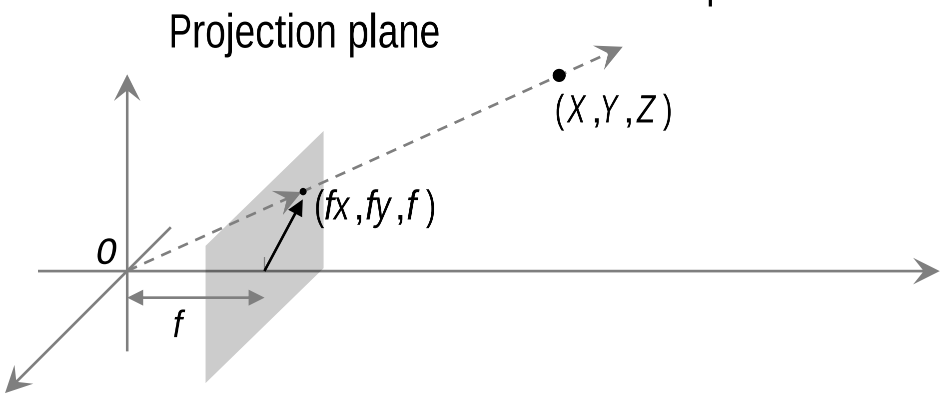
# Image Transform

# Homogeneous Coordinate

2D point =: 3D ray $\lambda(x, y, 1)$



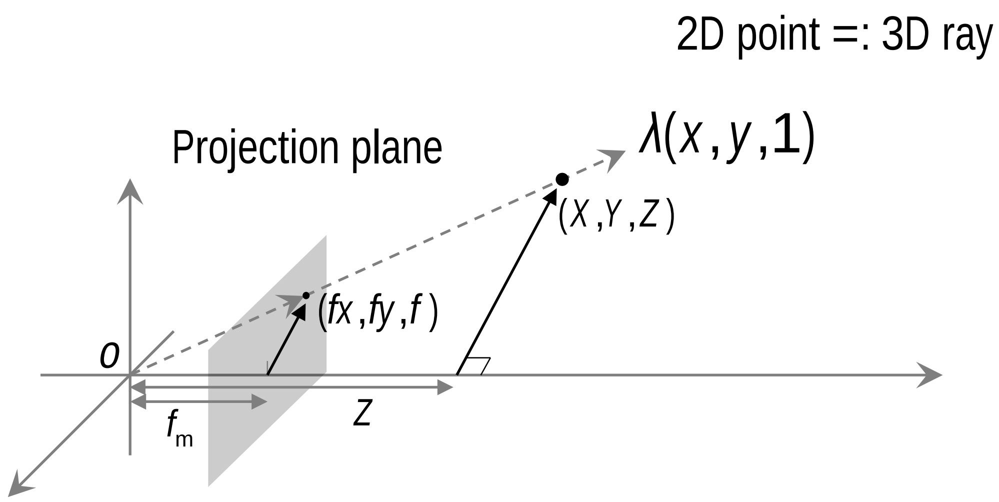

$$(x, y) \rightarrow (x, y, 1)$$

: A point in Euclidean space ( $\mathcal{R}^2$ ) can be represented by a homogeneous representation in Projective space ( $\mathcal{P}^2$ ) (3 numbers).

$$= f(x, y, 1)$$

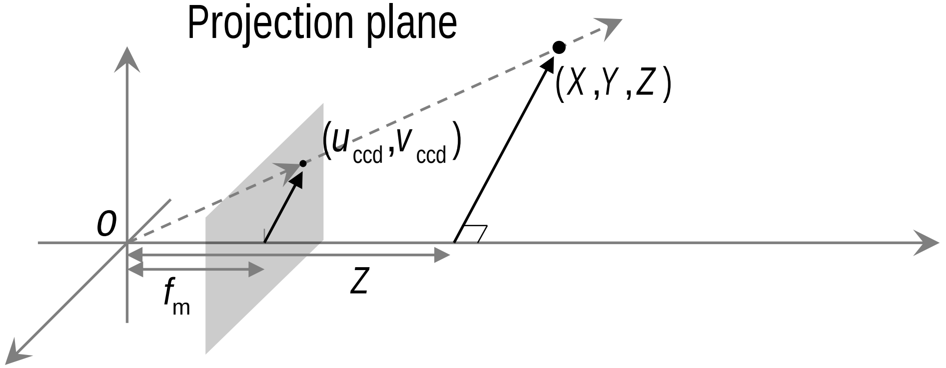
$$= \lambda(x, y, 1) = (X, Y, Z)$$

# 3D Point Projection (Metric Space)

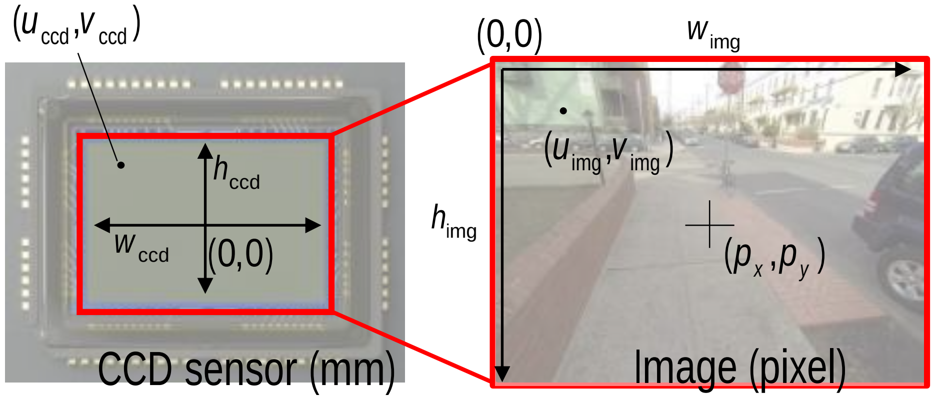



$$(x, y, 1) = (f_m x, f_m y, f_m) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}, f_m)$$

# 3D Point Projection (Pixel Space)

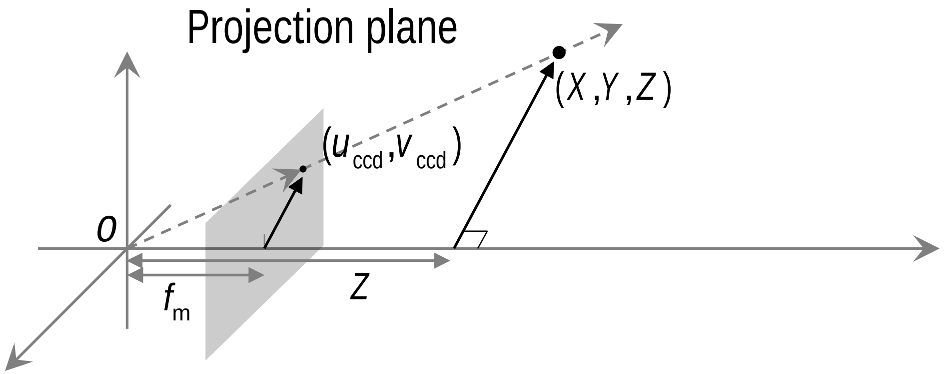



$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$



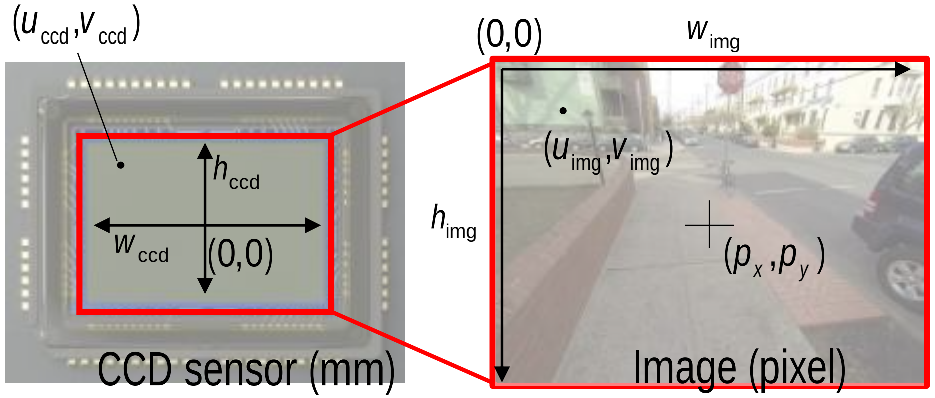

$$u_{img} = f_x \frac{X}{Z} + p_x \qquad v_{img} = f_y \frac{Y}{Z} + p_y$$

# 3D Point Projection (Pixel Space)




$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z}\right)$$

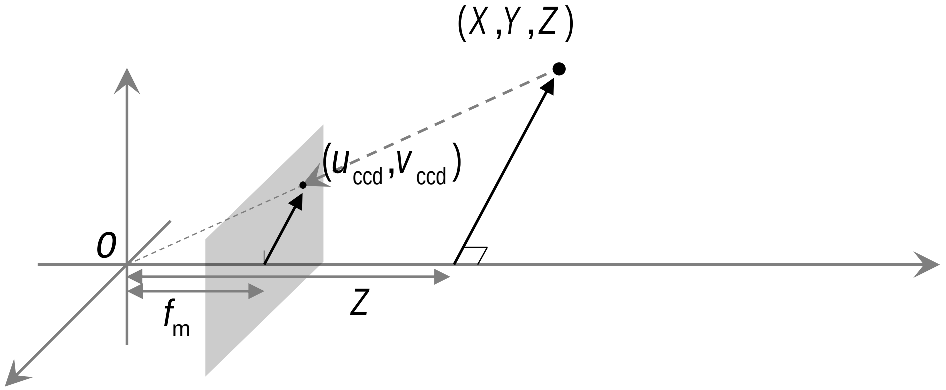
$$u_{img} = f_x \frac{X}{Z} + p_x \qquad v_{img} = f_y \frac{Y}{Z} + p_y$$




$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Homogeneous representation

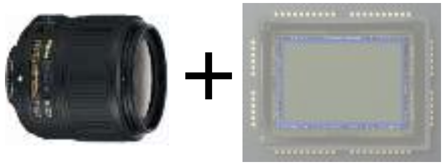
# Camera Intrinsic Parameter




Projection plane

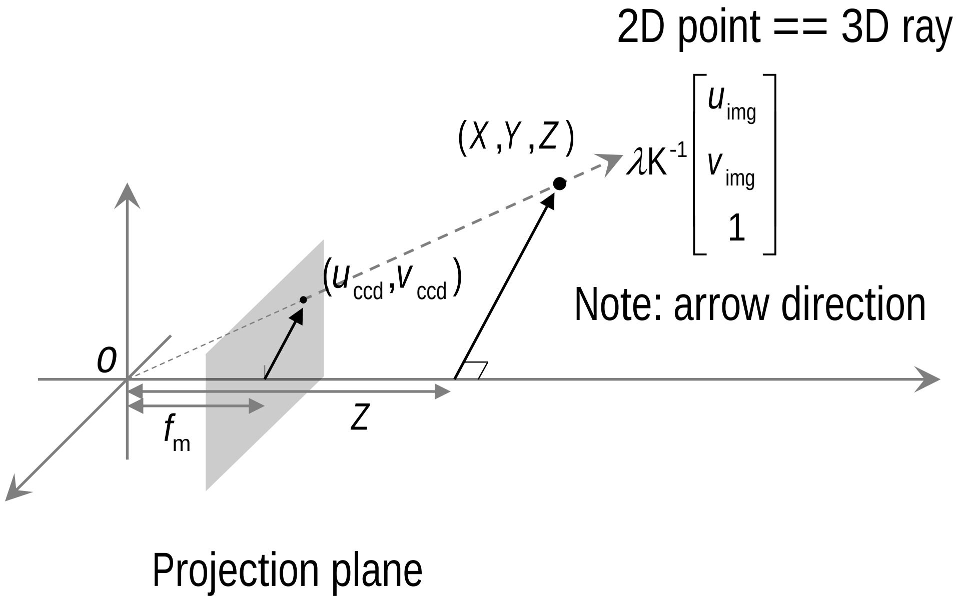
Pixel space　　　　Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

# 2D Inverse Projection




Pixel space Metric space

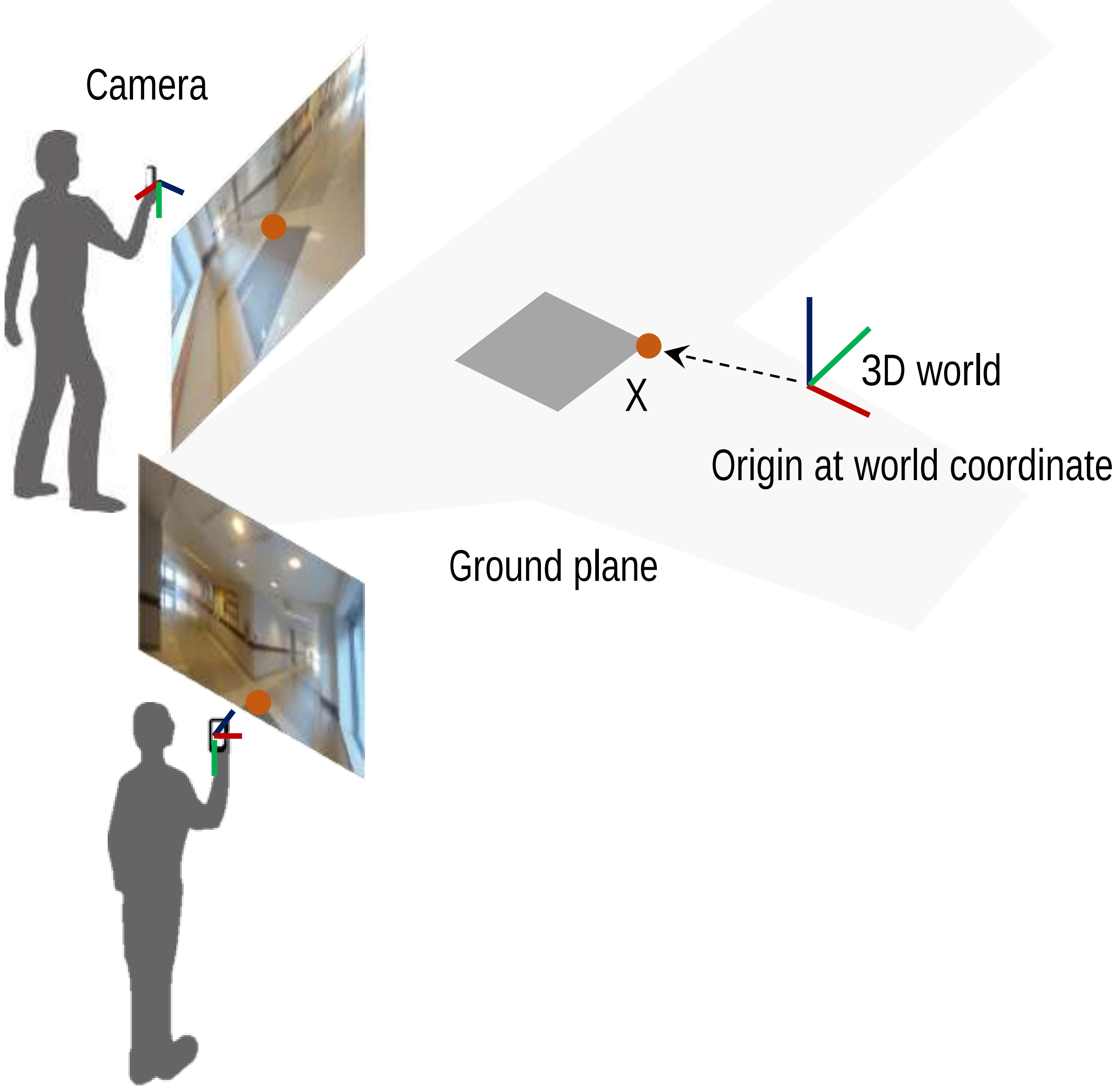
$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda K^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D ray

The 3D point must lie in
the *3D ray* passing through the origin and 2D image point.

# Camera Model (3rd Person Perspective)




Recall camera projection matrix:

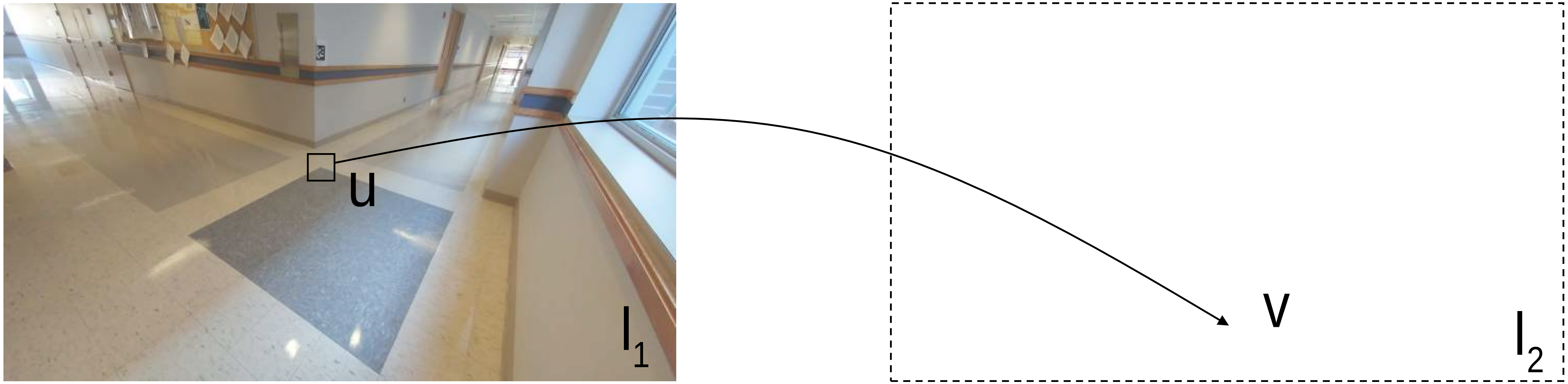
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K} \, \mathrm{X}$$

2D image (pix)

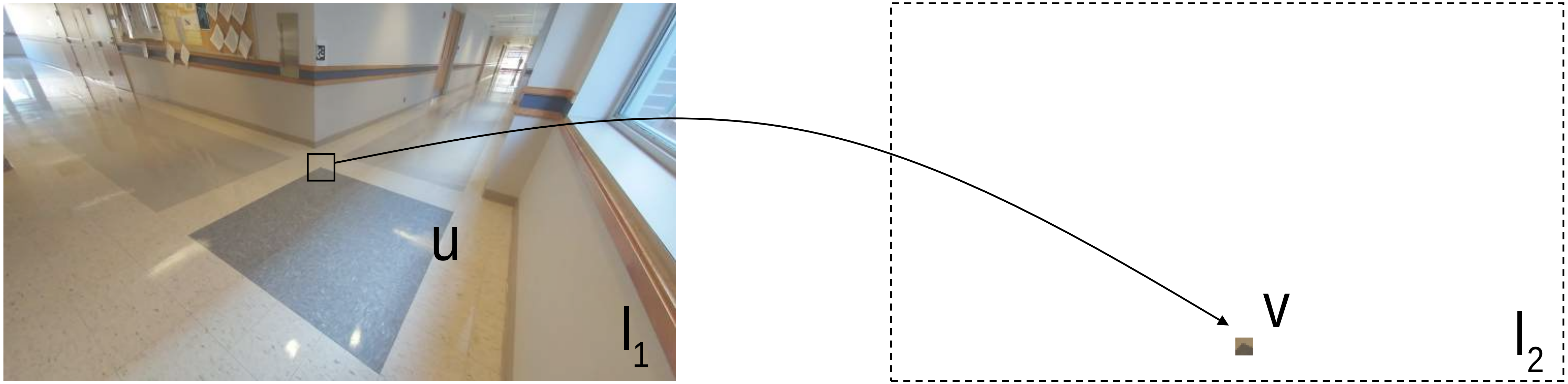
3D world (metric)

# Image Warping (Coordinate Transform)



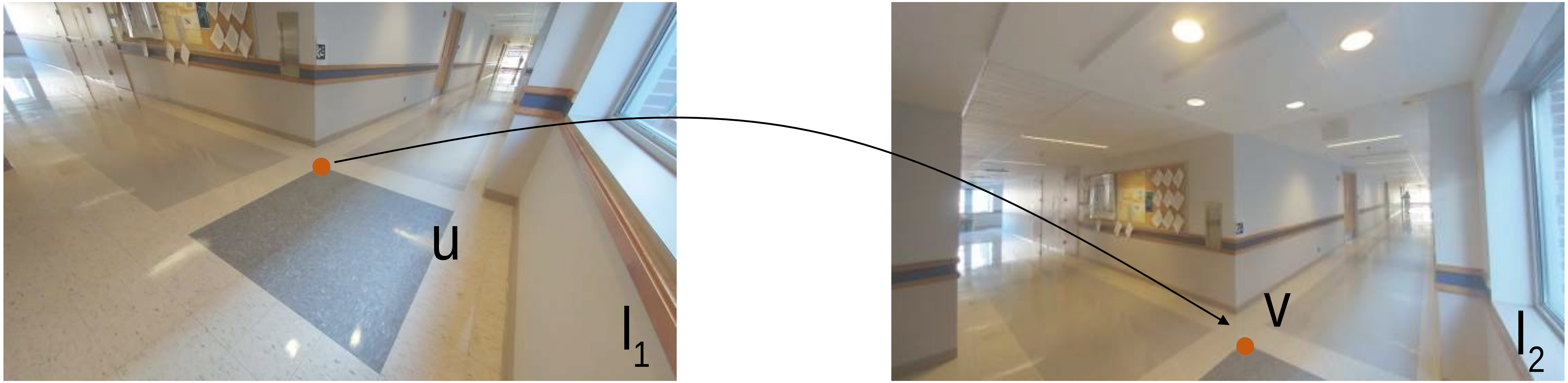

$$I_2(v) = I_1(u)$$

# Image Warping (Coordinate Transform)




$$I_2(v) = I_1(u)$$ : Pixel transport

# Image Warping (Coordinate Transform)




$I_2(v) = I_1(u)$ : Pixel transport

# Uniform Scaling


$$I_2(v) = I_1(u)$$

# Uniform Scaling


$$I_2(v) = I_1(u)$$

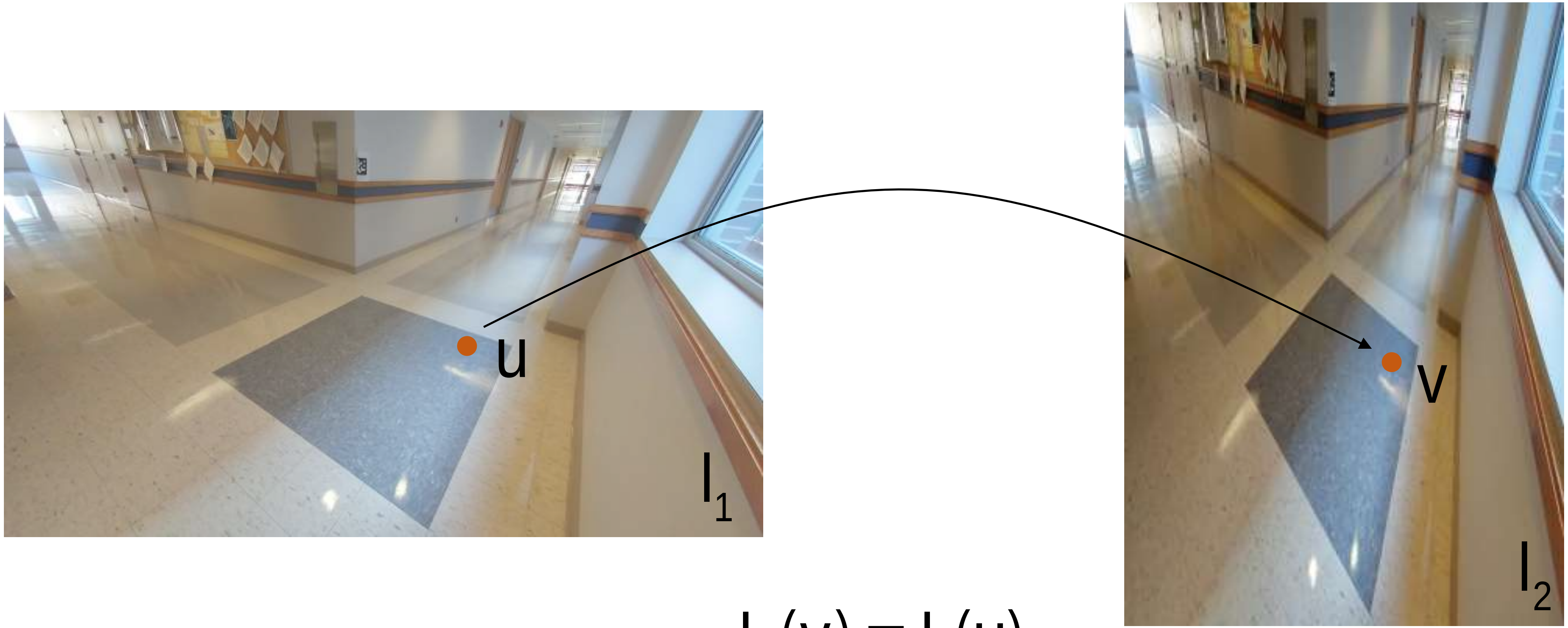
$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Uniform Scaling


$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & & \\ & s_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \qquad s_x = s_y$$

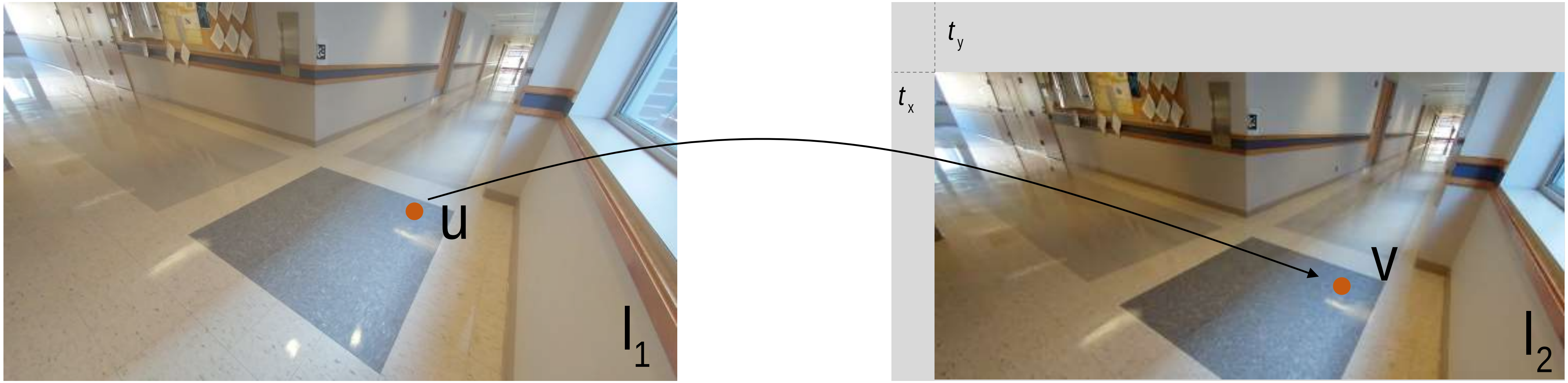
# Aspect Ratio Change




$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & & \\ & s_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \qquad s_x \neq s_y$$

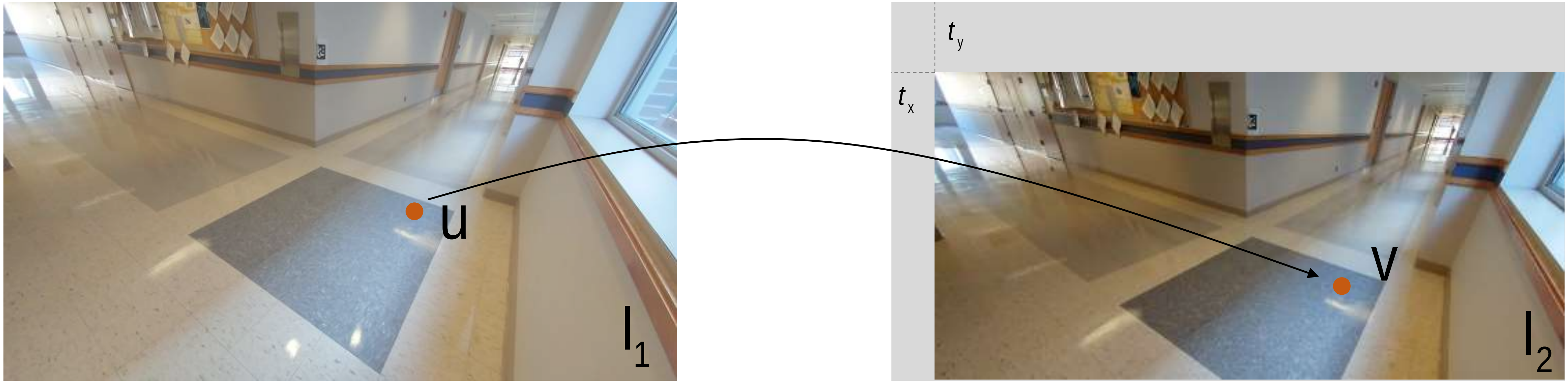
# Translation




$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

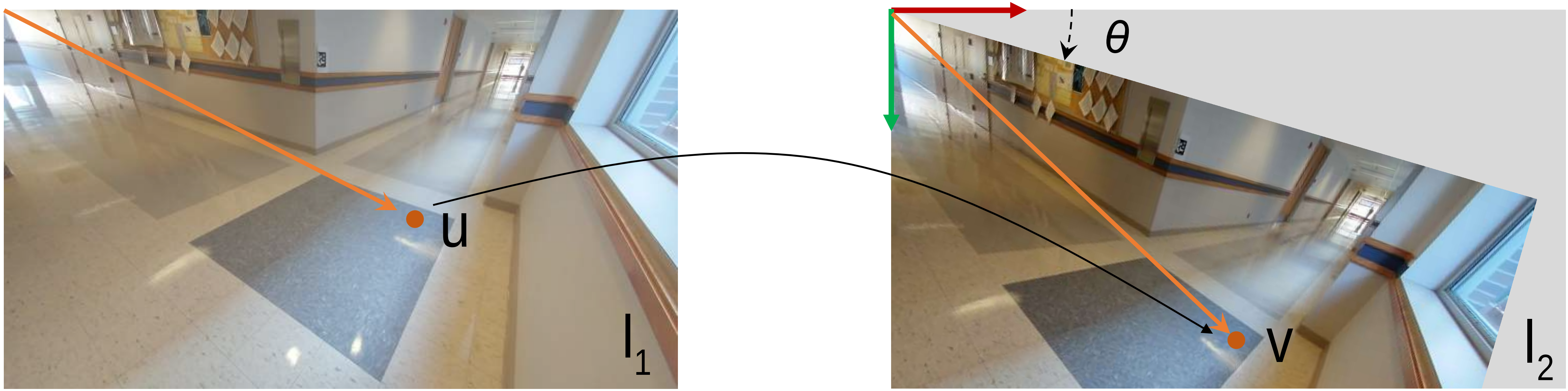
# Translation




$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & t_x \\ & 1 & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

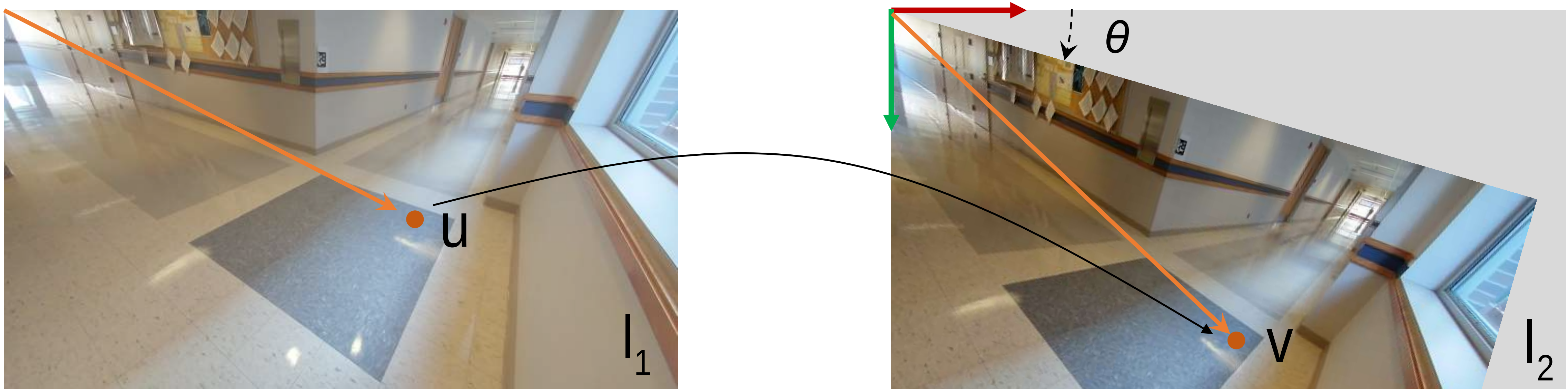
# Rotation




$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

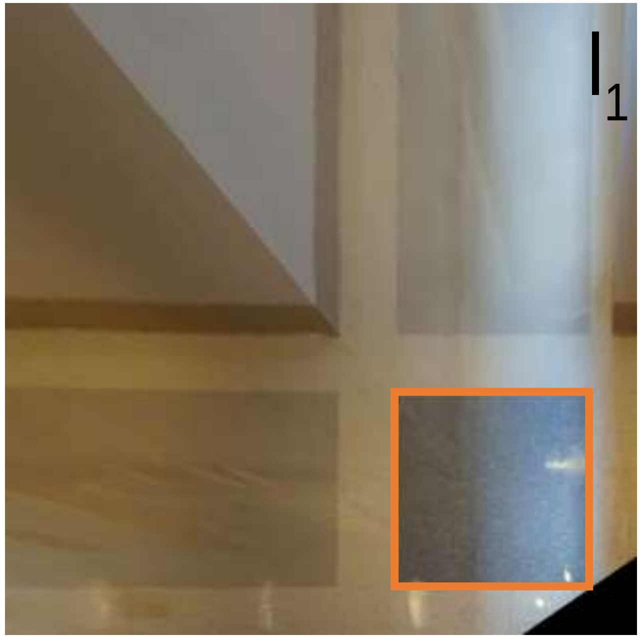
# Rotation

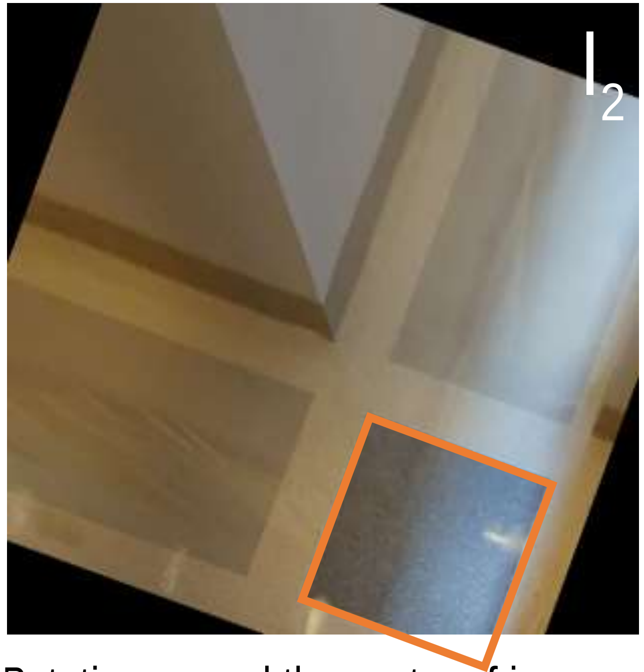



$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Euclidean Transform SE(3)

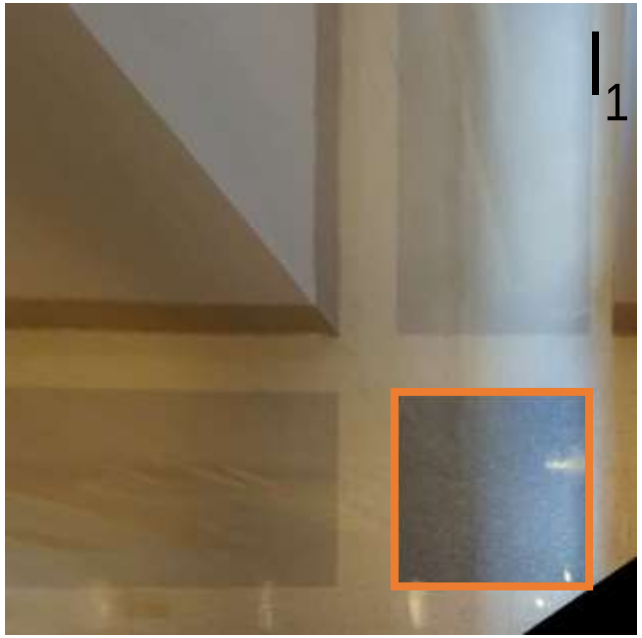


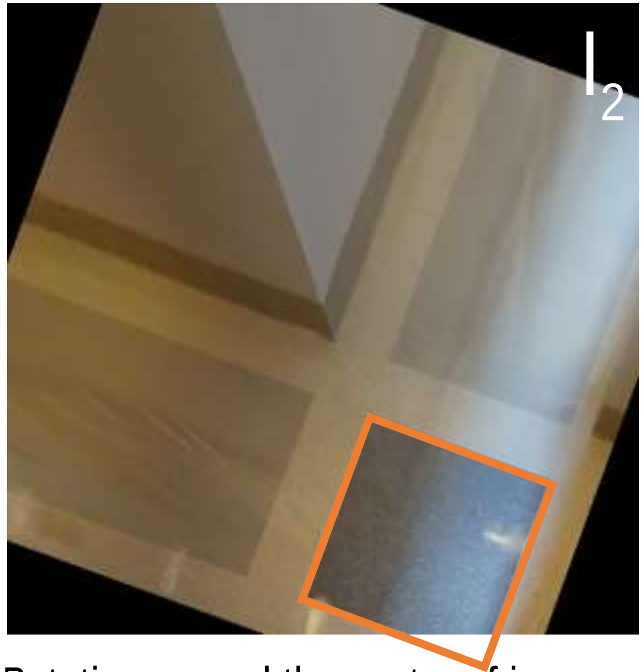




Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Euclidean Transform SE(3)

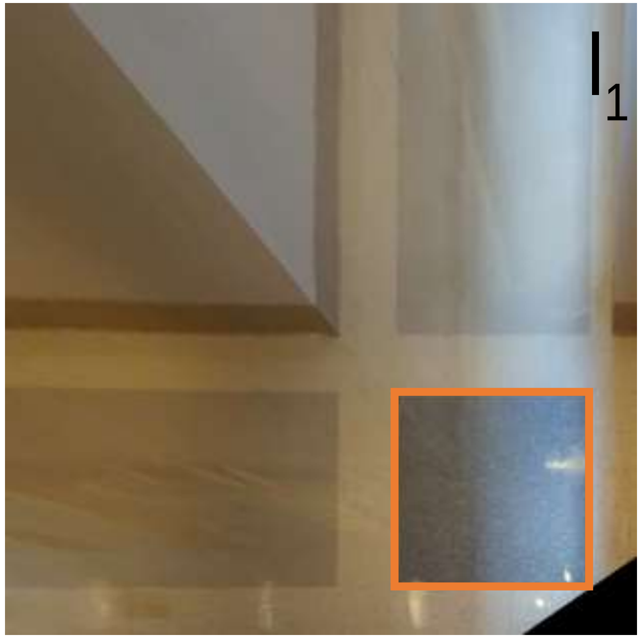


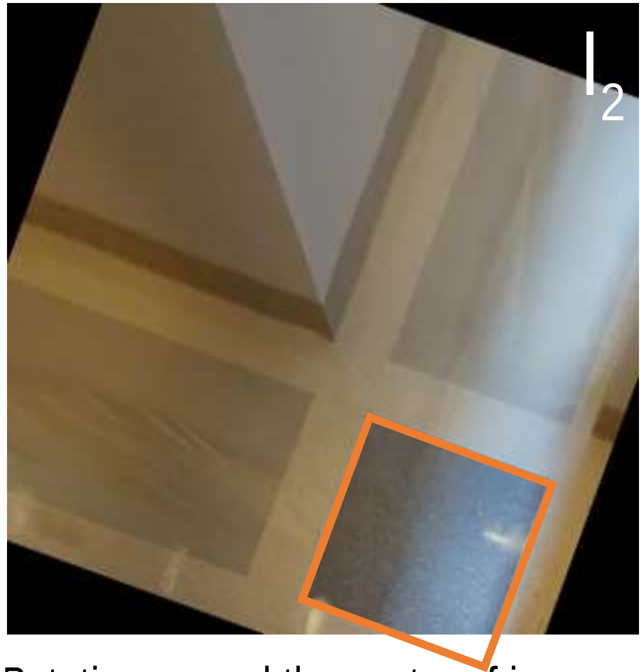




Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Euclidean Transform SE(3)




Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

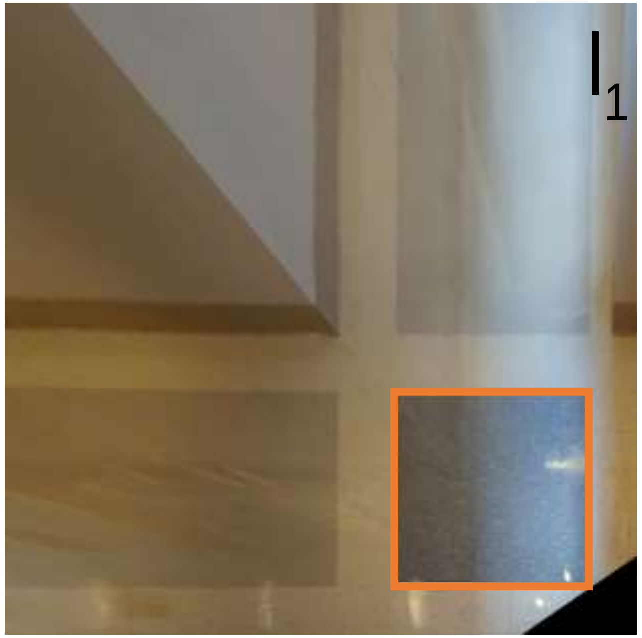
Invariant properties

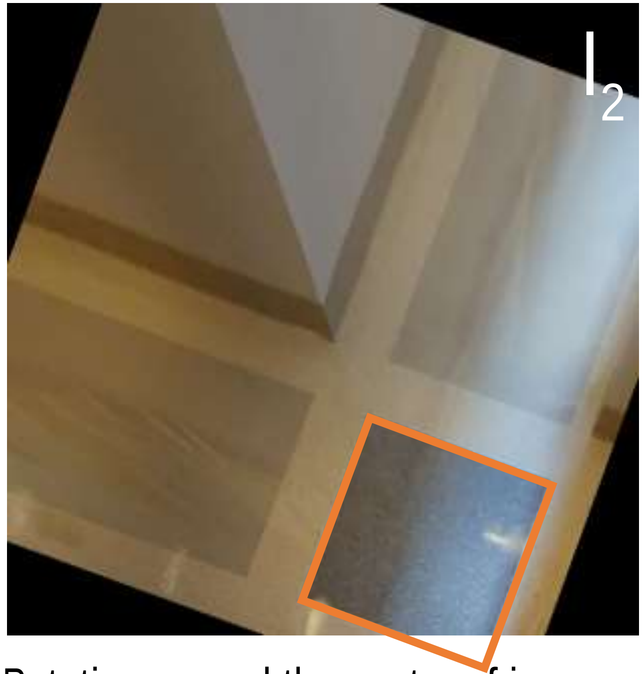
- Length
- Angle
- Area

Degree of freedom

3 (2 translation+1 rotation)

# Euclidean Transform SE(3)




Rotation around the center of image

Invariant properties

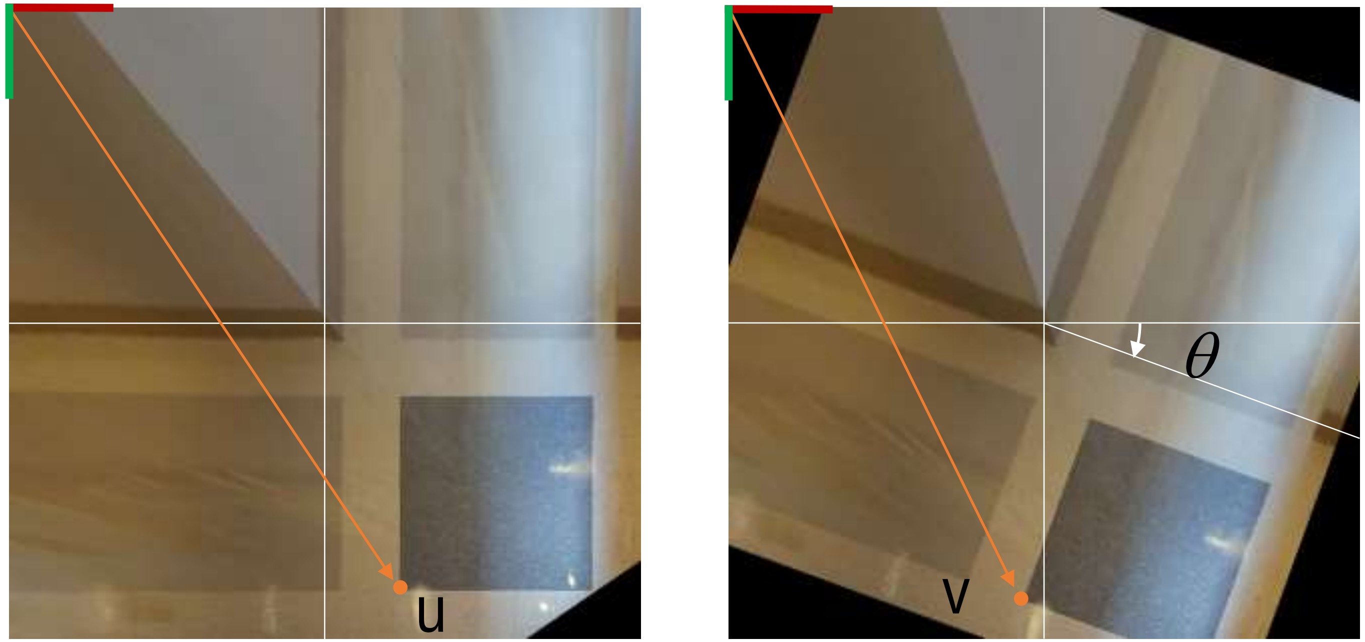
- Length
- Angle
- Area

Degree of freedom

3 (2 translation+1 rotation)

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathrm{v} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{R} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{u} \\ 1 \end{bmatrix}$$

# Euclidean Transform SE(3)

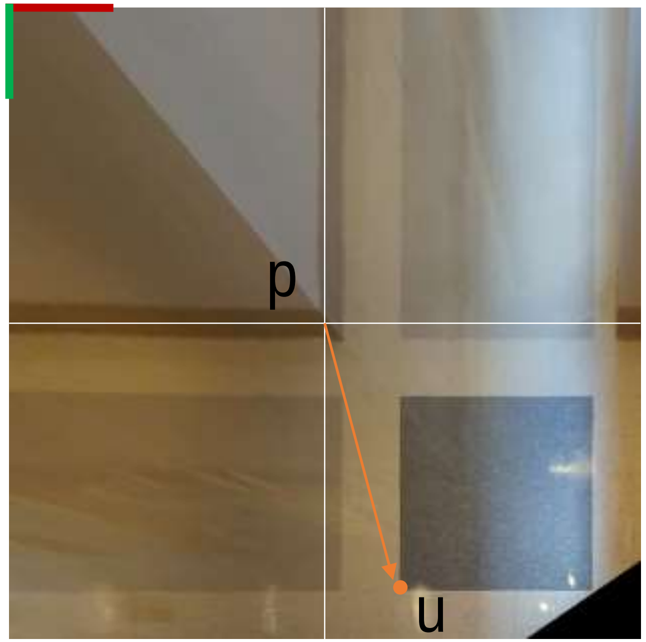


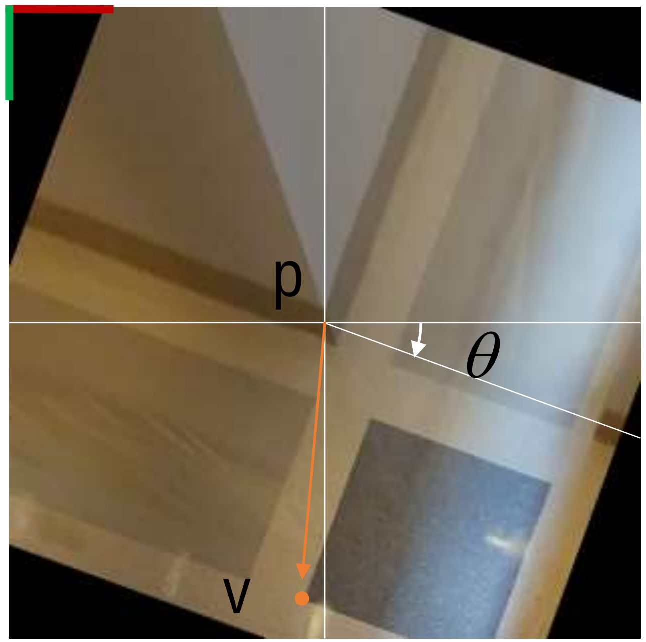

Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

$t$ ?

# Euclidean Transform SE(3)

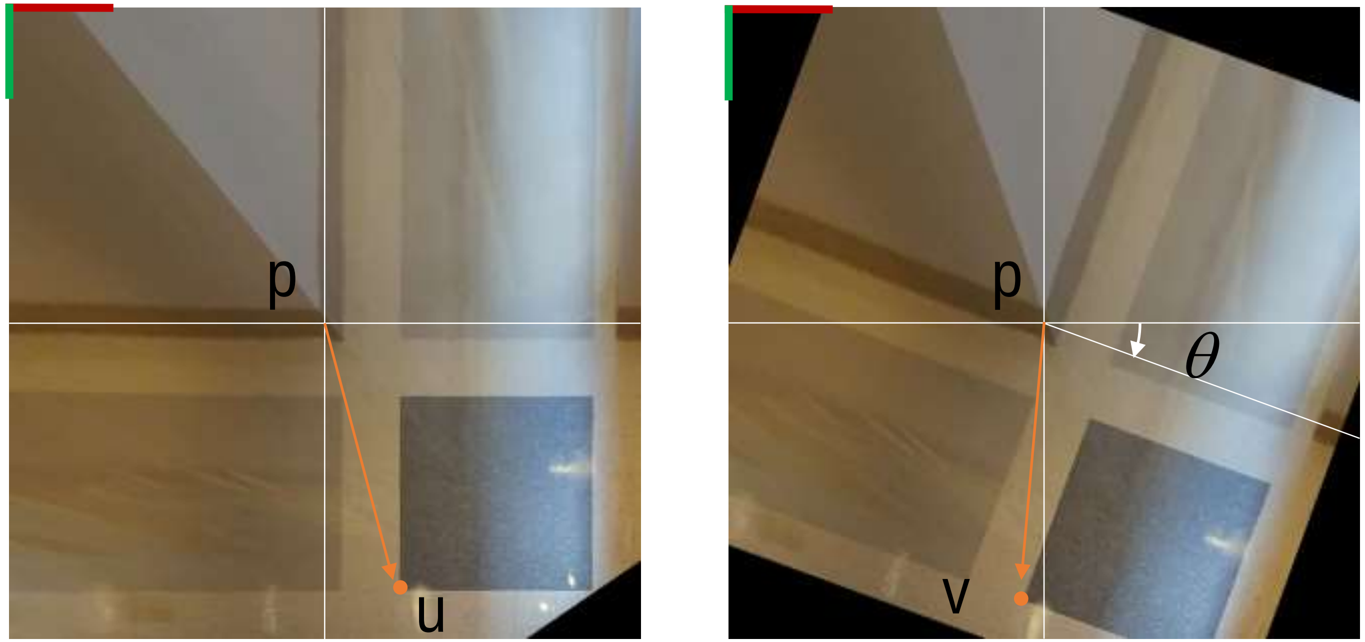



Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

# Euclidean Transform SE(3)



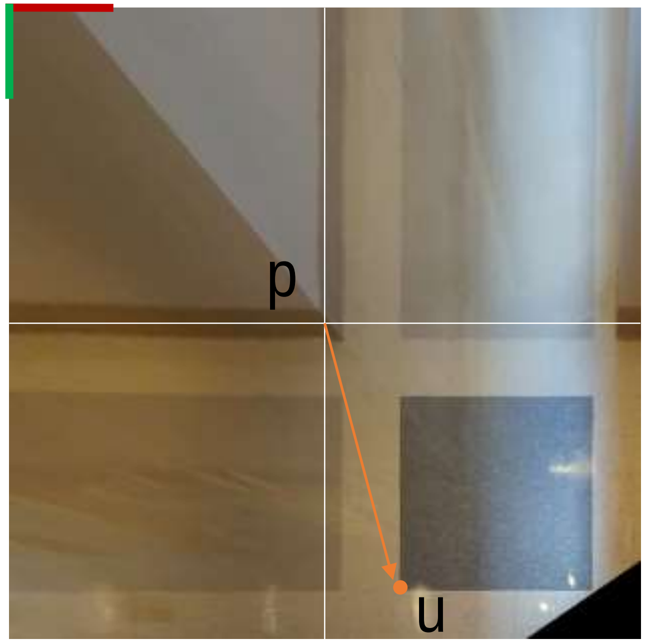

Rotate about the image center

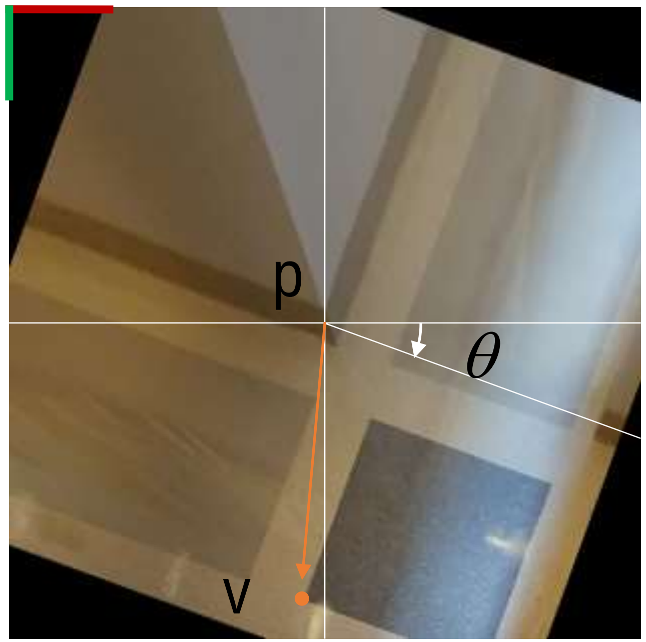
$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

$$\bar{u} = u - p \qquad \bar{v} = v - p$$

$$\longrightarrow \bar{v} = R\bar{u}$$

# Euclidean Transform SE(3)




Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

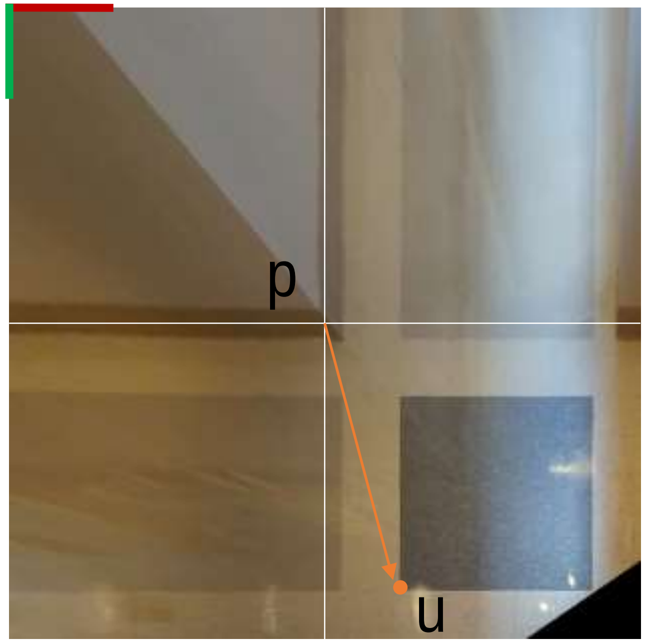
$$\bar{u} = u - p \qquad \bar{v} = v - p$$

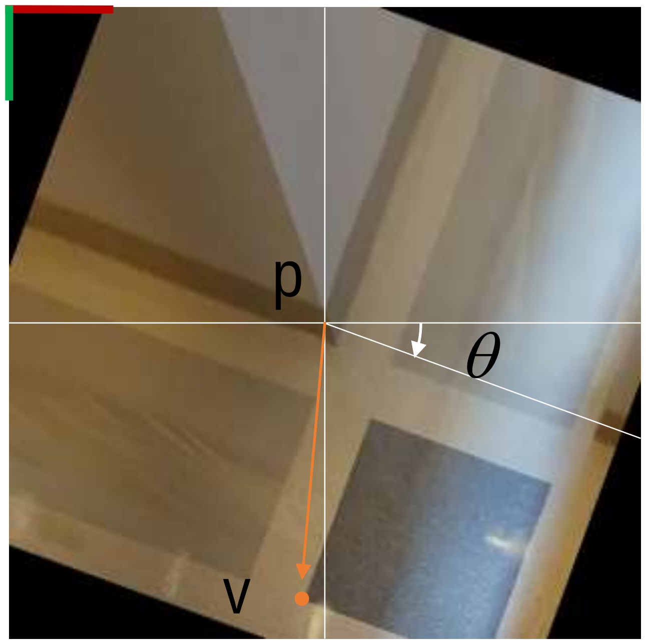
$$\longrightarrow \bar{v} = R\bar{u}$$

$$\longrightarrow v - p = R(u - p)$$

$$\longrightarrow v = Ru - Rp + p$$

# Euclidean Transform SE(3)




Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

$$\bar{u} = u - p \qquad \bar{v} = v - p$$

$$\longrightarrow \bar{v} = R\bar{u}$$

$$\longrightarrow v - p = R(u - p)$$

$$\longrightarrow v = Ru - Rp + p$$

$$\longrightarrow t = -Rp + p$$

# Euclidean Transform SE(3)

```
im = imread('rect.png');

theta = 20/180*pi;


R = [cos(theta) -sin(theta);
     sin(theta) cos(theta)];
p = [size(im,2)/2; size(im,1)/2];
```

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotation around the center of image

# Euclidean Transform SE(3)

RectificationViaEuclidean.m

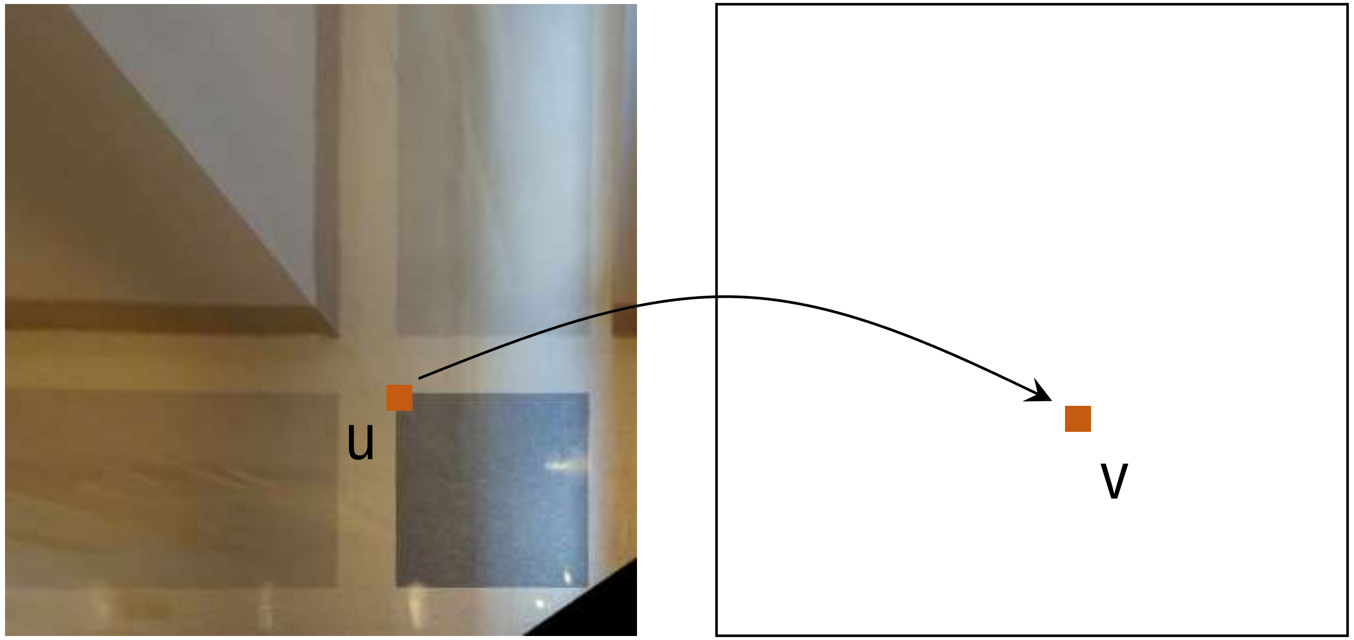
```
im = imread('rect.png');

theta = 20/180*pi;


R = [cos(theta) -sin(theta);
     sin(theta) cos(theta)];
p = [size(im,2)/2; size(im,1)/2];


T = [R -R*t+t;0 0 1];


im_warped = ImageWarpingEuclidean(im, T);
```

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} R & -Rp+p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Rotation around the center of image

# Euclidean Transform SE(3)

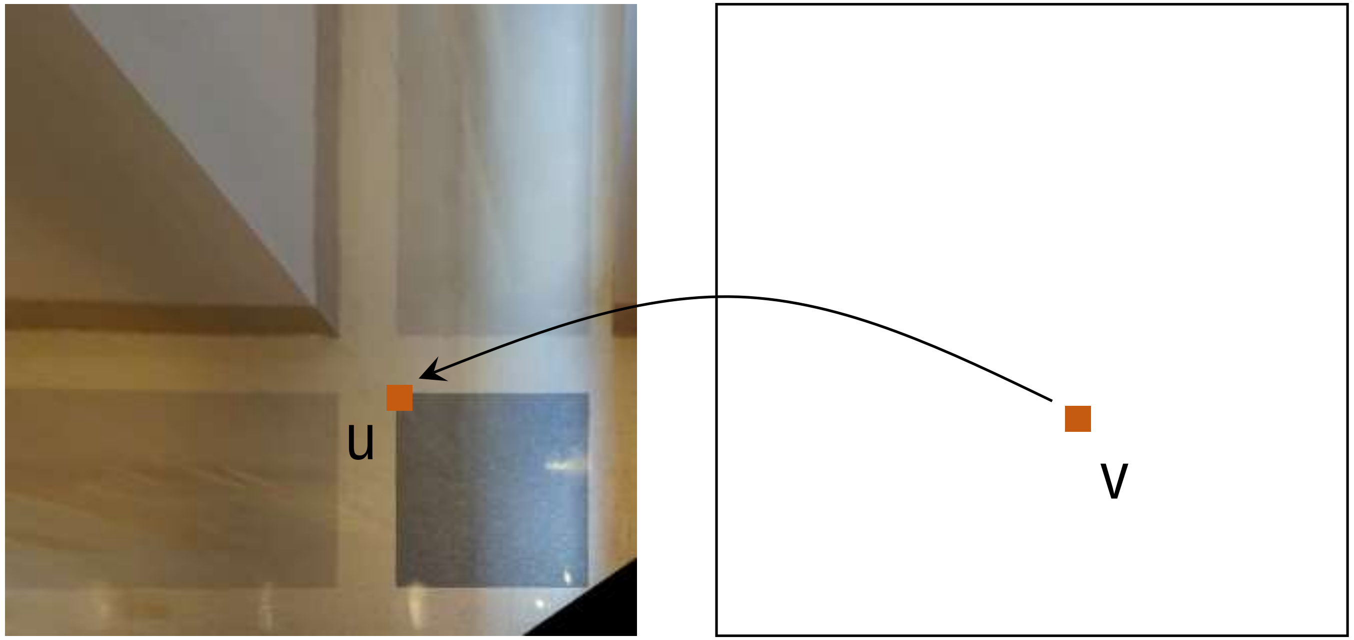



$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = H \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

ImageWarpingEuclidean.m

```
function im_warped = ImageWarpingEuclidean(im, H)

im = double(im);
```

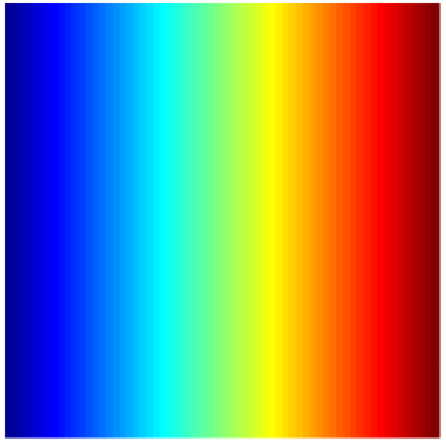
# Euclidean Transform SE(3)



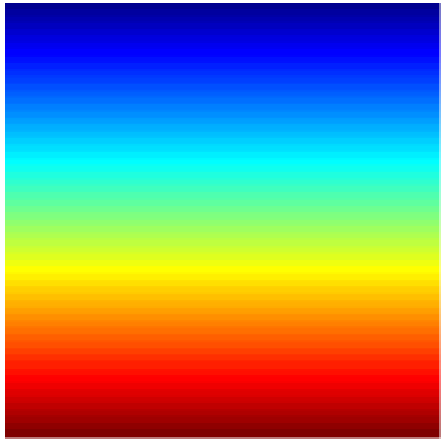

$$H^{-1}\begin{bmatrix}v_x\\v_y\\1\end{bmatrix}=\begin{bmatrix}u_x\\u_y\\1\end{bmatrix}$$

ImageWarpingEuclidean.m

```
function im_warped = ImageWarpingEuclidean(im, H)

im = double(im);

H = inv(H);

[v_x, v_y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(v_x, 1); w = size(v_x,2);
```

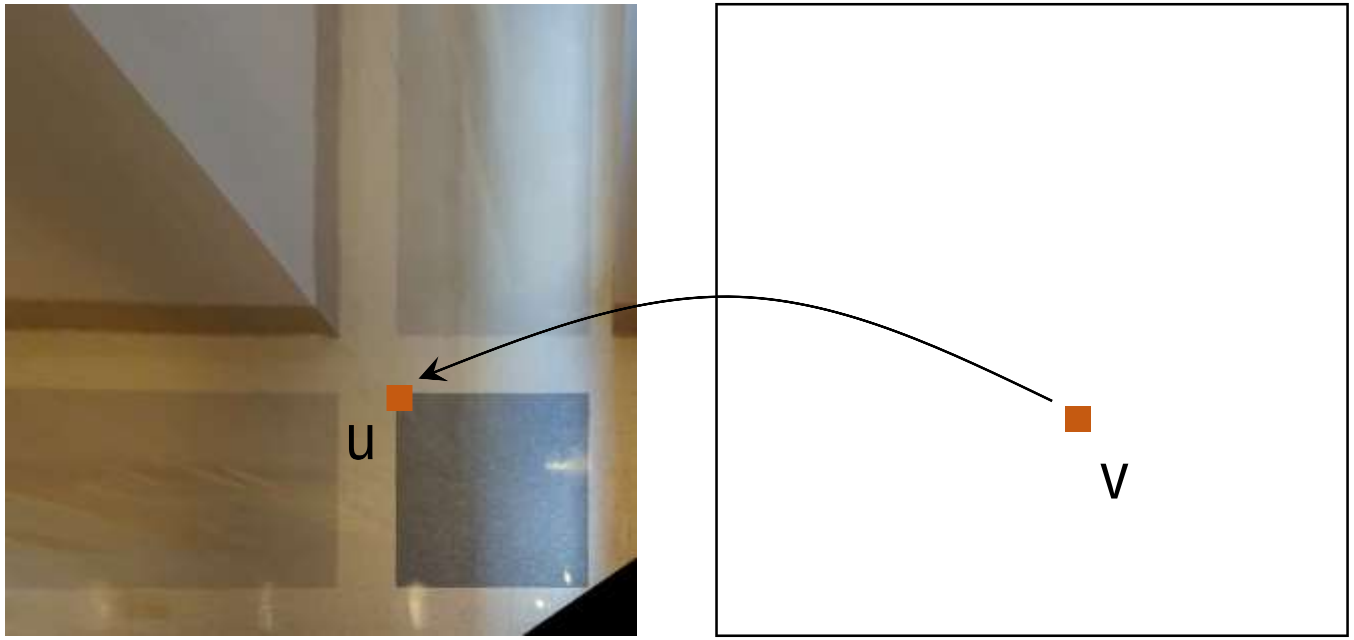


v_x



v_y

# Euclidean Transform SE(3)

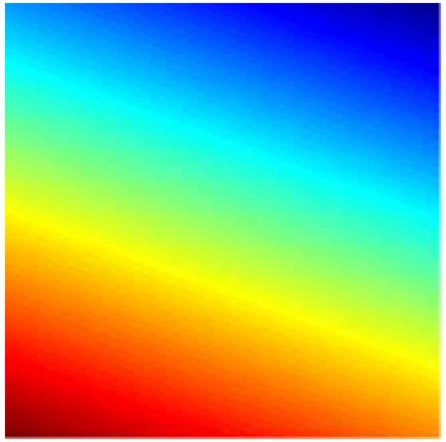



$$H^{-1}\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

ImageWarpingEuclidean.m

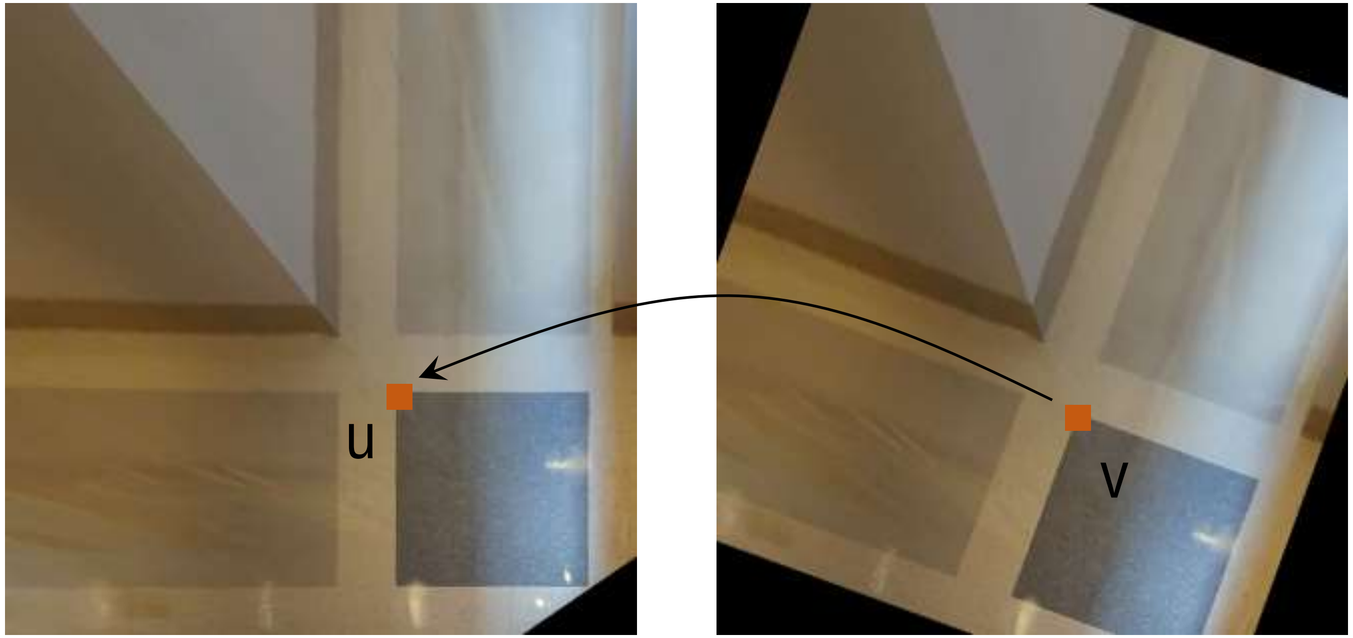
```
function im_warped = ImageWarpingEuclidean(im, H)

im = double(im);

H = inv(H);

[v_x, v_y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(v_x, 1); w = size(v_x,2);

u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
```

u_x



u_y

# Euclidean Transform SE(3)

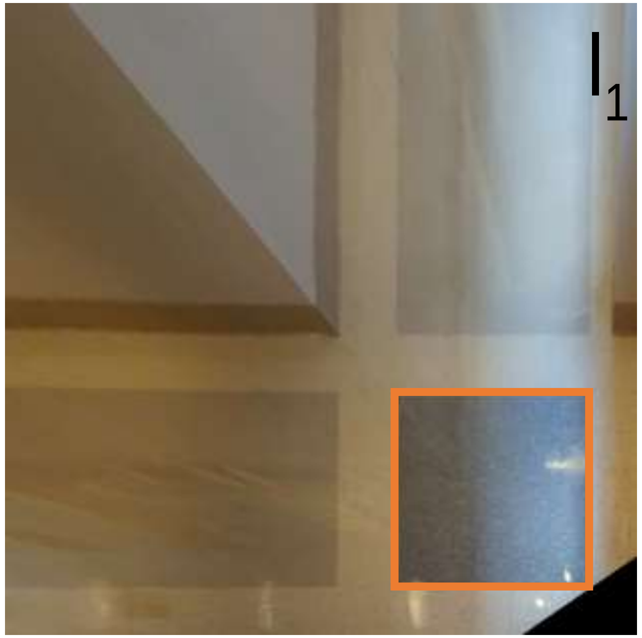


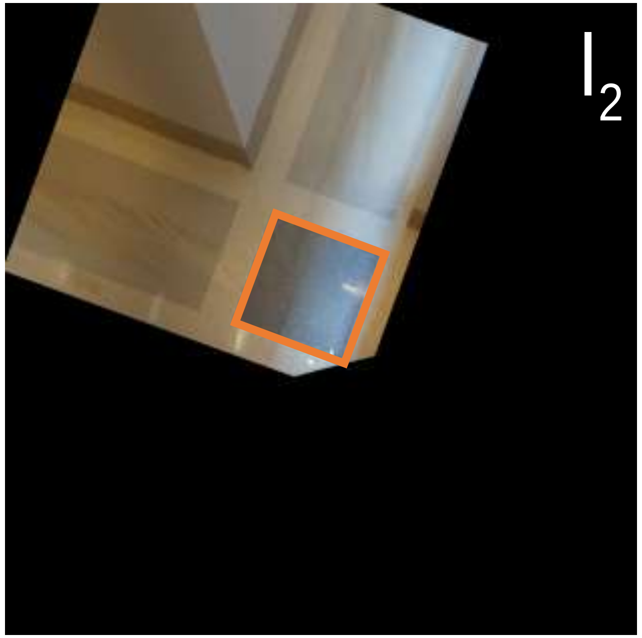

$$H^{-1}\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

ImageWarpingEuclidean.m

```
function im_warped = ImageWarpingEuclidean(im, H)

im = double(im);

H = inv(H);

[v_x, v_y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(v_x, 1); w = size(v_x,2);

u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);

im_warped(:,:,1) = reshape(interp2(im(:,:,1), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,2) = reshape(interp2(im(:,:,2), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,3) = reshape(interp2(im(:,:,3), u_x(:), u_y(:)), [h, w]);

im_warped = uint8(im_warped);
```

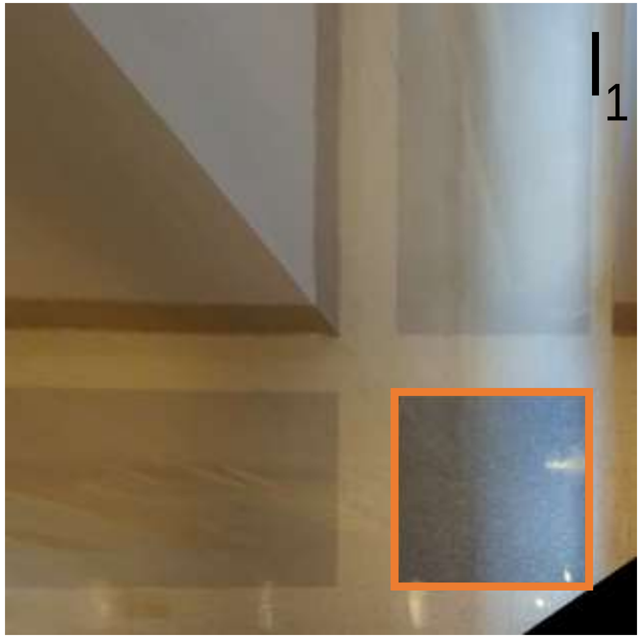
# Similarity Transform

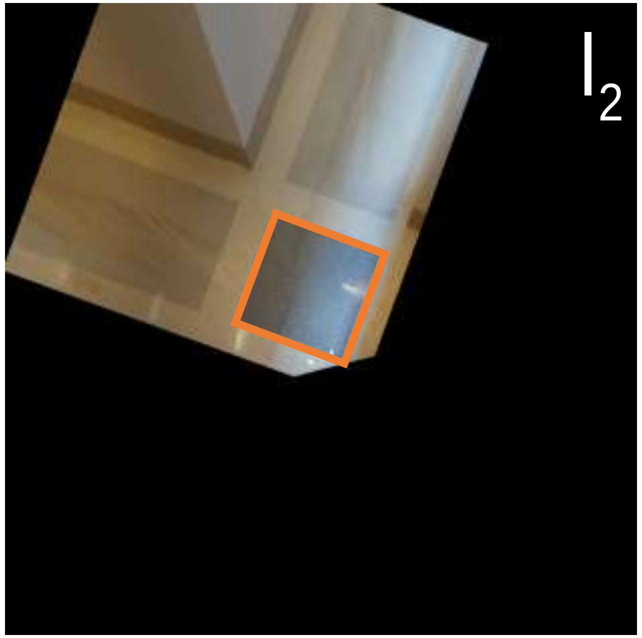



$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & & \\ & \alpha & \\ & & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Similarity Transform




Invariant properties

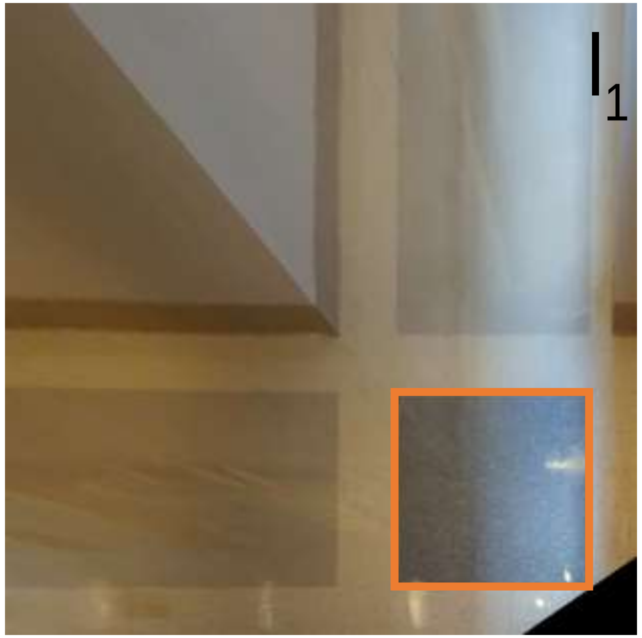
- Length ratio
- Angle

Degree of freedom

4 (2 translation+1 rotation+1 scale)

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & & \\ & \alpha & \\ & & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \mathrm{R} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Affine Transform

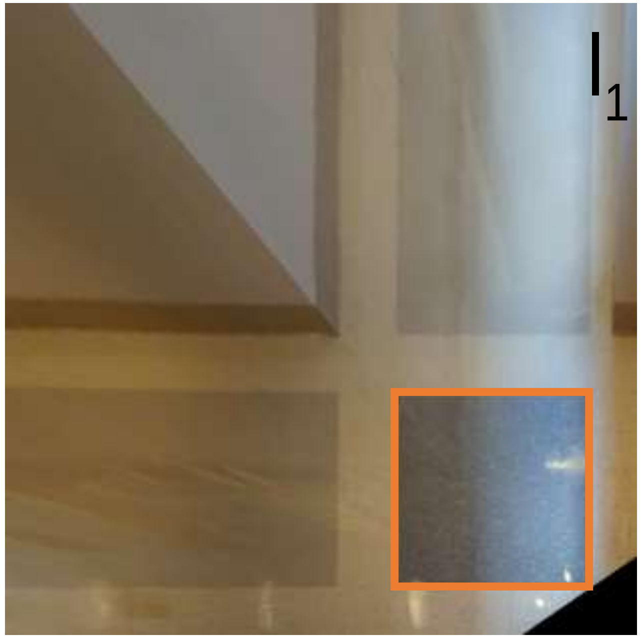




$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

# Affine Transform



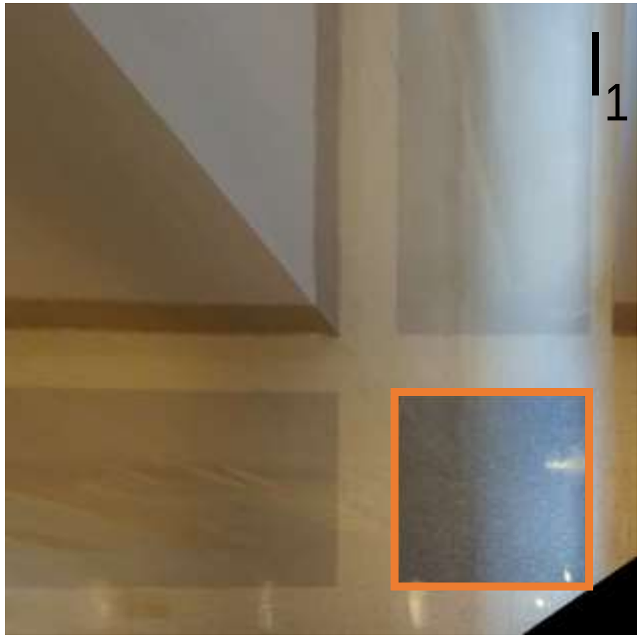


$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \mathrm{R} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Affine Transform





Invariant properties

- Parallelism
- Ratio of area
- Ratio of length

Degree of freedom

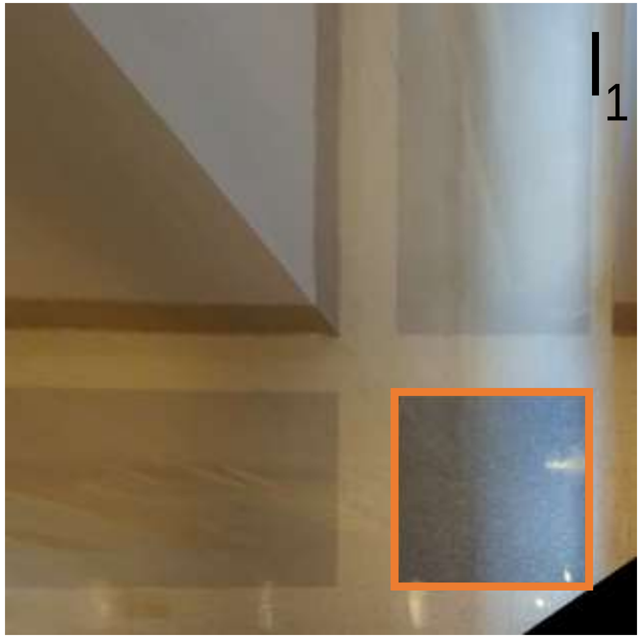
6

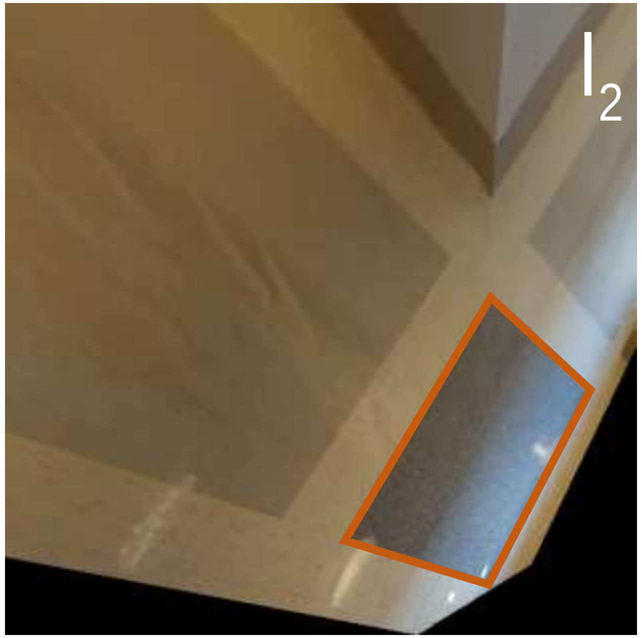
$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

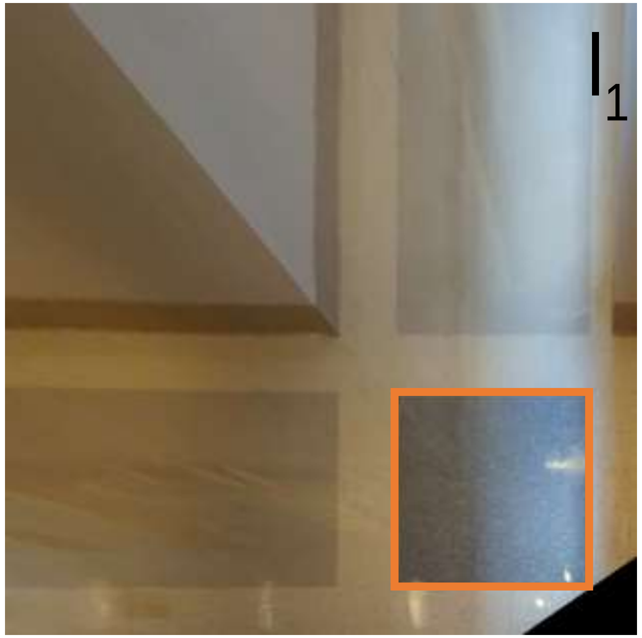
# Perspective Transform (Homography)

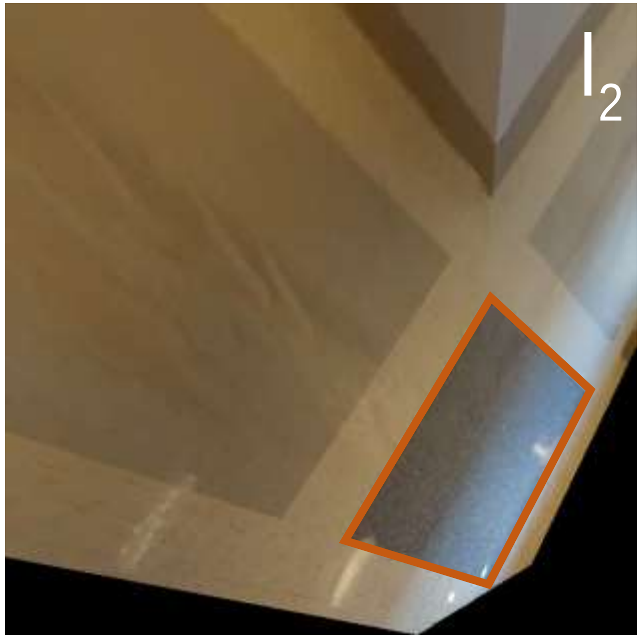



$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Perspective Transform (Homography)

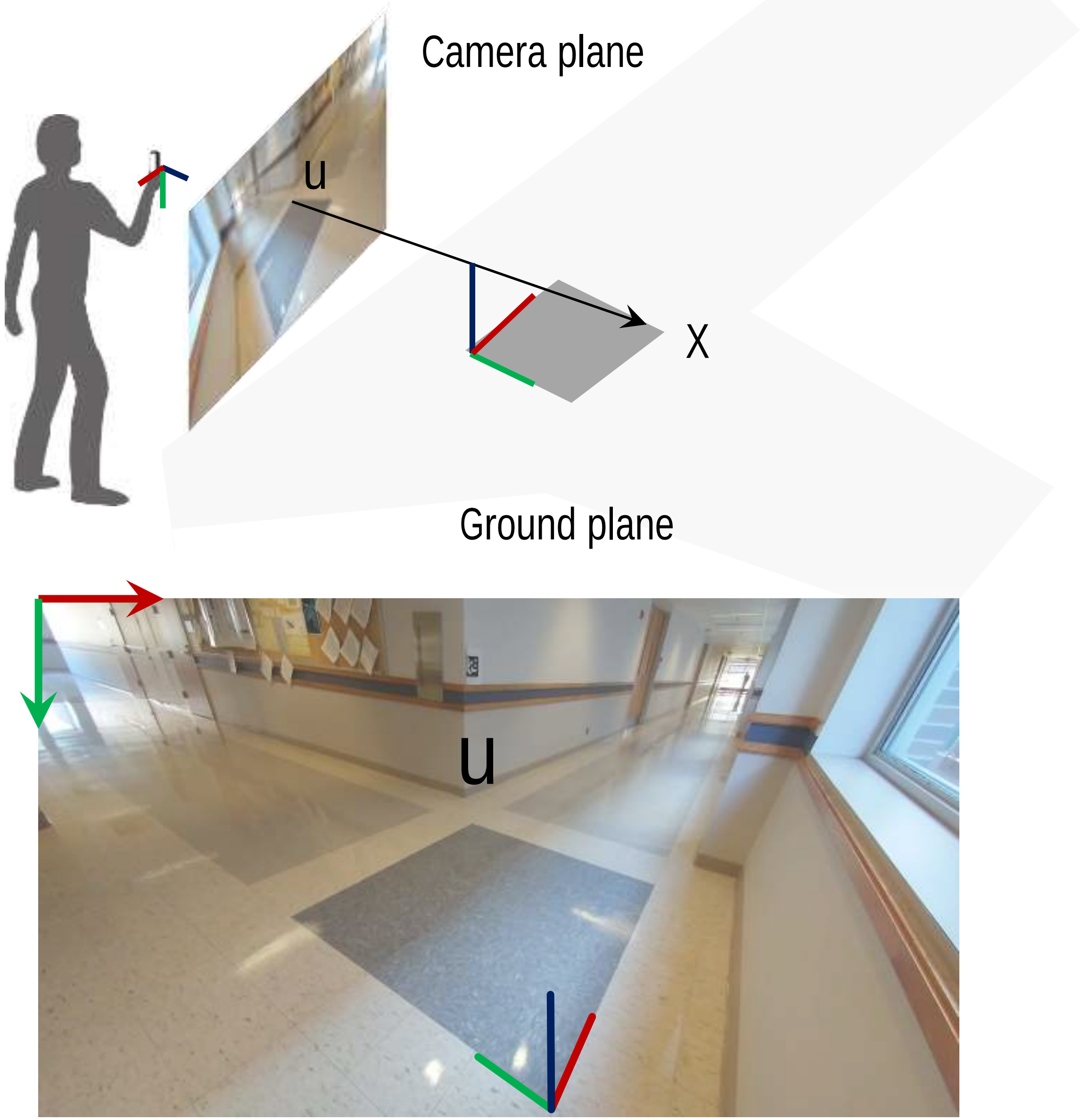



$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

: General form of plane to plane linear mapping

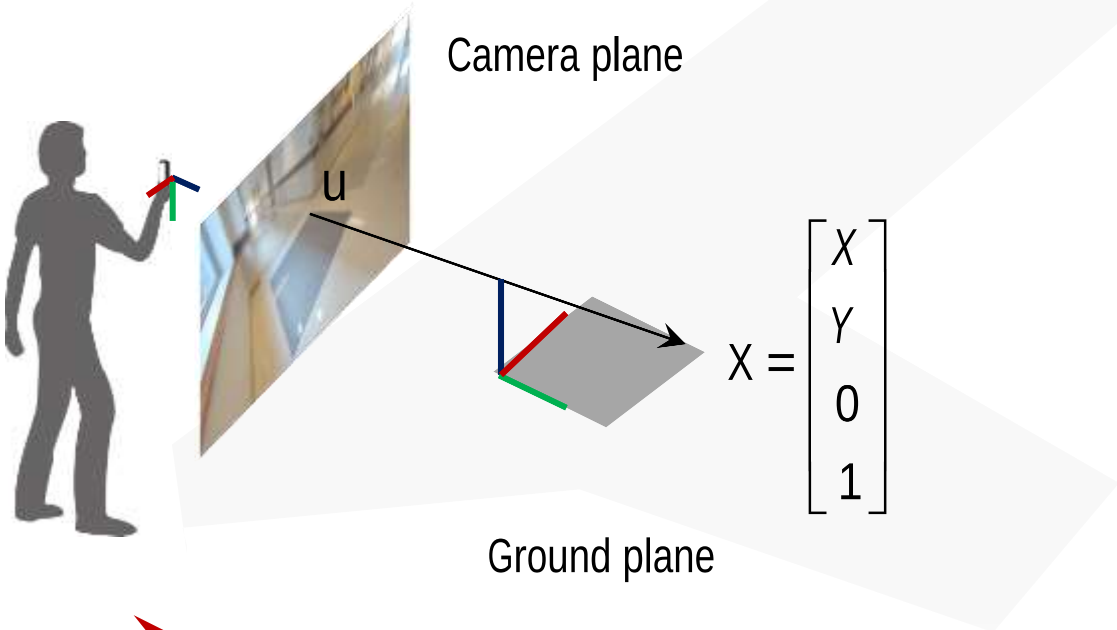
# Perspective Transform (Homography)




$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

Camera plane    Ground plane

# Perspective Transform (Homography)

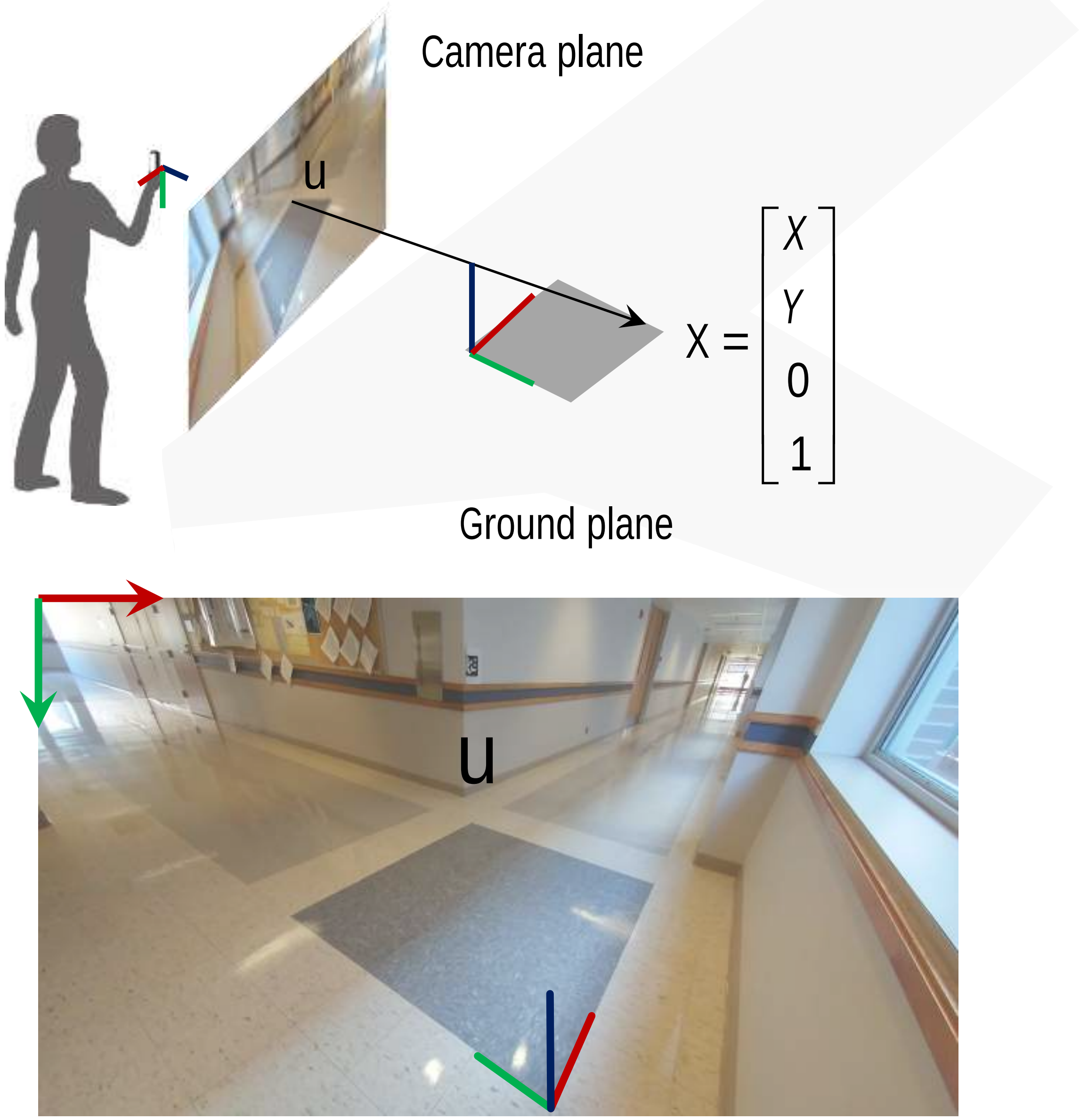




$$\lambda \mathrm{u} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \mathrm{X}$$

Camera plane    Ground plane

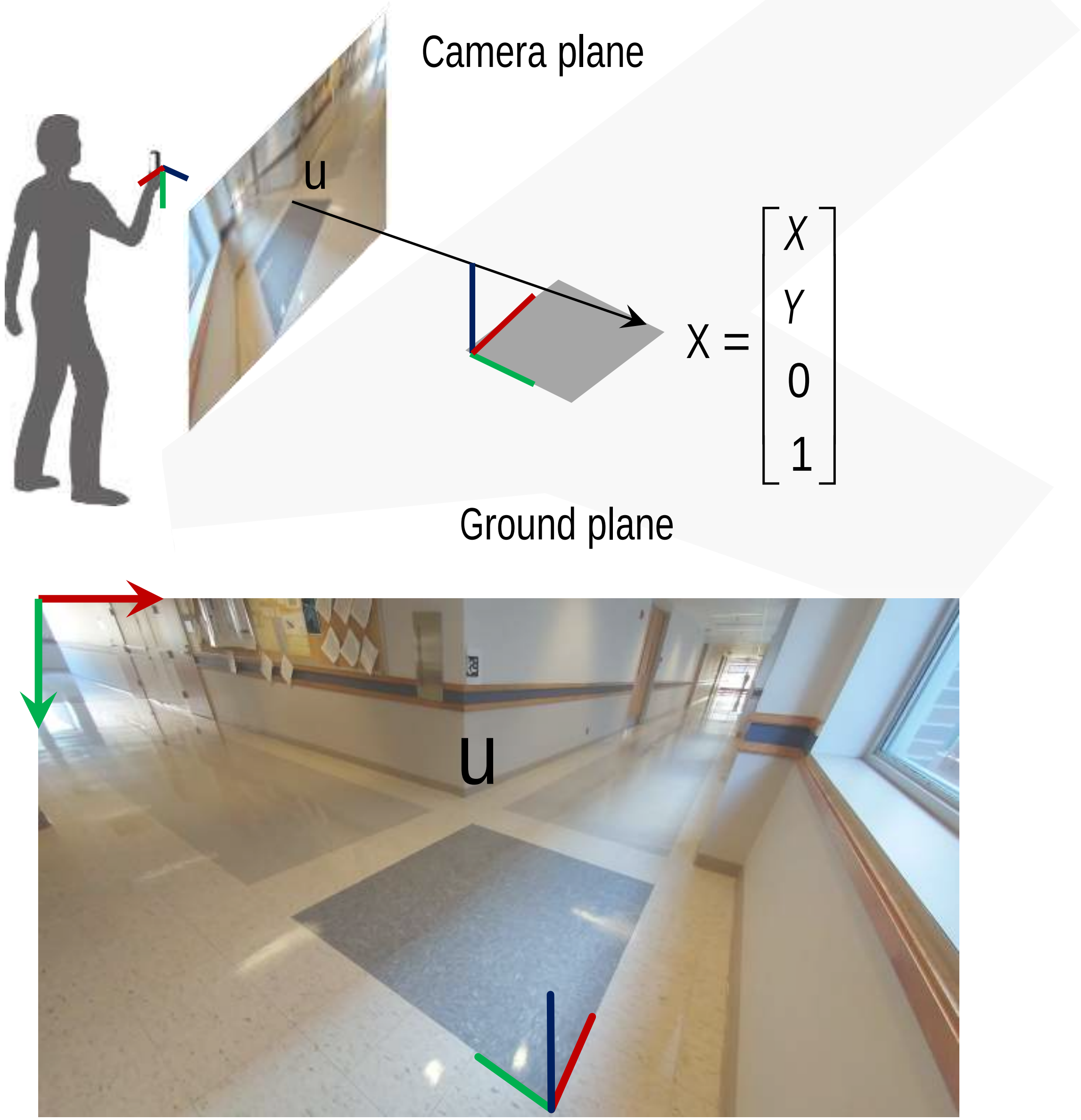
# Perspective Transform (Homography)




$$\underbrace{\lambda u}_{\text{Camera plane}} = K\begin{bmatrix} R & t \end{bmatrix}\underbrace{X}_{\text{Ground plane}}$$

$$\longrightarrow \lambda u = K\begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

# Perspective Transform (Homography)

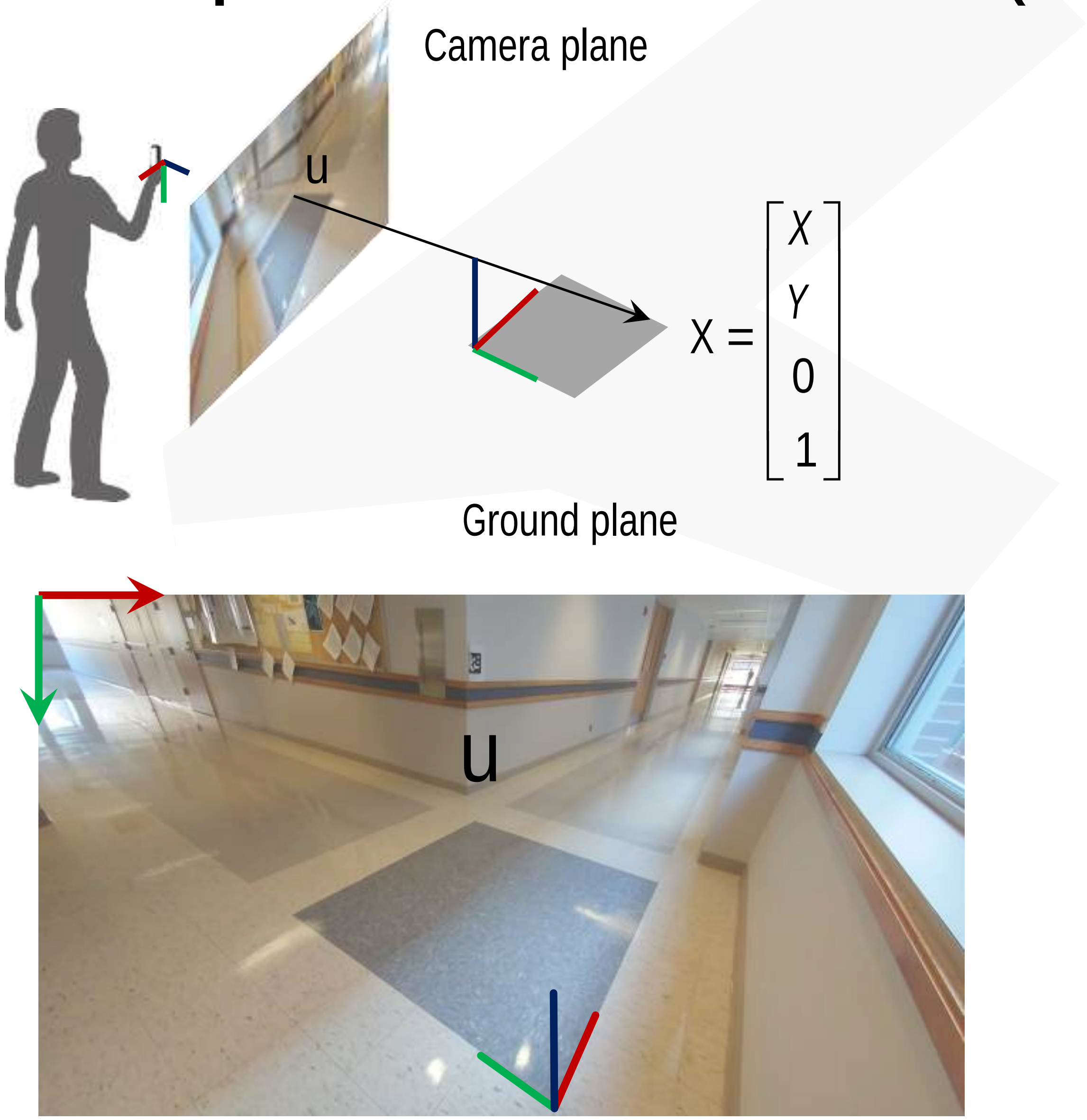



$$\underbrace{\lambda u}_{\text{Camera plane}} = K\begin{bmatrix}R & t\end{bmatrix}\underbrace{X}_{\text{Ground plane}}$$

$$\longrightarrow \lambda u = K\begin{bmatrix}r_1 & r_2 & r_3 & t\end{bmatrix}\begin{bmatrix}X\\Y\\0\\1\end{bmatrix}$$

$$\longrightarrow \lambda u = K\begin{bmatrix}r_1 & r_2 & t\end{bmatrix}\begin{bmatrix}X\\Y\\1\end{bmatrix}$$

# Perspective Transform (Homography)




$$\lambda \underline{u} = K\begin{bmatrix} R & t \end{bmatrix}\underline{X}$$

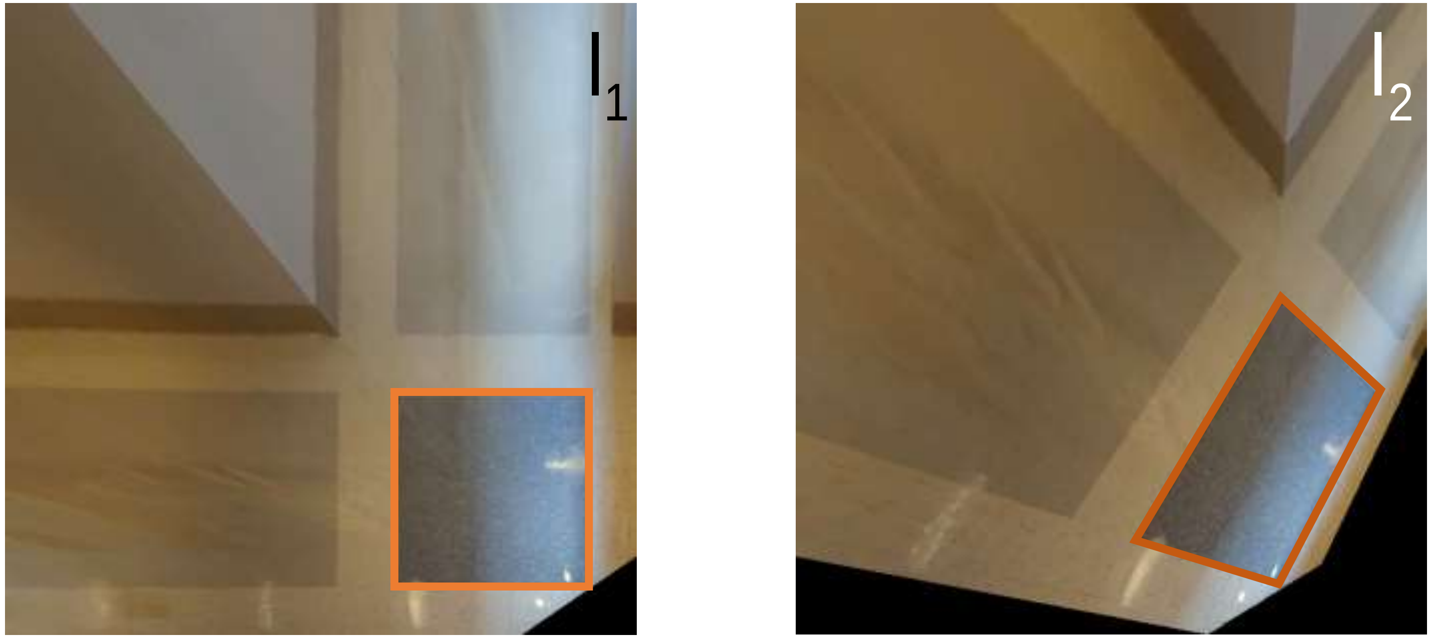
Camera plane — Ground plane

$$\longrightarrow \lambda u = K\begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda u = K\begin{bmatrix} r_1 & r_2 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = H\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

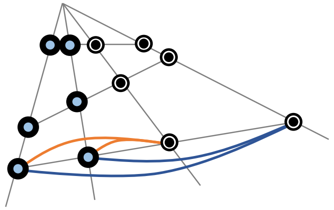
# Perspective Transform (Homography)



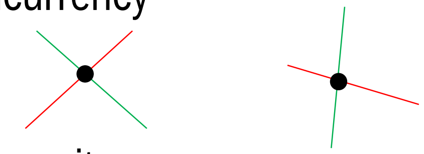

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

## Invariant properties

- Cross ratio



- Concurrency

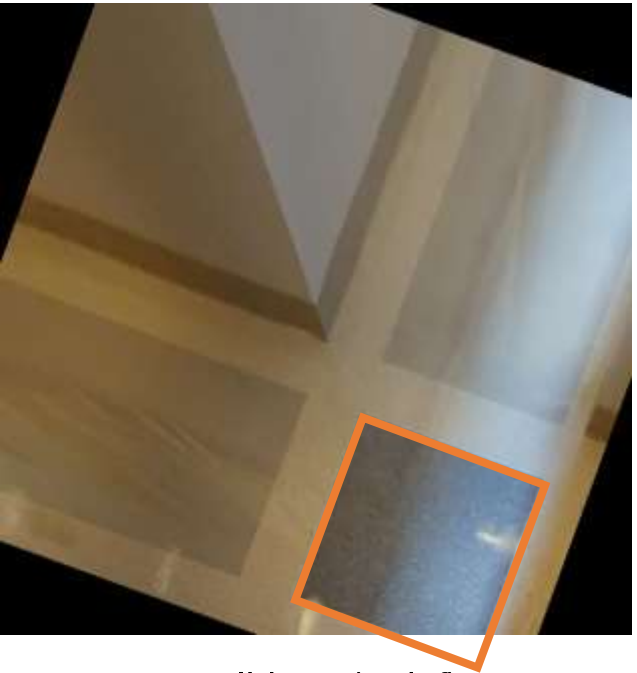


- Colinearity

## Degree of freedom

8 (9 variables – 1 scale)

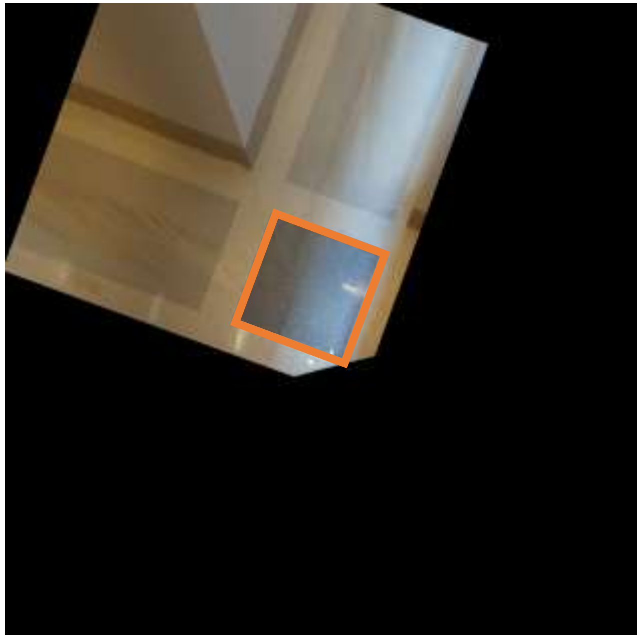
# Hierarchy of Transformations



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Similarity (4 dof)

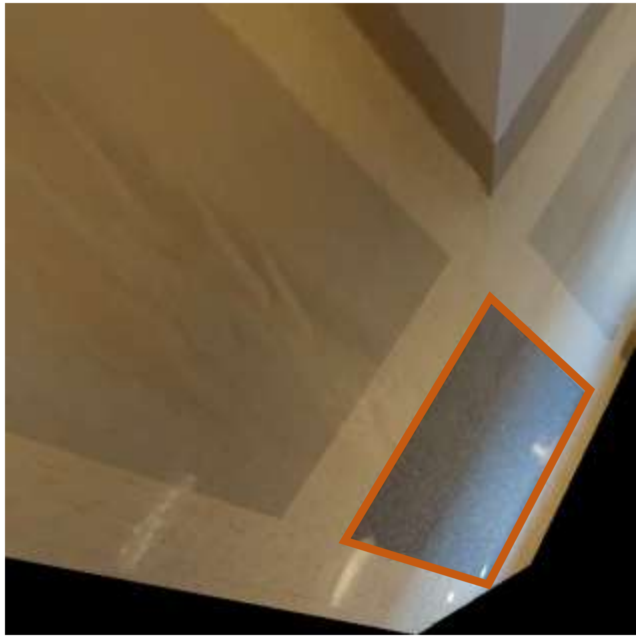
- Length ratio
- Angle

$$\begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

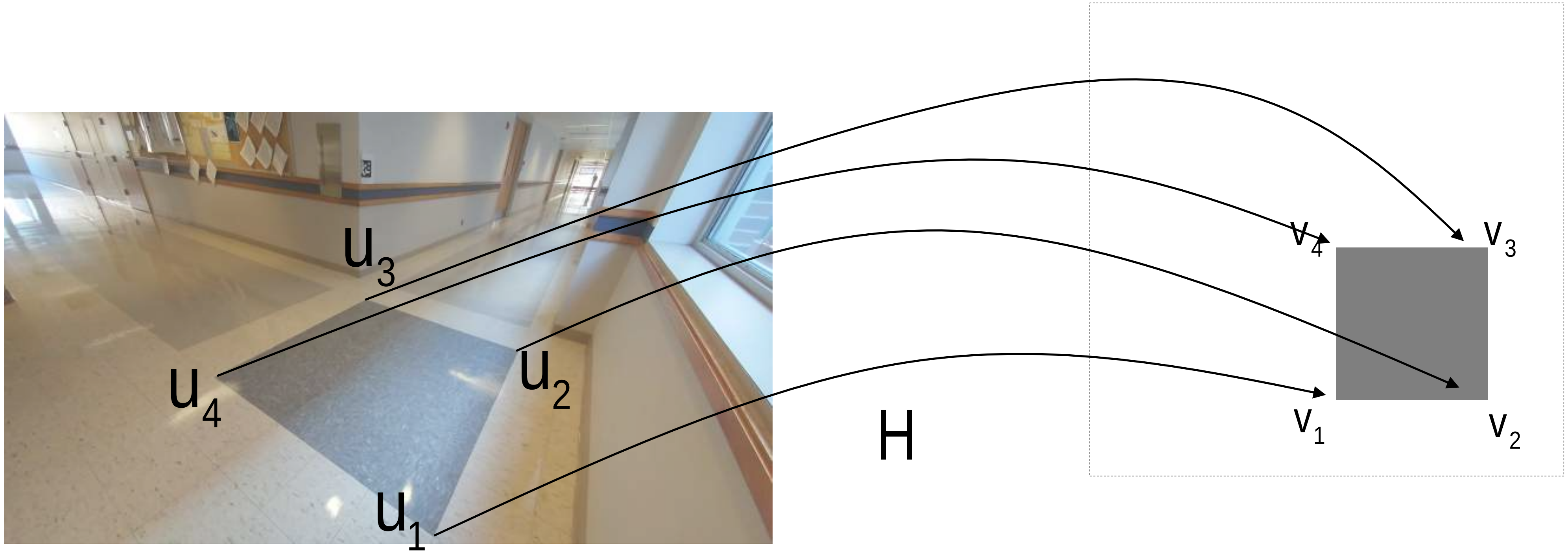


Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

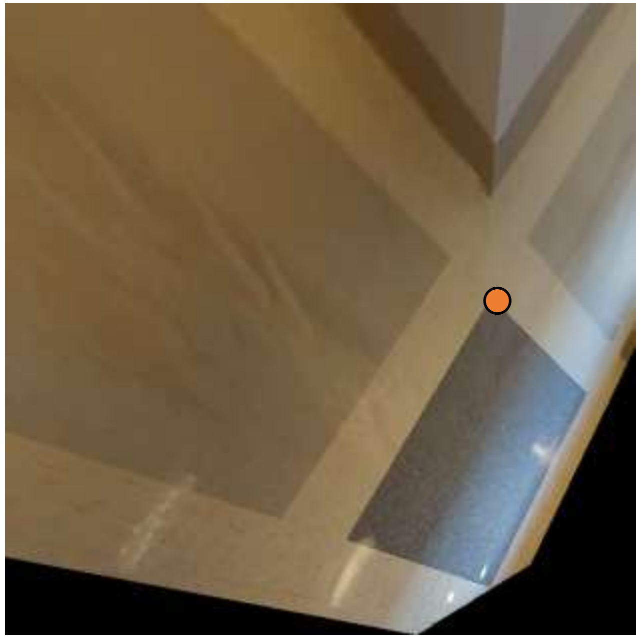
$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

# Fun with Homography




The image can be rectified as if it is seen from top view.

# Homography Computation

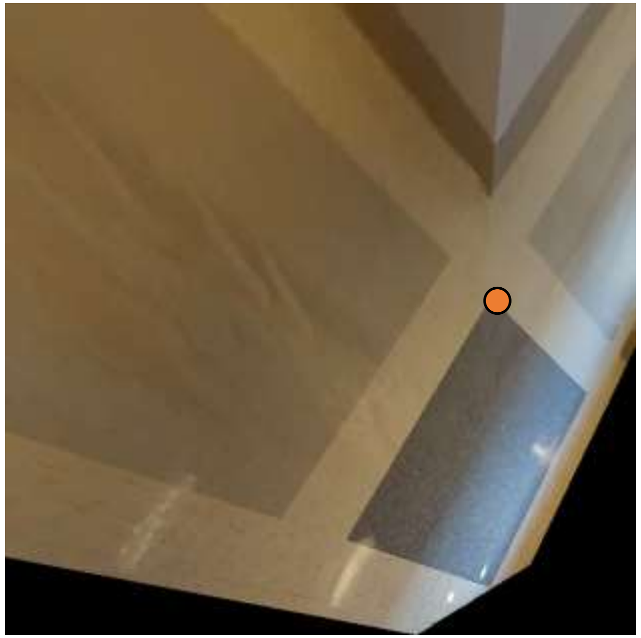


$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Homography Computation



$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

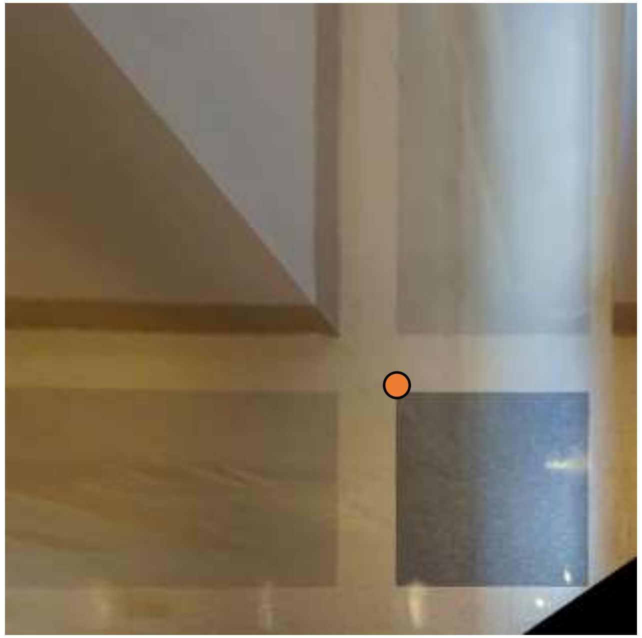
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\longrightarrow \begin{aligned} h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_x v_x - h_{32}u_y v_x - h_{33}v_x &= 0 \\ h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_x v_y - h_{32}u_y v_y - h_{33}v_y &= 0 \end{aligned}$$

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Homography Computation



$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\rightarrow \quad h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_x v_x - h_{32}u_y v_x - h_{33}v_x = 0$$

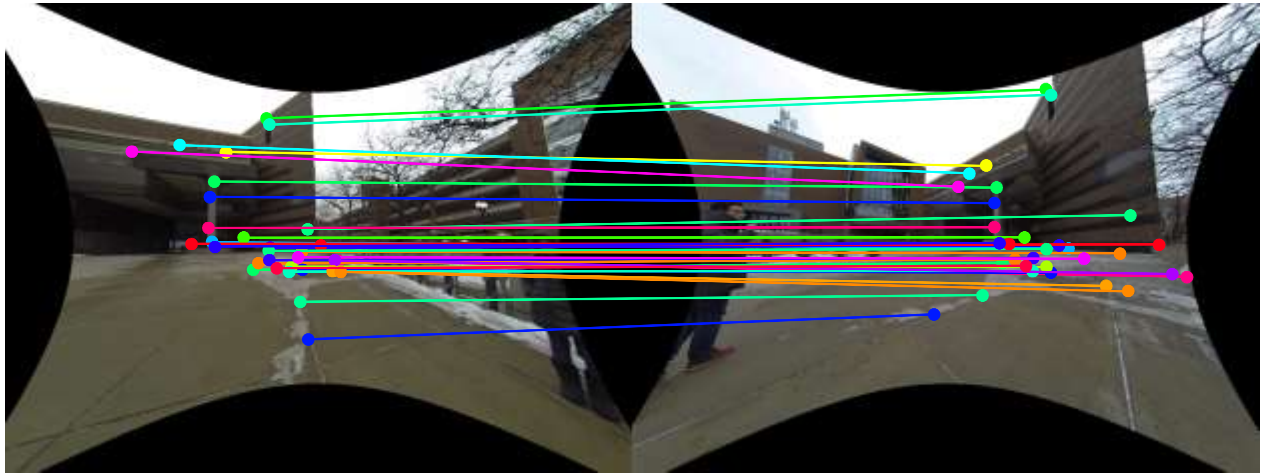
$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_x v_y - h_{32}u_y v_y - h_{33}v_y = 0$$

$$\rightarrow \begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \, x = 0$$

2x9

# Linear System for Homography Matrix

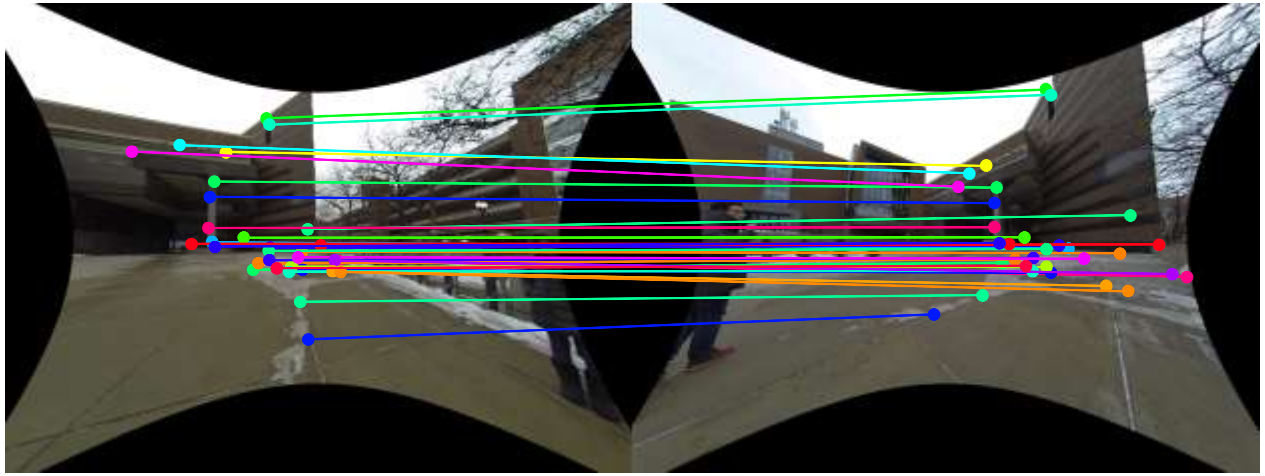


$$\begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \, x = 0$$

2x9

# How Many Correspondences?



$$\underbrace{\begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

2x9

What is minimum m?

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{\begin{array}{c} v_1 \leftrightarrow u_1 \\ v_2 \leftrightarrow u_2 \\ v_3 \leftrightarrow u_3 \\ v_4 \leftrightarrow u_4 \end{array}\right\} \rightarrow H$$

Homography computation

$$\begin{bmatrix} u^1_x & u^1_y & 1 & & & & -u^1_x v^1_x & -u^1_y v^1_x & -v^1_x \\ & & & u^1_x & u^1_y & 1 & -u^1_x v^1_y & -u^1_y v^1_y & -v^1_y \\ u^2_x & u^2_y & 1 & & & & -u^2_x v^2_x & -u^2_y v^2_x & -v^2_x \\ & & & u^2_x & u^2_y & 1 & -u^2_x v^2_y & -u^2_y v^2_y & -v^2_y \\ u^3_x & u^3_y & 1 & & & & -u^3_x v^3_x & -u^3_y v^3_x & -v^3_x \\ & & & u^3_x & u^3_y & 1 & -u^3_x v^3_y & -u^3_y v^3_y & -v^3_y \\ u^4_x & u^4_y & 1 & & & & -u^4_x v^4_x & -u^4_y v^4_x & -v^4_x \\ & & & u^4_x & u^4_y & 1 & -u^4_x v^4_y & -u^4_y v^4_y & -v^4_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathcal{I}_1$ $\mathcal{I}_2$

$$\left\{\begin{matrix} v_1 \leftrightarrow u_1 \\ v_2 \leftrightarrow u_2 \\ v_3 \leftrightarrow u_3 \\ v_4 \leftrightarrow u_4 \end{matrix}\right\} \rightarrow H$$

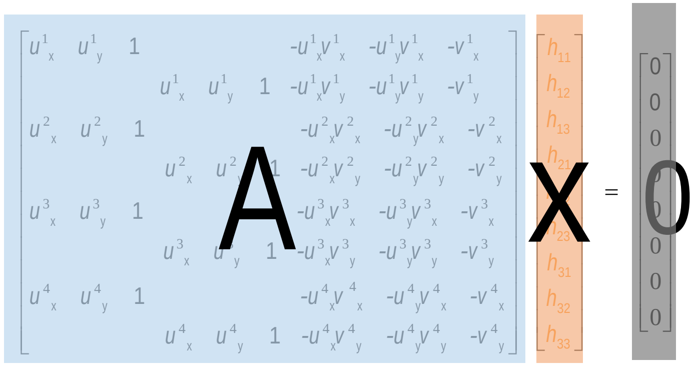
Homography computation

$$\underbrace{\begin{bmatrix} u^1_x & u^1_y & 1 & & & & -u^1_x v^1_x & -u^1_y v^1_x & -v^1_x \\ & & & u^1_x & u^1_y & 1 & -u^1_x v^1_y & -u^1_y v^1_y & -v^1_y \\ u^2_x & u^2_y & 1 & & & & -u^2_x v^2_x & -u^2_y v^2_x & -v^2_x \\ & & & u^2_x & u^2_y & 1 & -u^2_x v^2_y & -u^2_y v^2_y & -v^2_y \\ u^3_x & u^3_y & 1 & & & & -u^3_x v^3_x & -u^3_y v^3_x & -v^3_x \\ & & & u^3_x & u^3_y & 1 & -u^3_x v^3_y & -u^3_y v^3_y & -v^3_y \\ u^4_x & u^4_y & 1 & & & & -u^4_x v^4_x & -u^4_y v^4_x & -v^4_x \\ & & & u^4_x & u^4_y & 1 & -u^4_x v^4_y & -u^4_y v^4_y & -v^4_y \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{0}$$

$\mathcal{I}_1$ $\mathcal{I}_2$

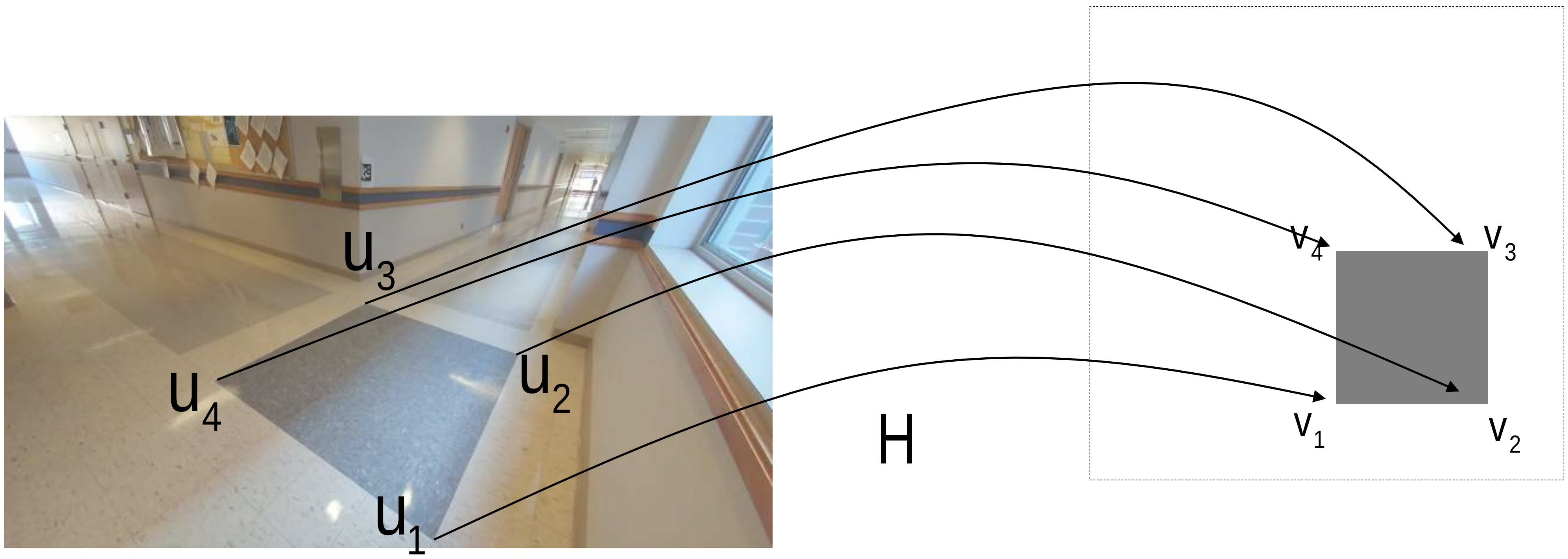
$$\left\{\begin{array}{l} v_1 \leftrightarrow u_1 \\ v_2 \leftrightarrow u_2 \\ v_3 \leftrightarrow u_3 \\ v_4 \leftrightarrow u_4 \end{array}\right\} \rightarrow H$$

Homography computation



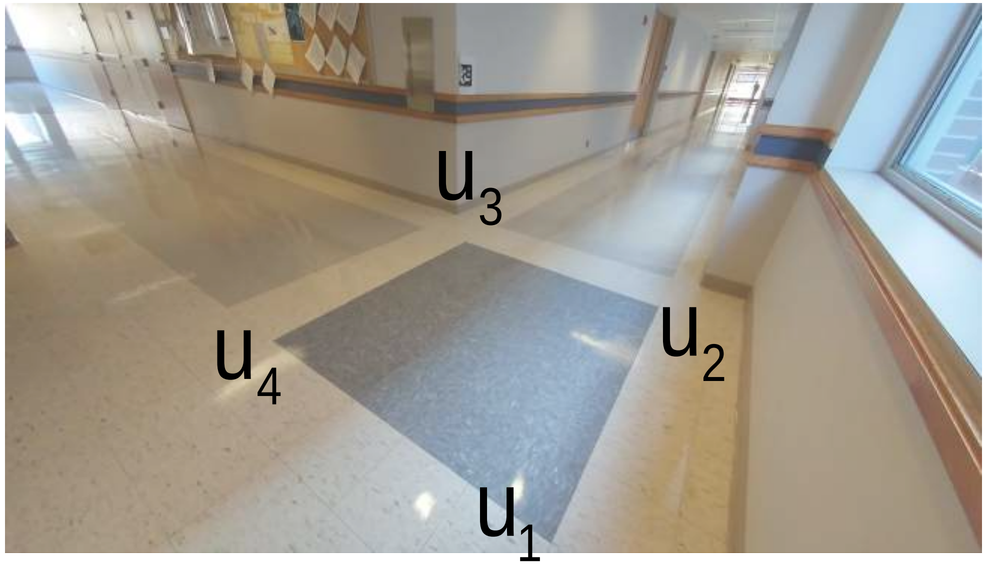

```
[u,d,v] = svd(A);
X = v(:,end)/v(end,end);
H = reshape(X,3,3)';
```

# Fun with Homography




The image can be rectified as if it is seen from top view.

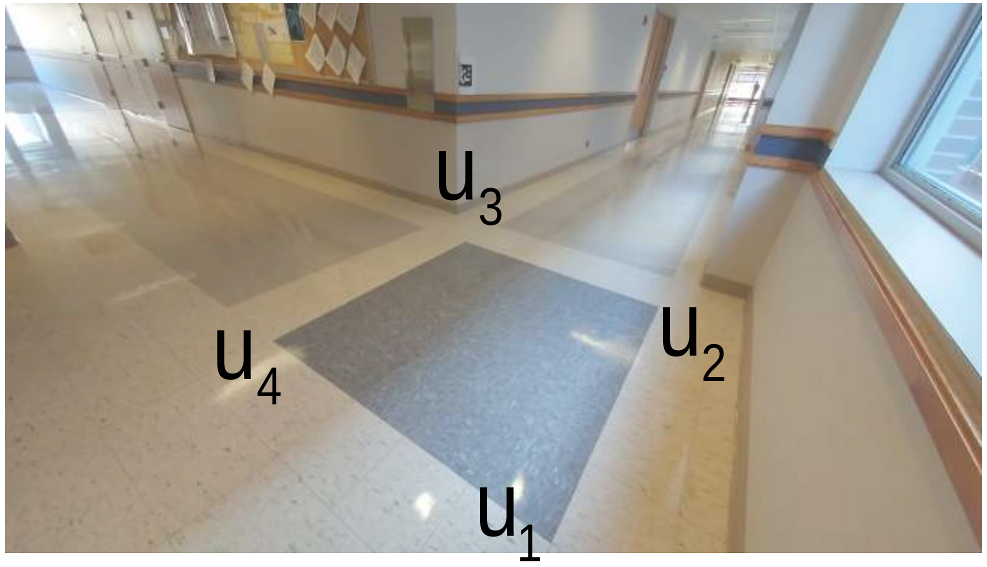
# Fun with Homography




## RectificationViaHomography.m

```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, inv(H));
```

# Fun with Homography




Cf) ImageWarpingEuclidean.m

```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
```

RectificationViaHomography.m

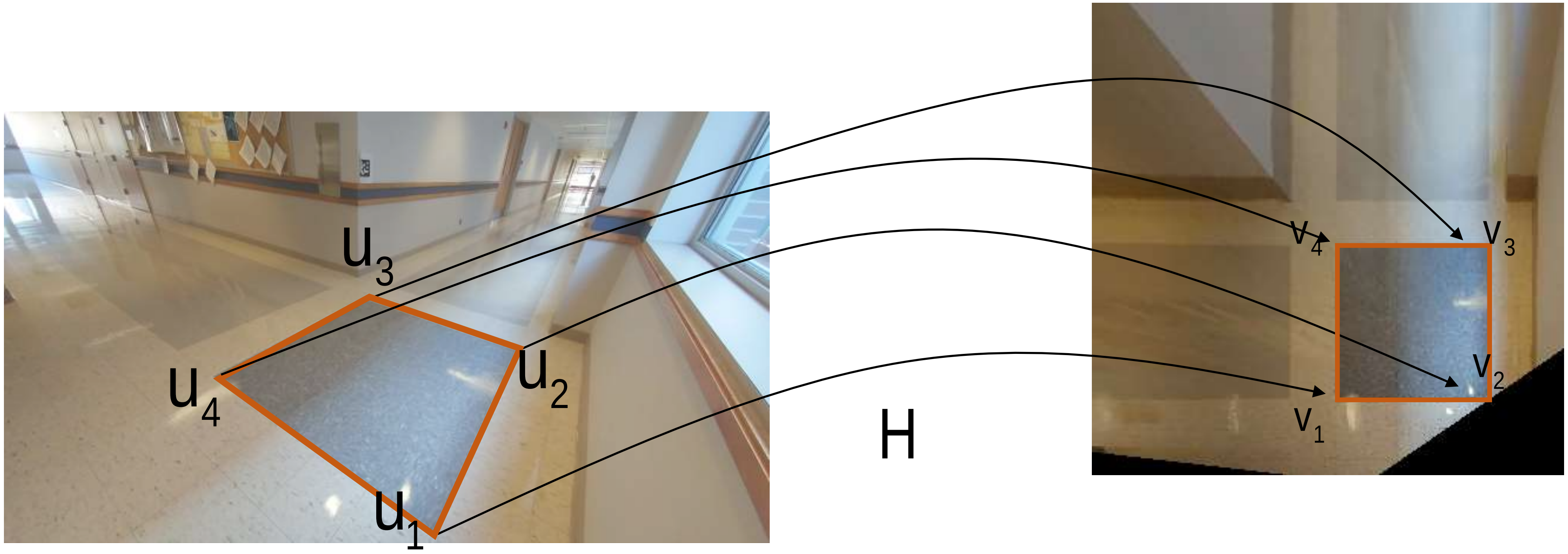
```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, inv(H));
```

ImageWarping.m

```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
u_z = H(3,1)*v_x + H(3,2)*v_y + H(3,3);

u_x = u_x./u_z;
u_y = u_y./u_z;

im_warped(:,:,1) = reshape(interp2(im(:,:,1), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,2) = reshape(interp2(im(:,:,2), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,3) = reshape(interp2(im(:,:,3), u_x(:), u_y(:)), [h, w]);

im_warped = uint8(im_warped);
```

$$\leftarrow \lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = H^{-1} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

# Fun with Homography



# Fun with Homography