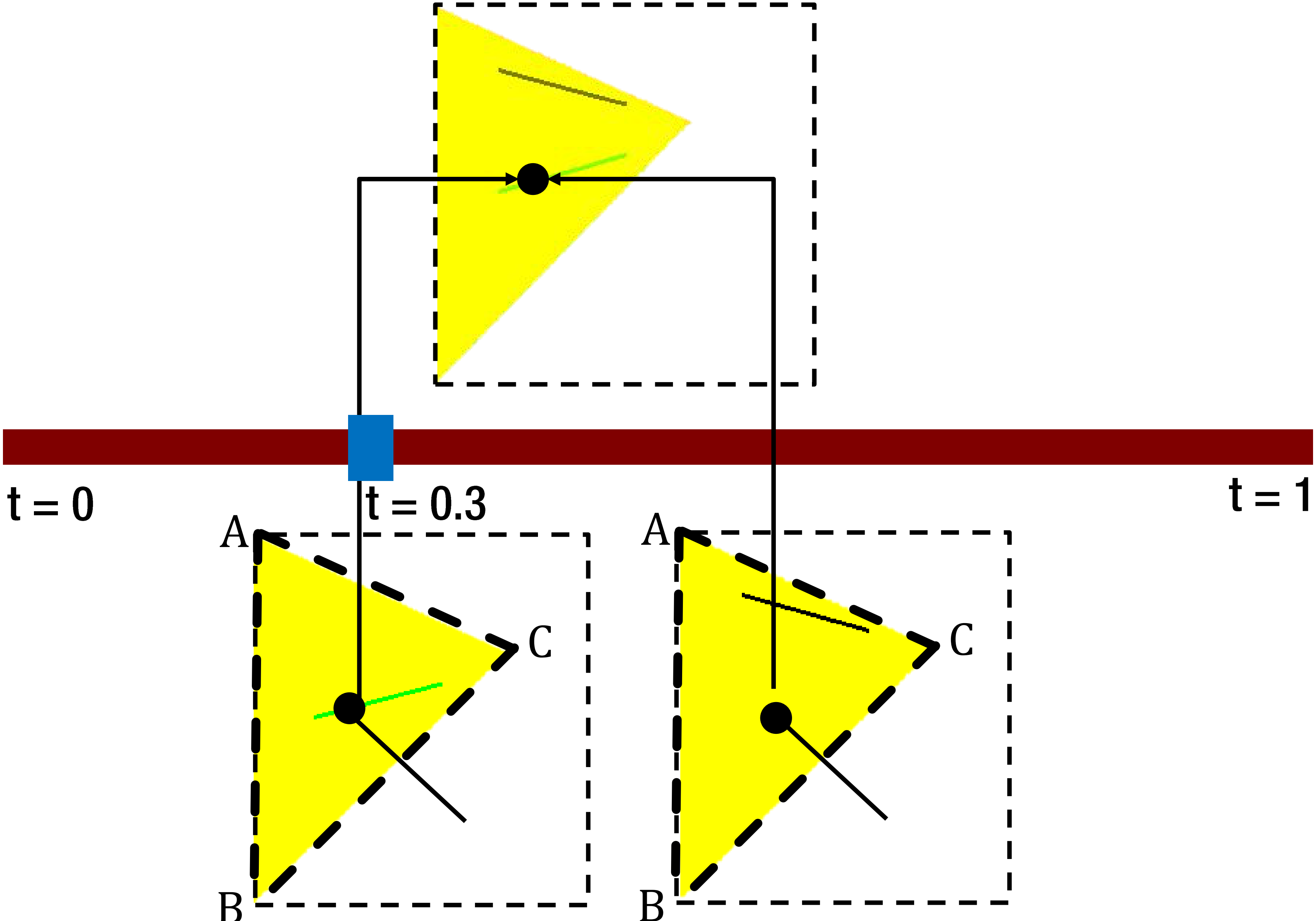
Inverse warping from the source image



Inverse warping from target image



Step 3: averaging warped image



Warping, then cross-dissolve

Morphing procedure:

for every t,

1. Find the average shape
2. Non-parametric warping
3. Find the average color
   – Cross-dissolve the warped images