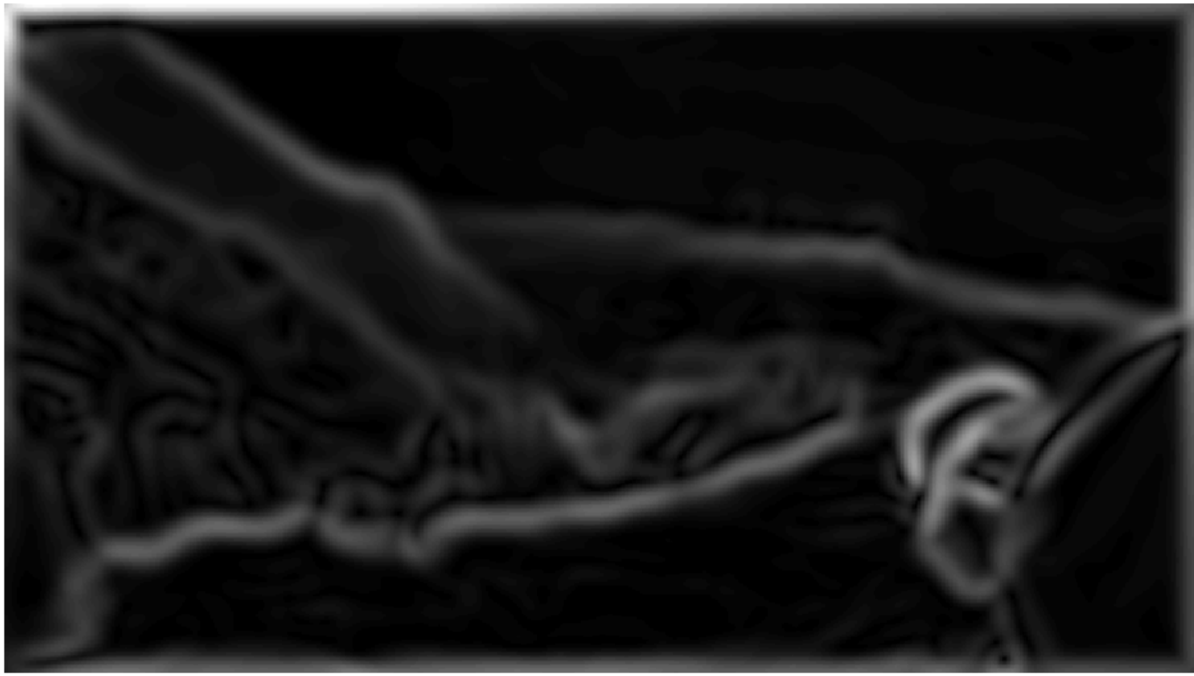
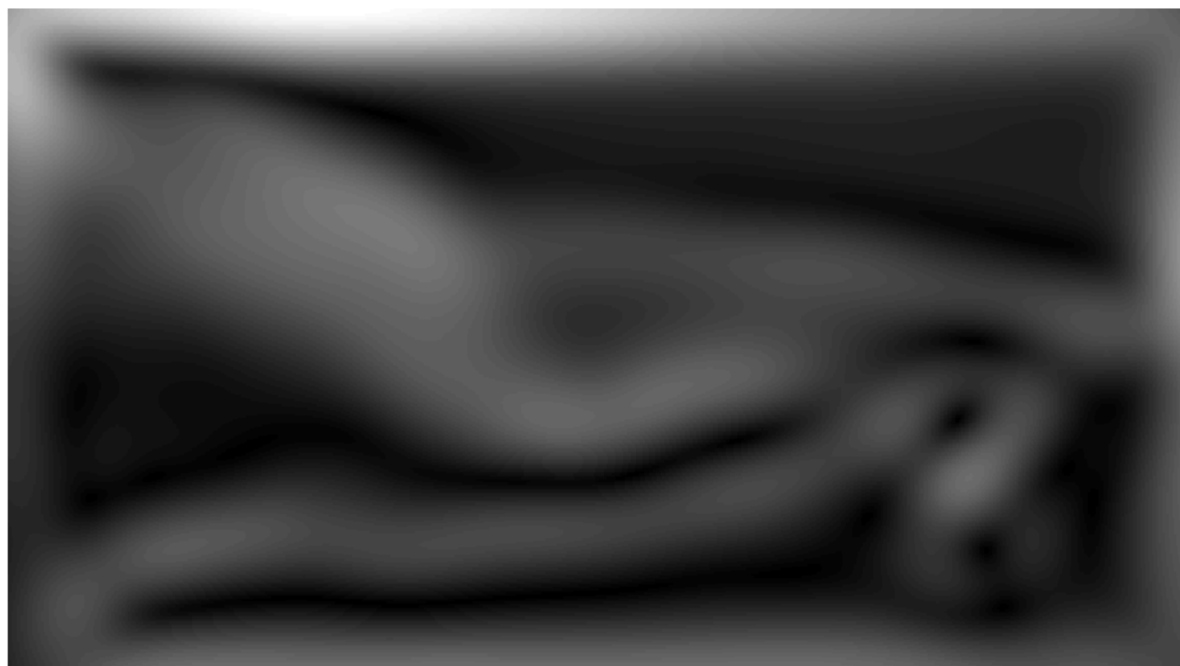
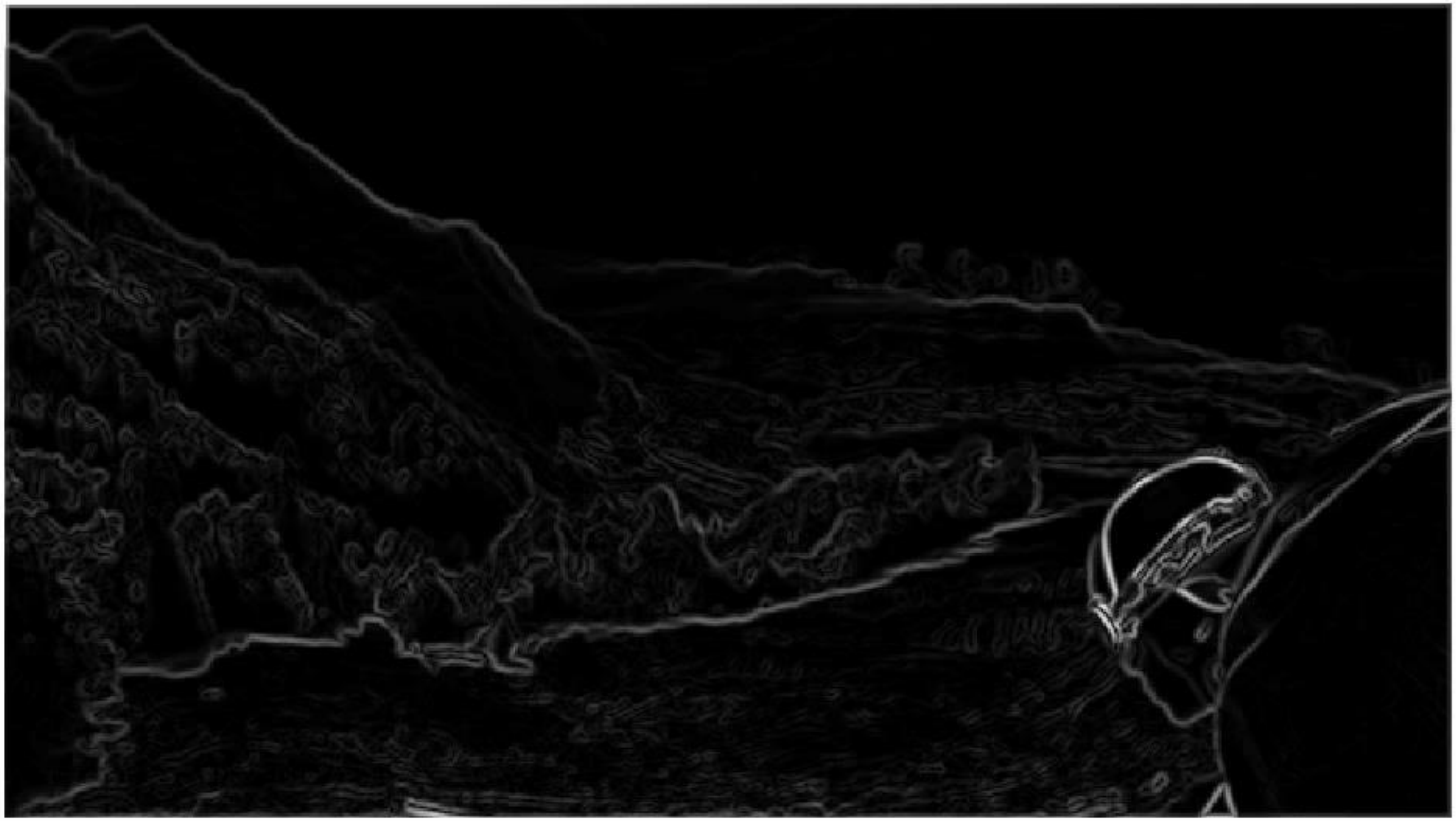
Image Scale

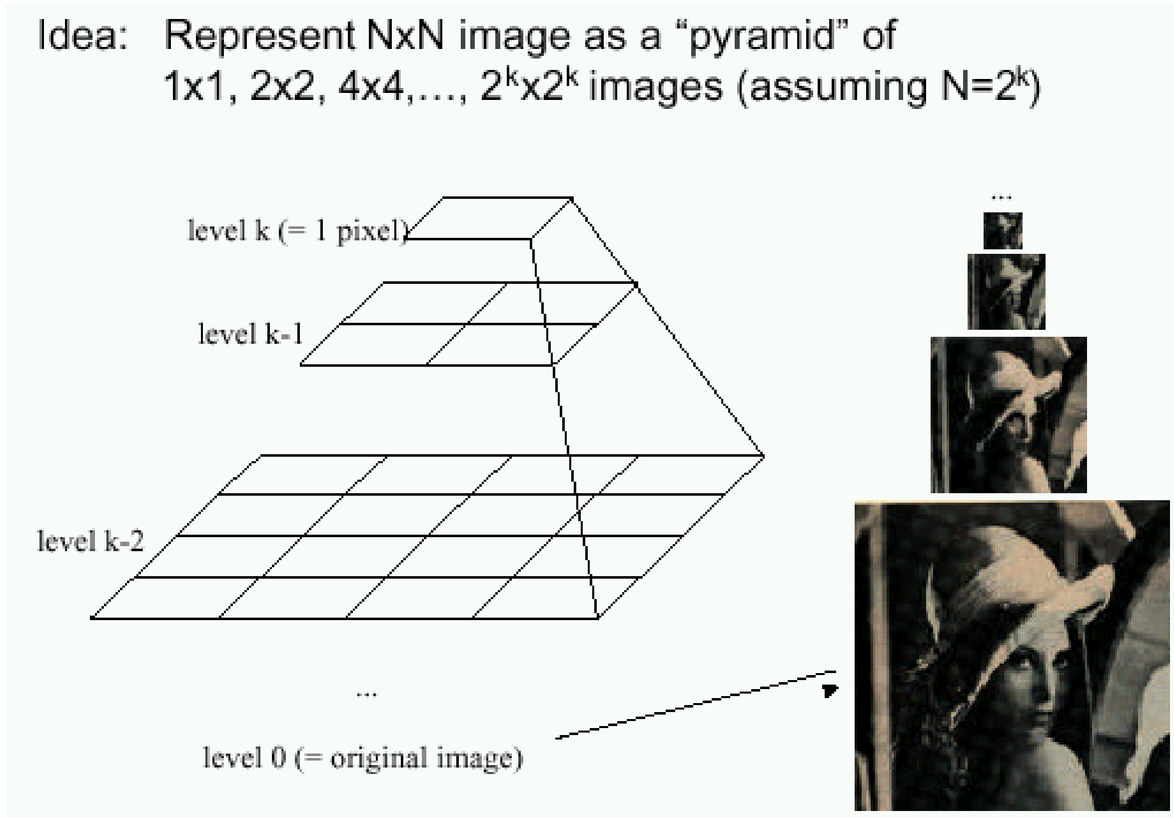






Different scale of image encodes different edge response.

Image Pyramids



Known as a Gaussian Pyramid [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

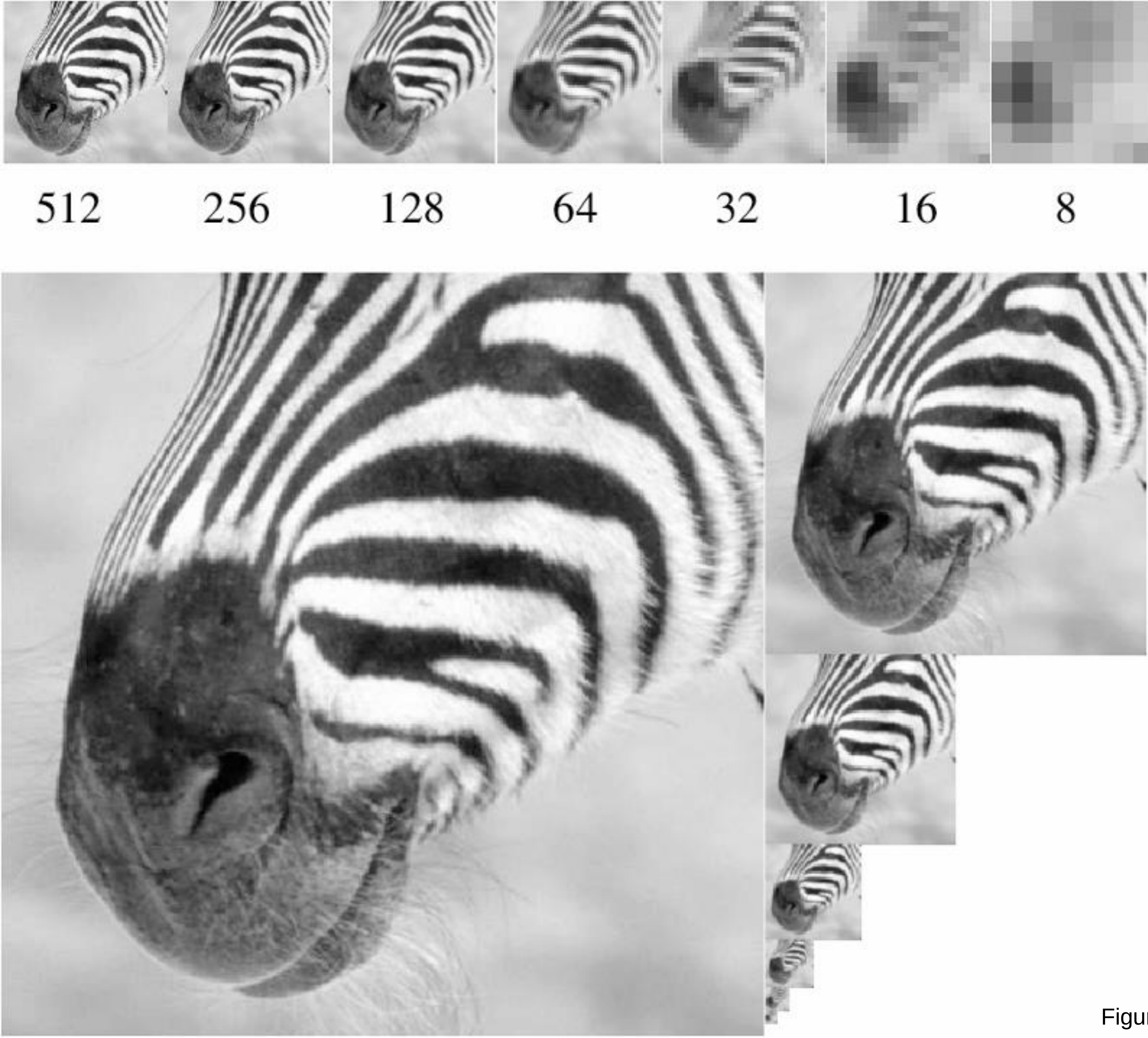
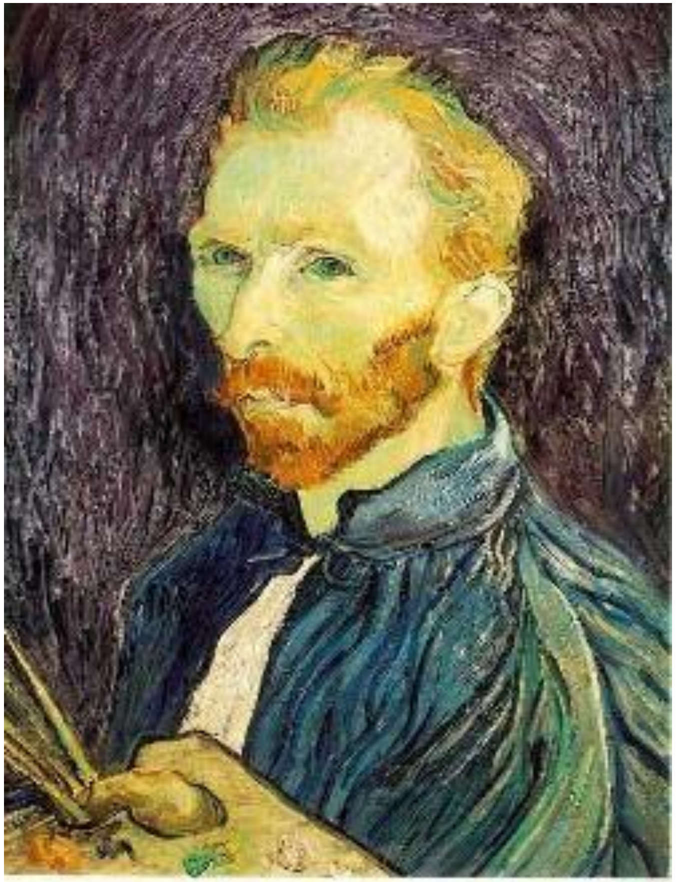


Figure from David Forsyth

Image sub-sampling

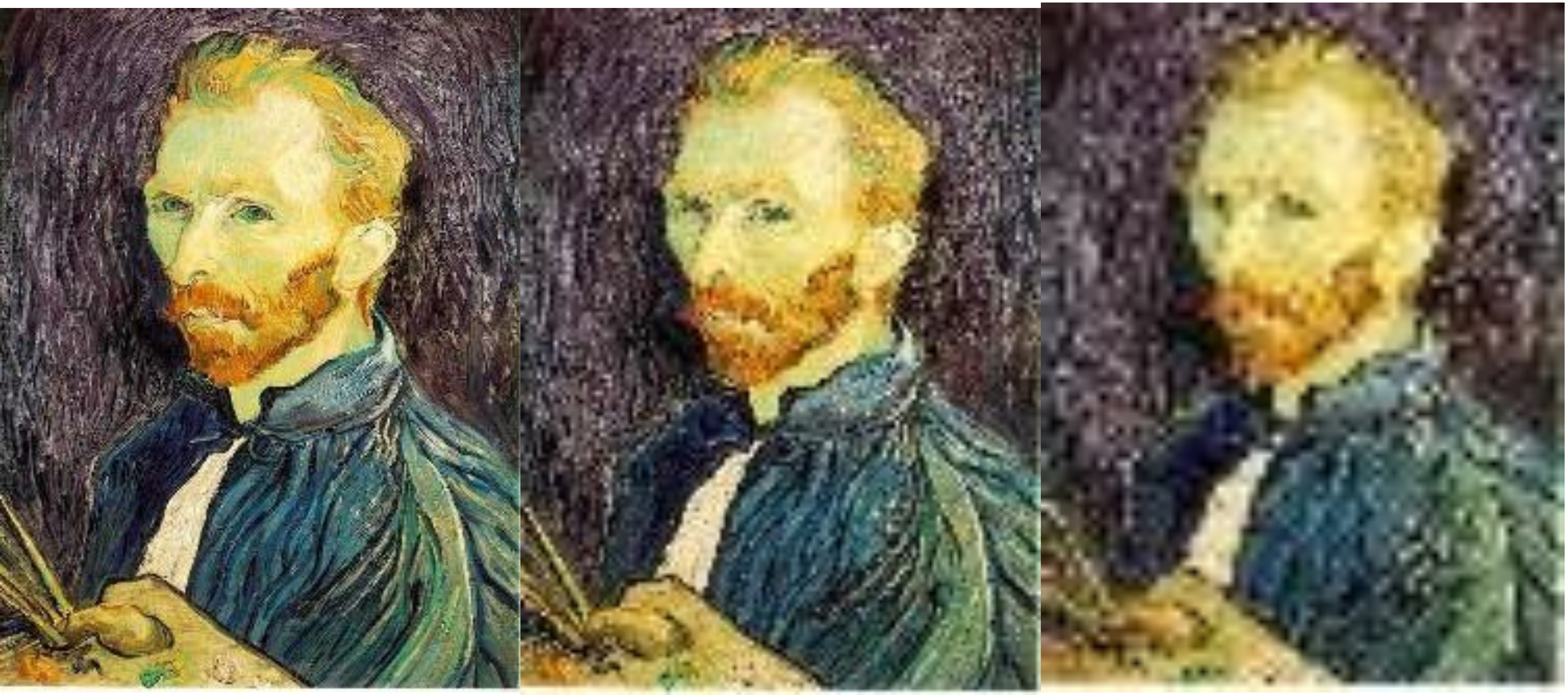


1/4

1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

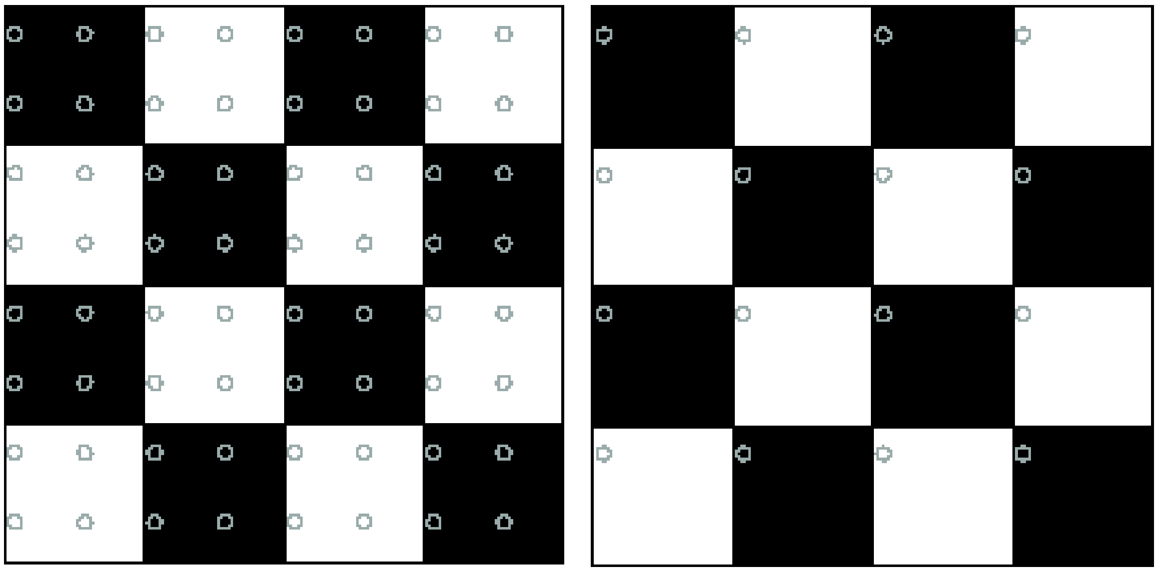
Image sub-sampling



1/2 1/4 (2x zoom) 1/8 (4x zoom)

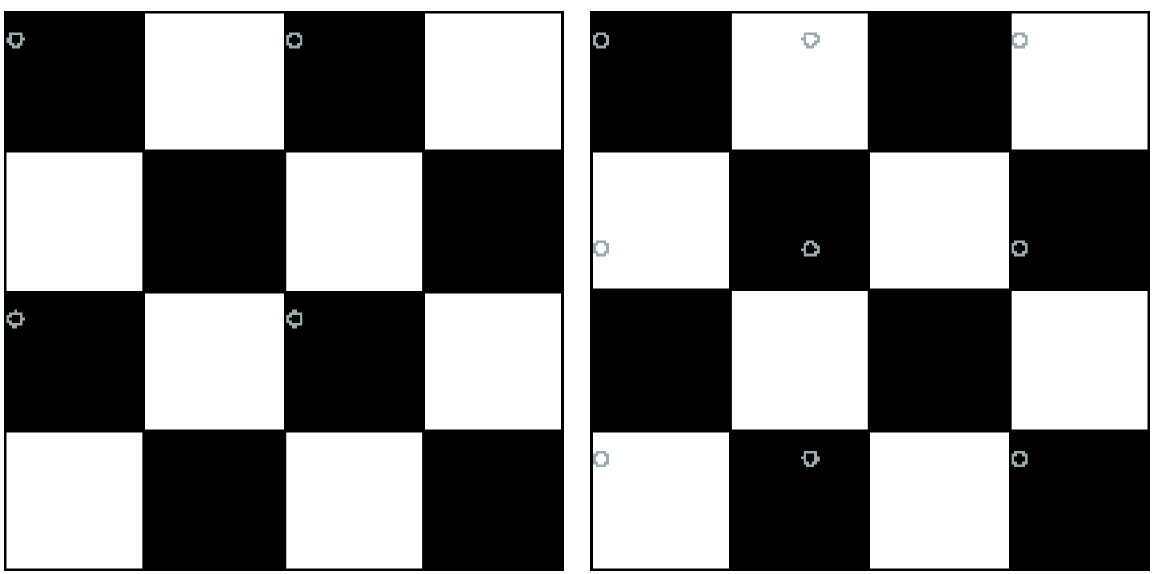
Why does this look so bad?

Sampling



Good sampling:

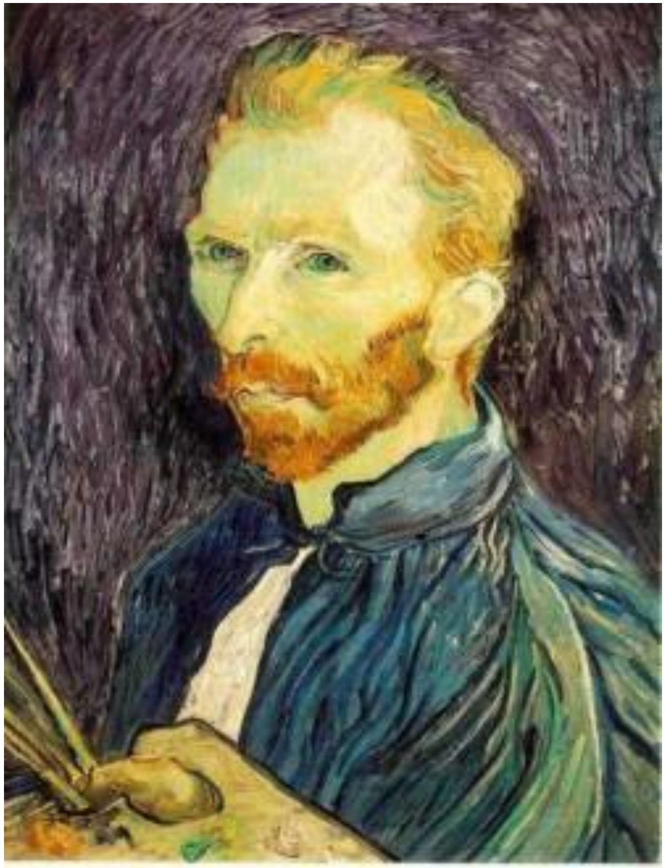
- Sample often or,
- Sample wisely



Bad sampling:

- see aliasing in action!

Gaussian pre-filtering



Gaussian 1/2

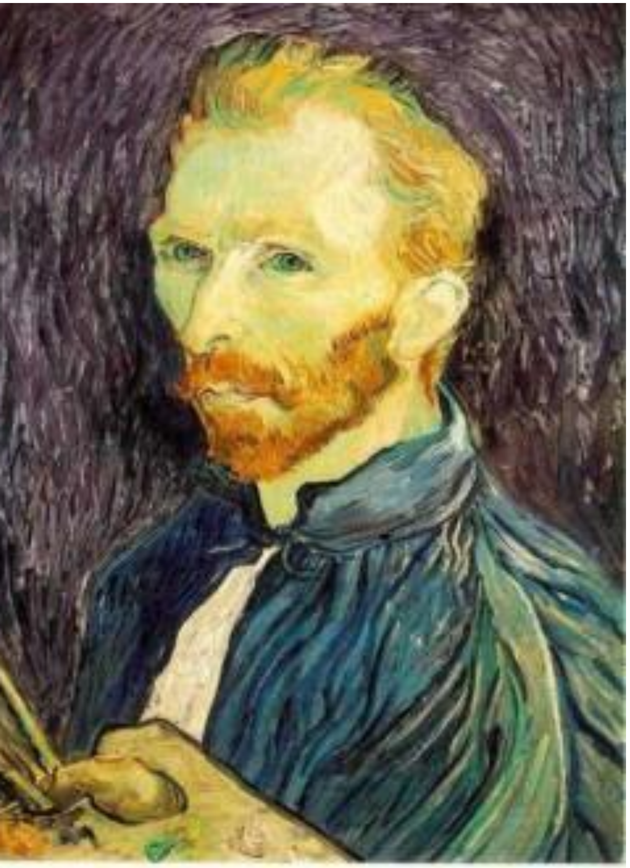
G 1/4

G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?

Subsampling with Gaussian pre-filtering



Gaussian 1/2

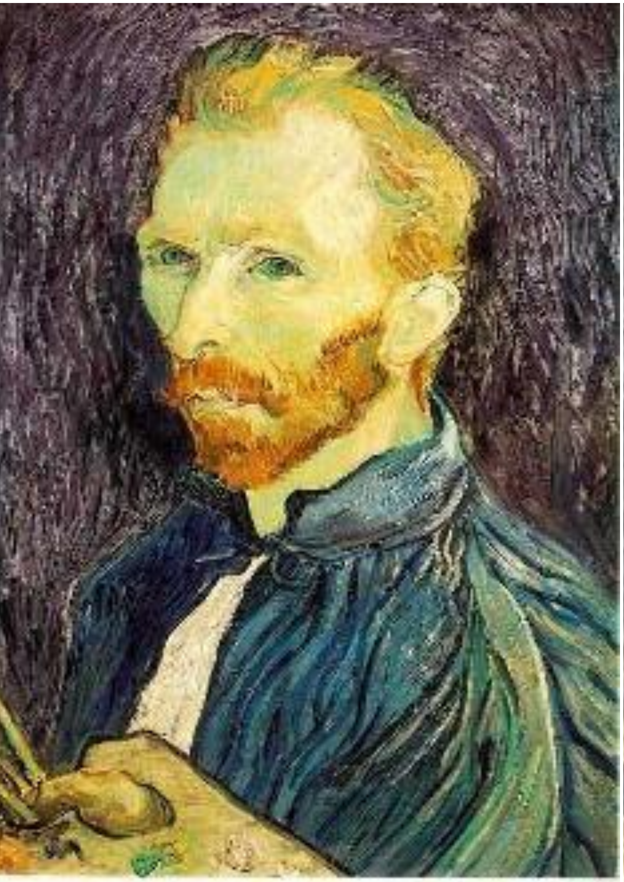
G 1/4

G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?
- How can we speed this up?

Comparison



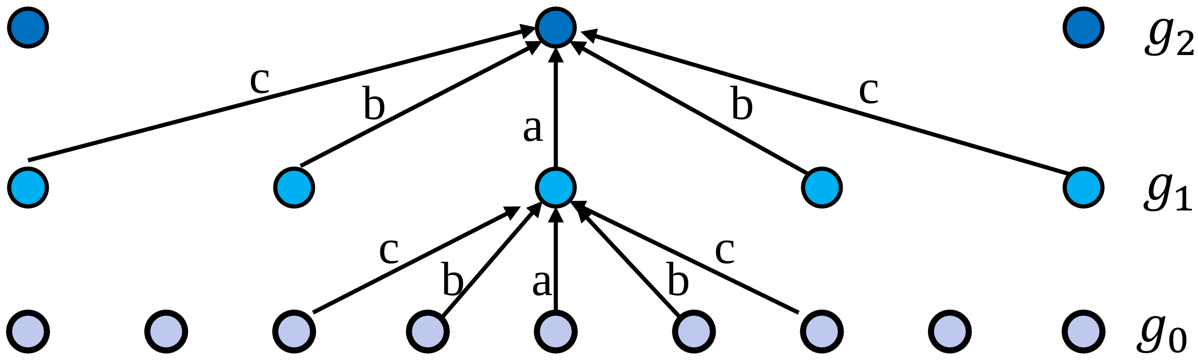
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Image Reduce

[Burt & Adelson, 1983]

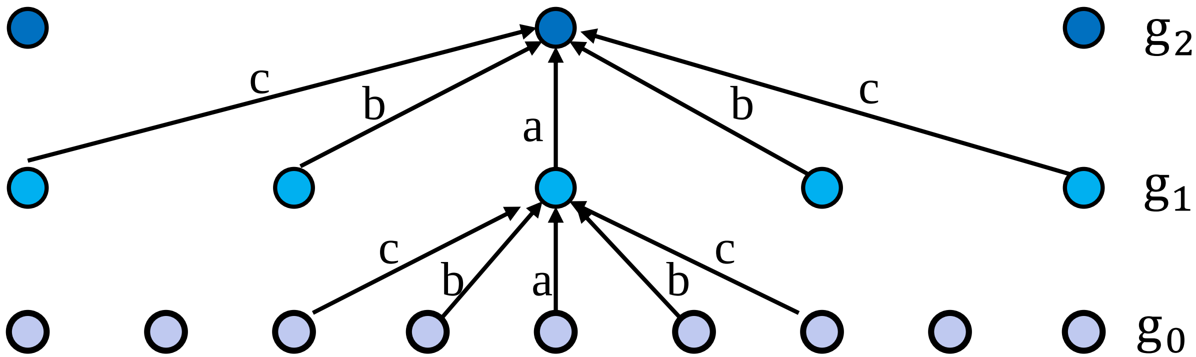


$$g_0 = \text{Image}$$

$$g_l = \text{REDUCE}[g_{l-1}]$$

Image Reduce

[Burt & Adelson, 1983]



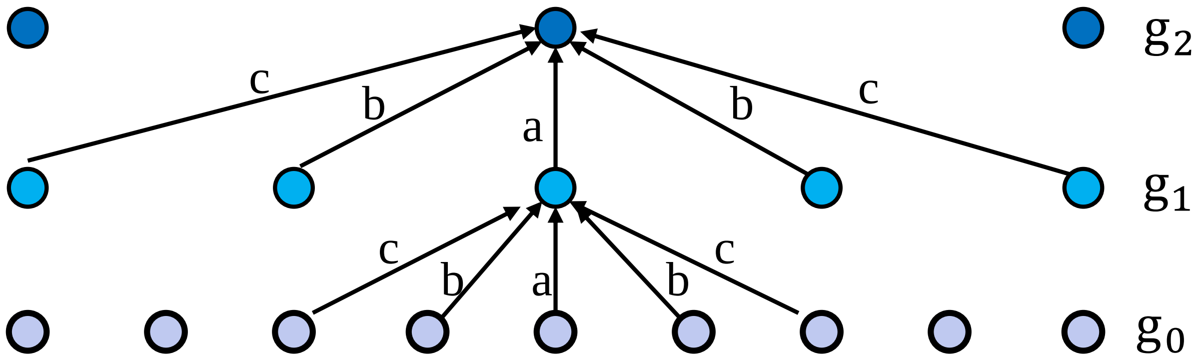
$$g_0 = \text{Image}$$

$$g_l = \text{REDUCE}[g_{l-1}]$$

$$g_l(i,j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) g_{l-1}(2i+m, 2j+n)$$

$$w(m,n) = \hat{w}(m)\hat{w}(n) \qquad \sum_{m}^{2} \hat{w}(m) = 1 \qquad \hat{w}(m) = \hat{w}(-m)$$

Image Reduce



$$g_0 = \mathrm{IMAGE}$$

$$g_l = \mathrm{REDUCE}[g_{l-1}]$$

$$g_l(i,j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) g_{l-1}(2i+m, 2j+n)$$

$$g_l = [g_{l-1} \otimes w] \downarrow 2$$

Choice in weighting function

$$\hat{w}(0) = a$$
$$\hat{w}(1) = \hat{w}(-1) = {}^{1}\!/_{4}$$
$$\hat{w}(1) = \hat{w}(-1) = {}^{1}\!/_{4} - {}^{a}\!/_{2}$$

Gaussian

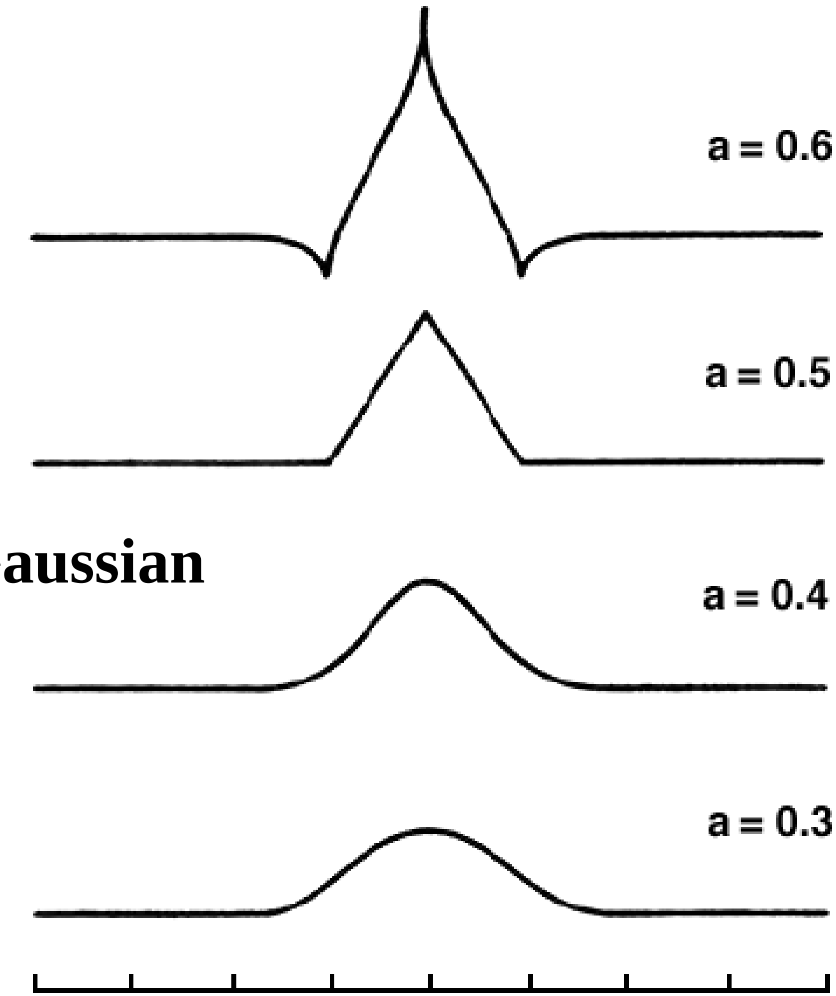
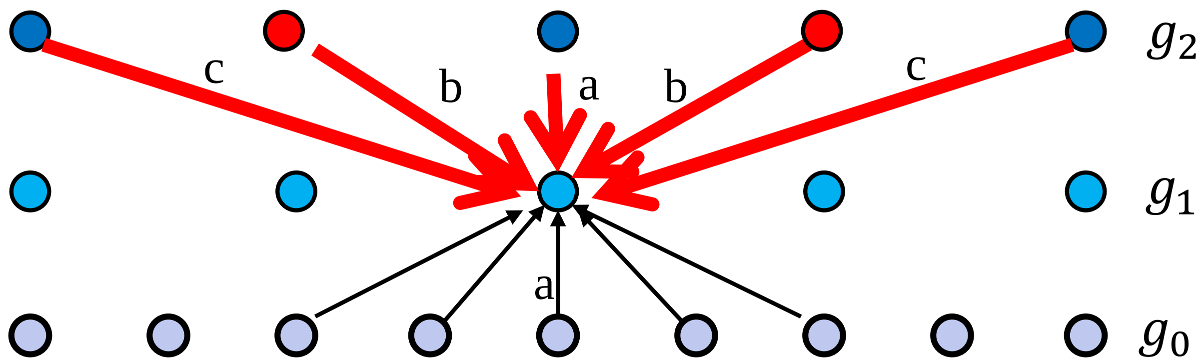


Image Expansion

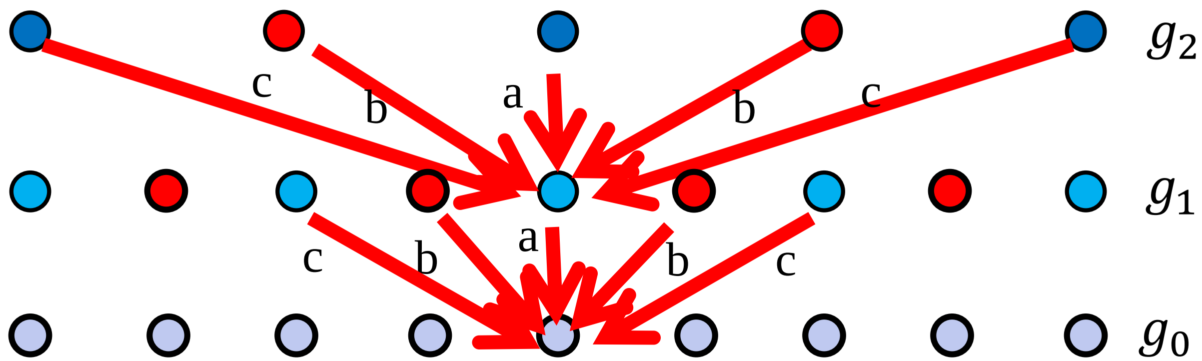


$$g_{l-1} = \mathrm{EXPAND}[g_l]$$

$$g_l(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) \cdot g_{l-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

$^{(i-m)}/_{2}$ and $^{(j-n)}/_{2}$ are integers

Image Expansion

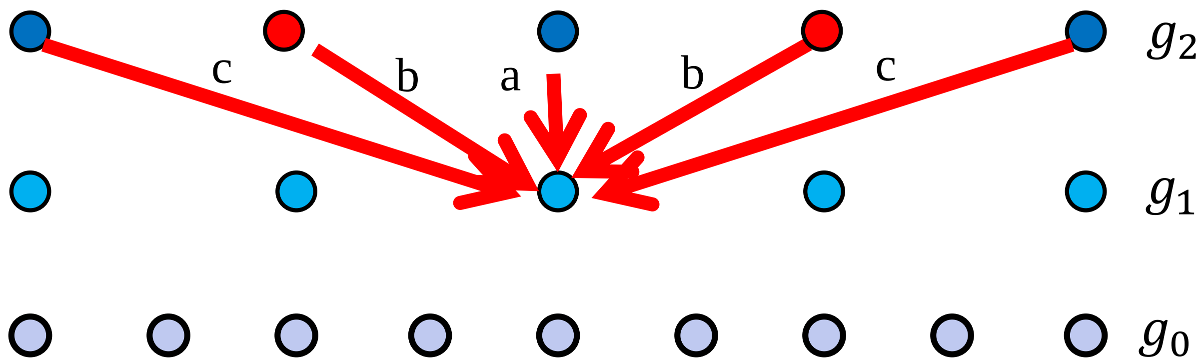


$$g_{l-1} = \mathrm{EXPAND}[g_l]$$

$$g_l(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) \cdot g_{l-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

$^{(i-m)}/_2$ and $^{(j-n)}/_2$ are integers

Image Expansion

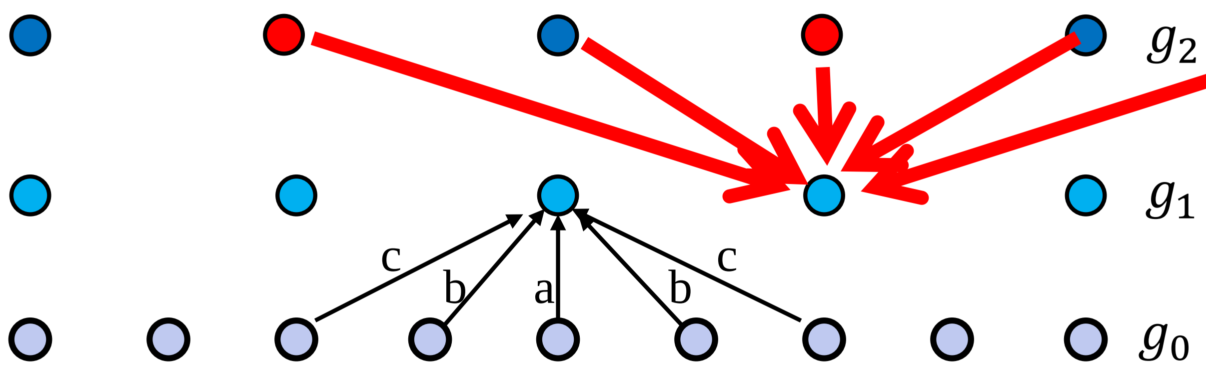


$$g_{l-1} = \mathrm{EXPAND}[g_l]$$

$$g_l(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) \cdot g_{l-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

${}^{(i-m)}/_{2}$ and ${}^{(j-n)}/_{2}$ are integers

Image Expansion

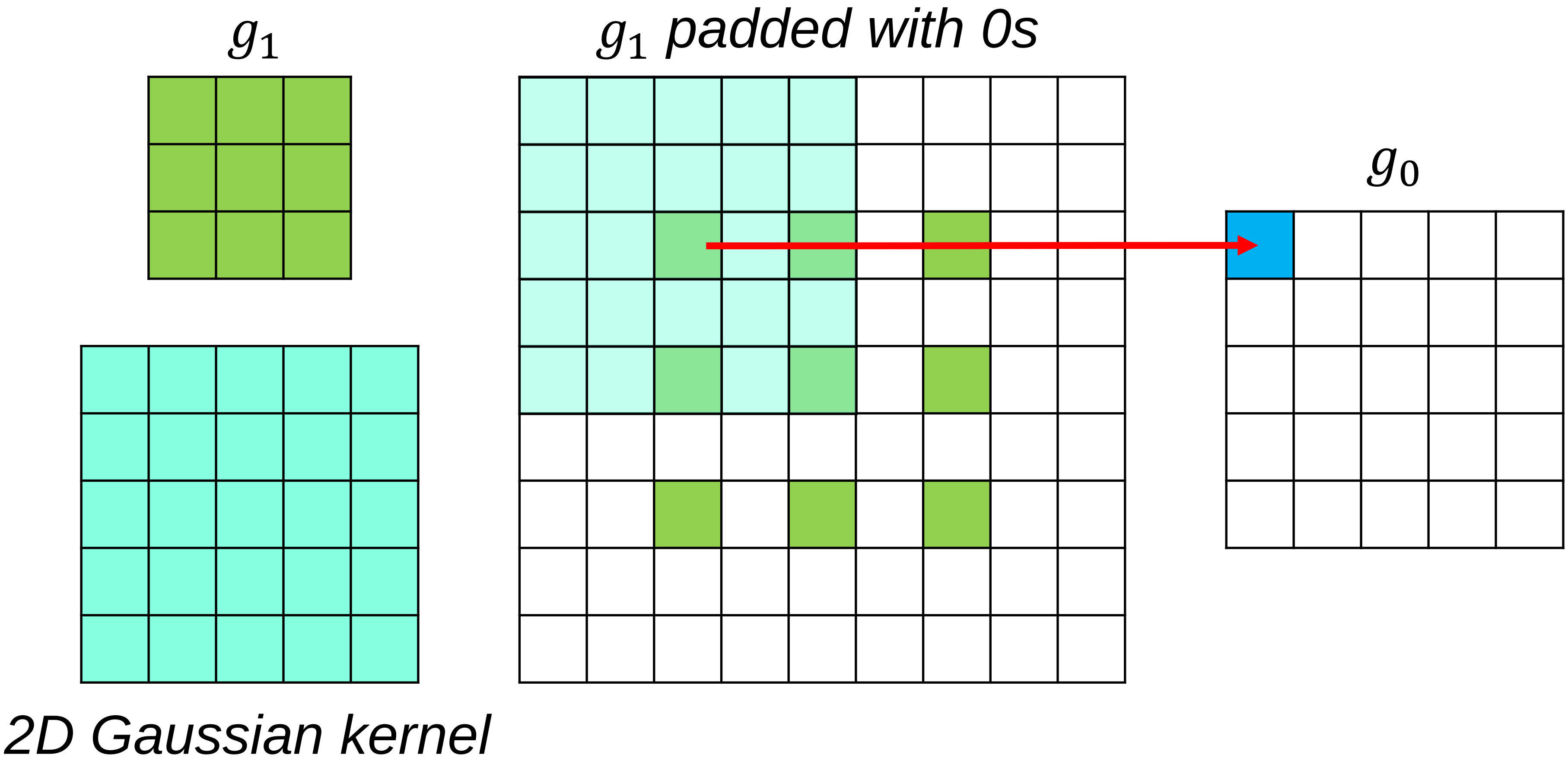


$$g_{l-1} = \mathrm{EXPAND}[g_l]$$

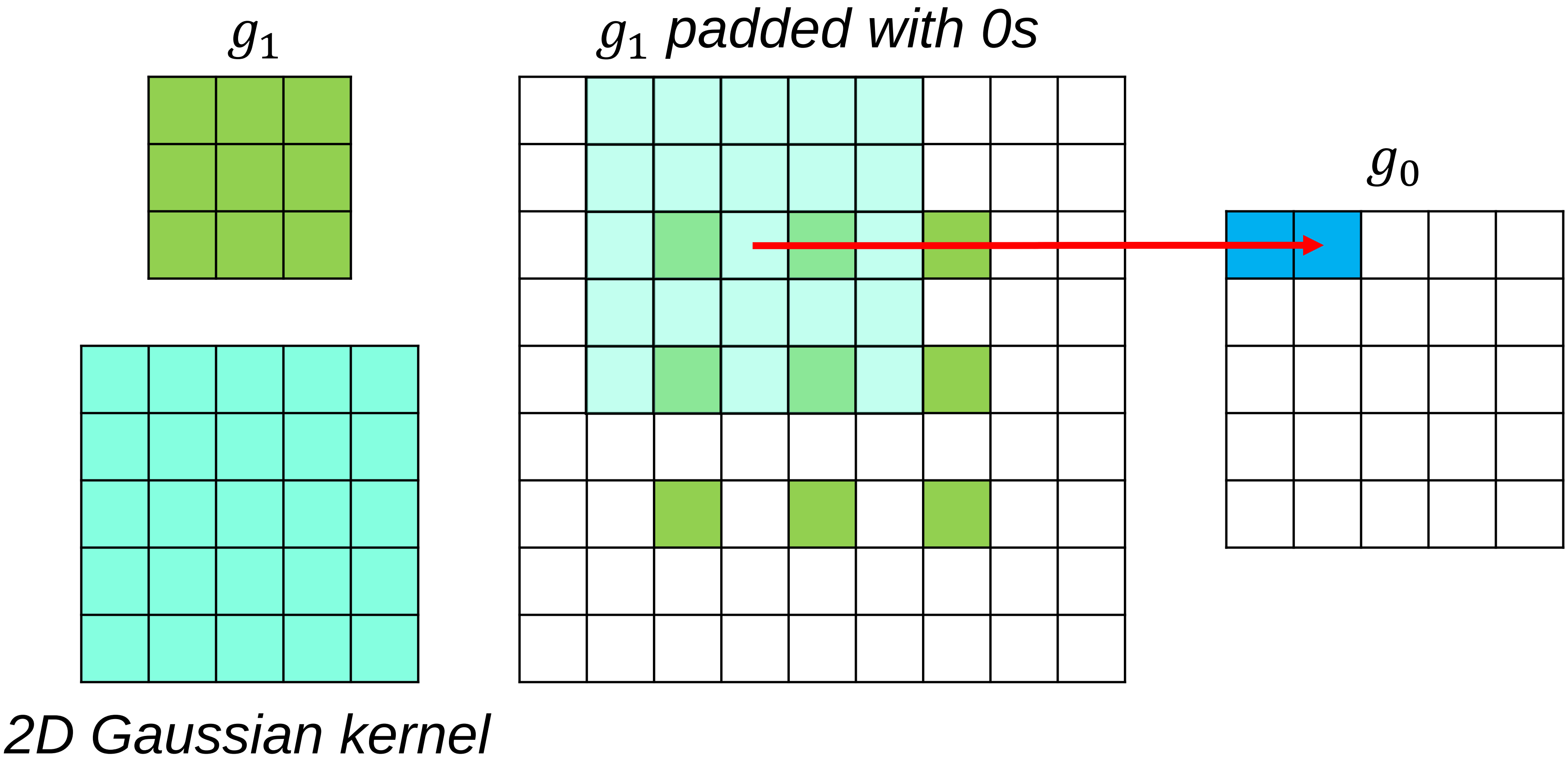
$$g_l(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) \cdot g_{l-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right)$$

${}^{(i-m)}/_{2}$ and ${}^{(j-n)}/_{2}$ are integers

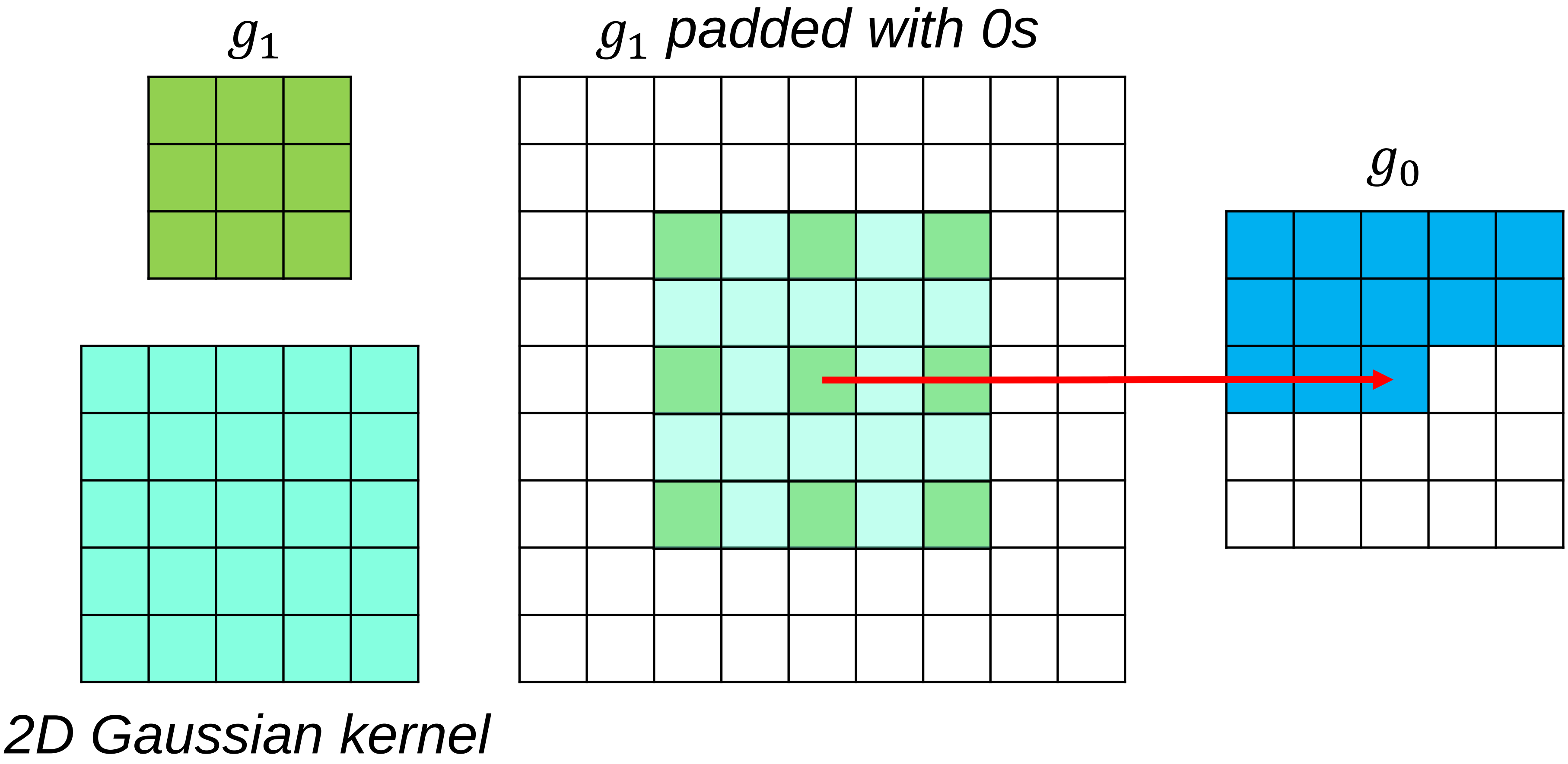
2D Image Expansion (part1)



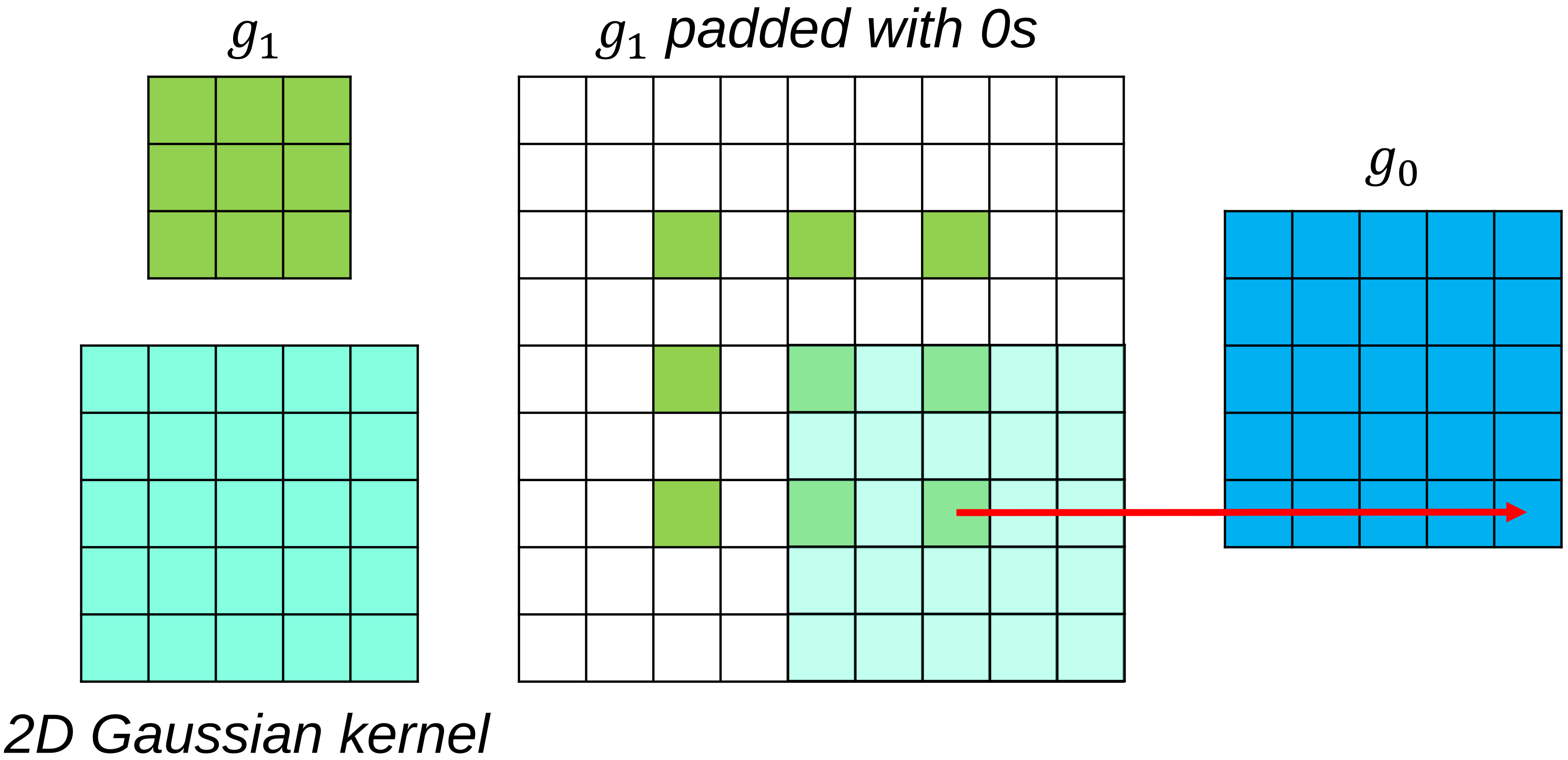
2D Image Expansion (part2)



2D Image Expansion (part3)



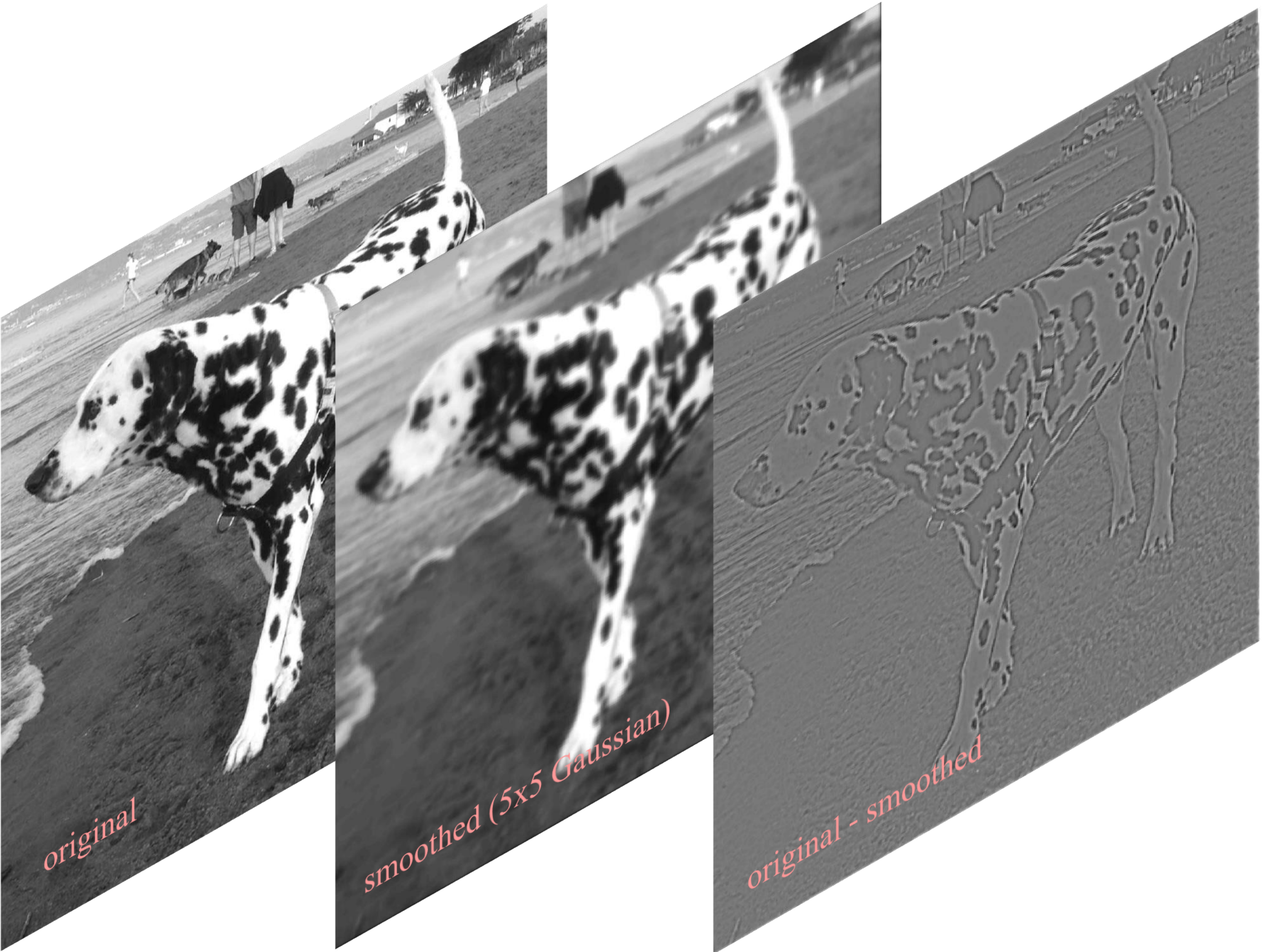
2D Image Expansion (part4)



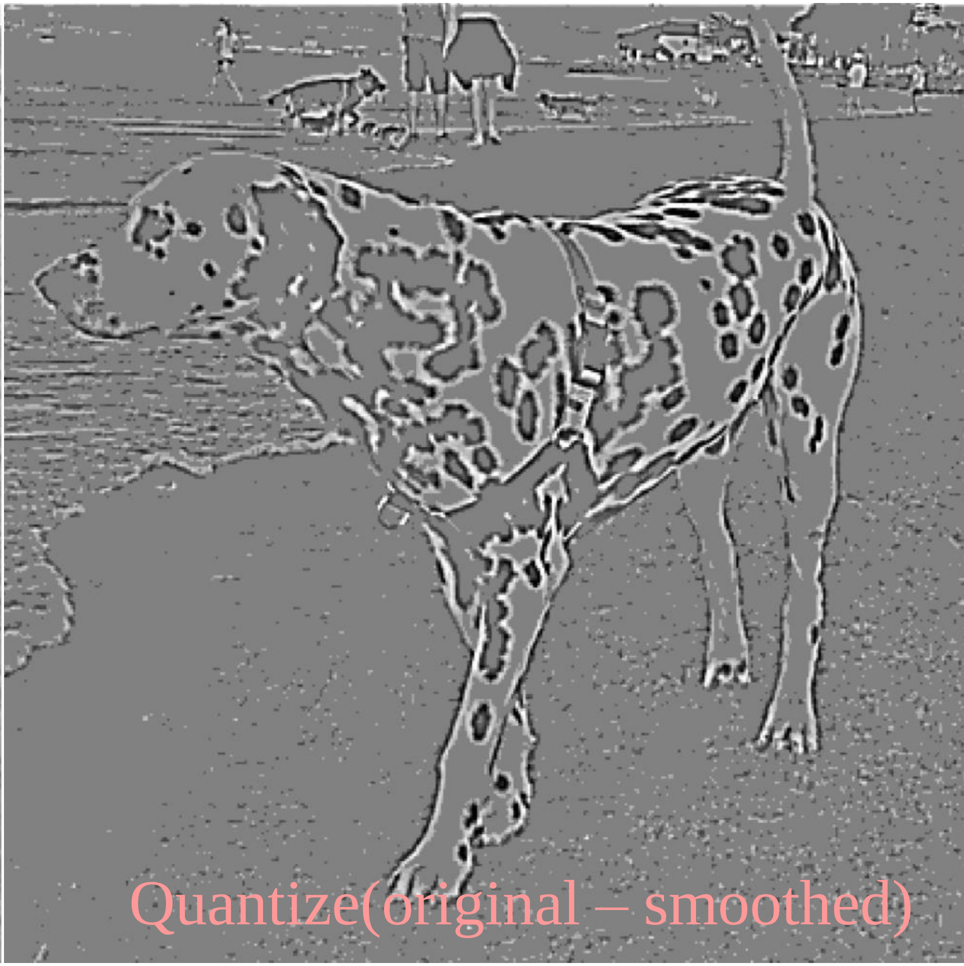
What does blurring take away?

What does blurring take away?

Difference as result of smoothing

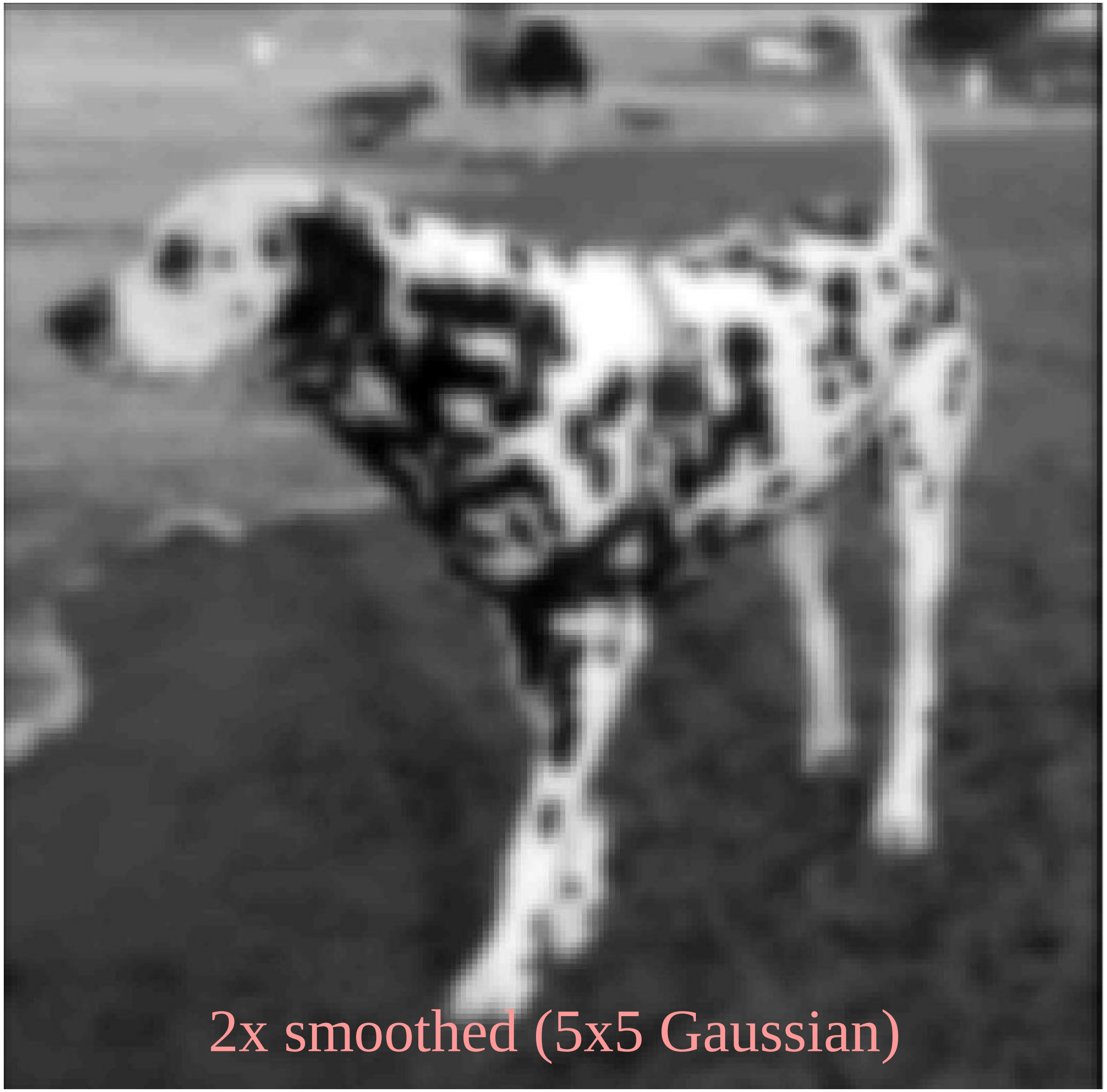

original
smoothed (5x5 Gaussian)
original - smoothed


original - smoothed


Quantize(original – smoothed)

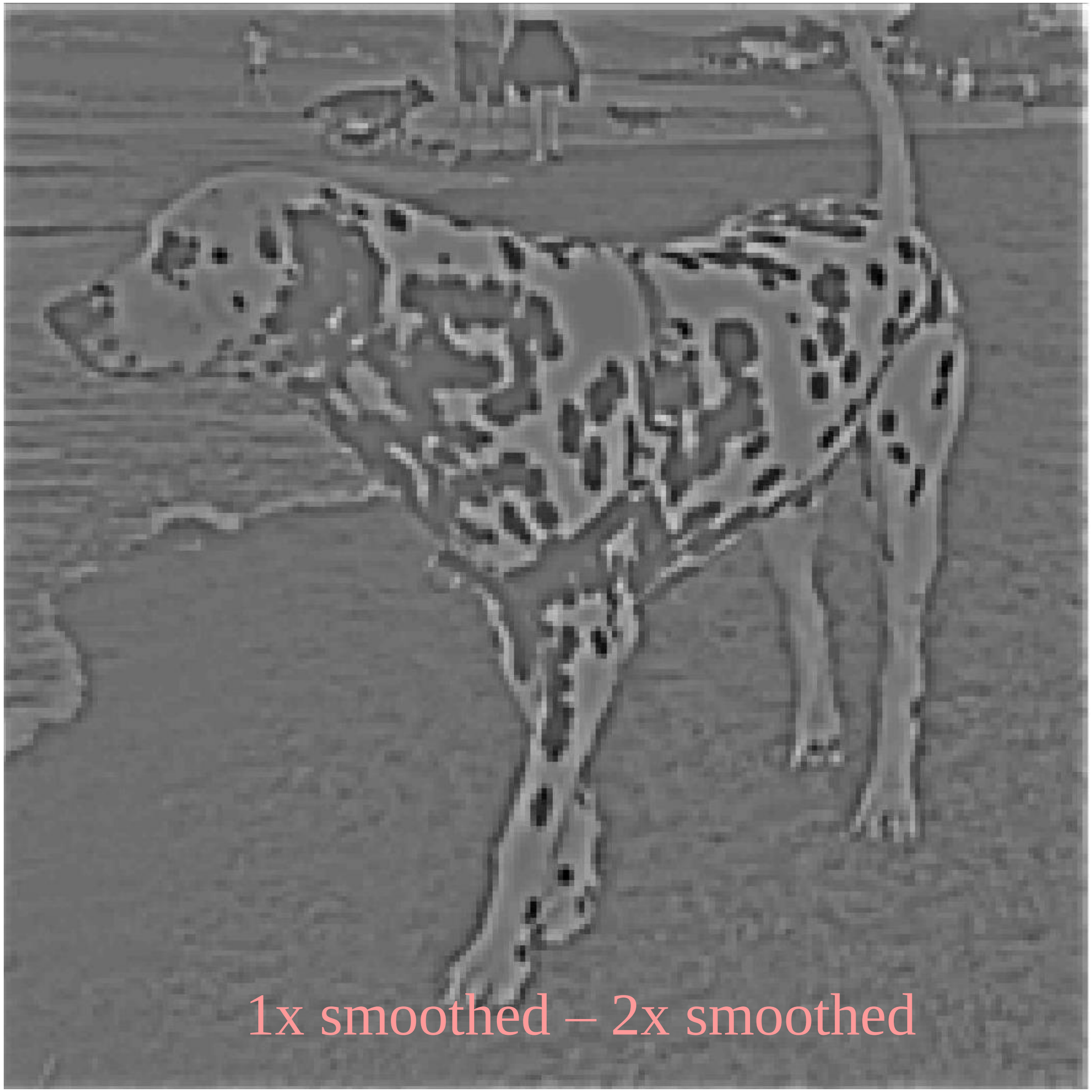
What does blurring take away?

What does blurring take away?

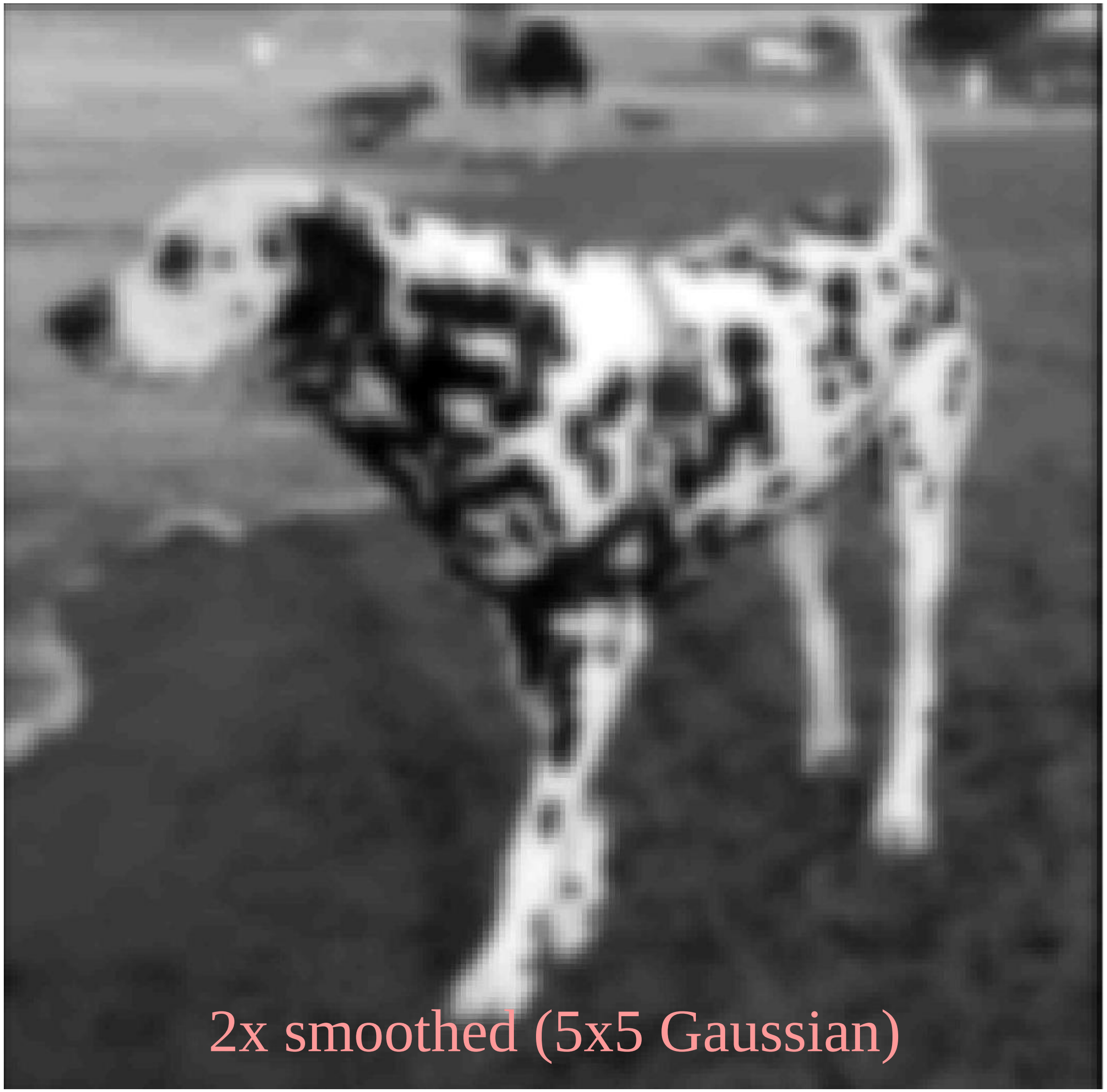


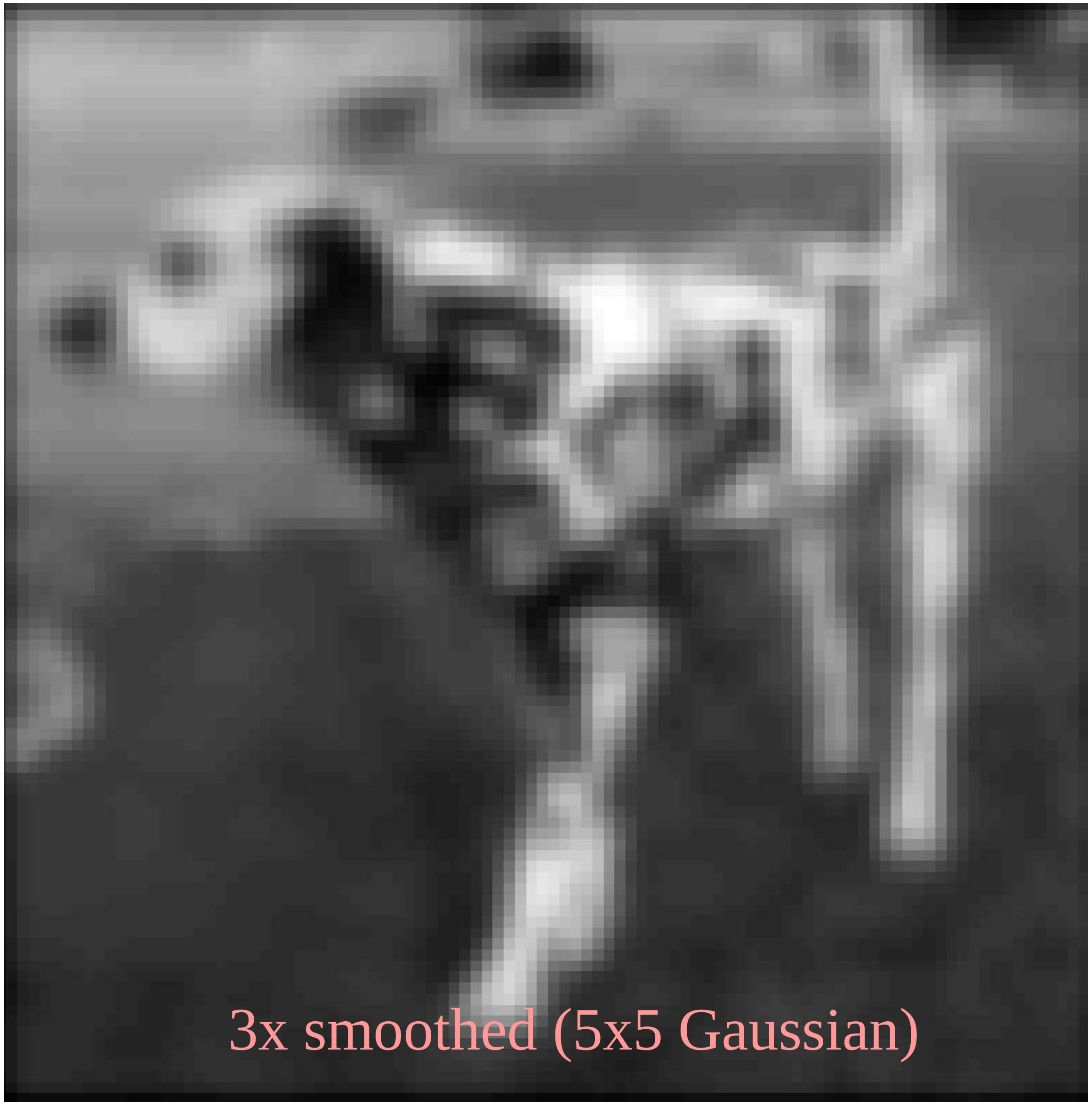
2x smoothed (5x5 Gaussian)

Difference of Gaussian

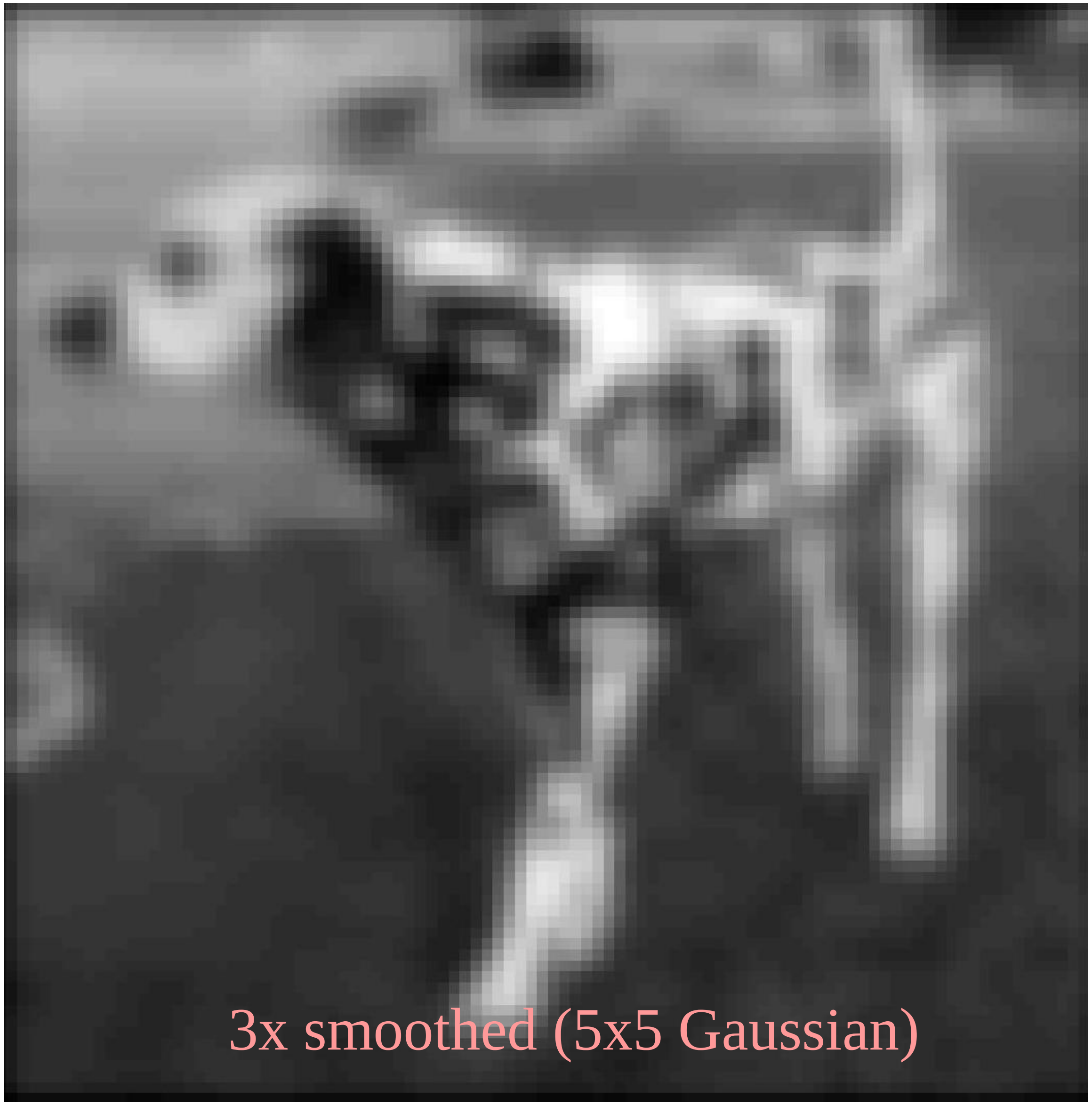


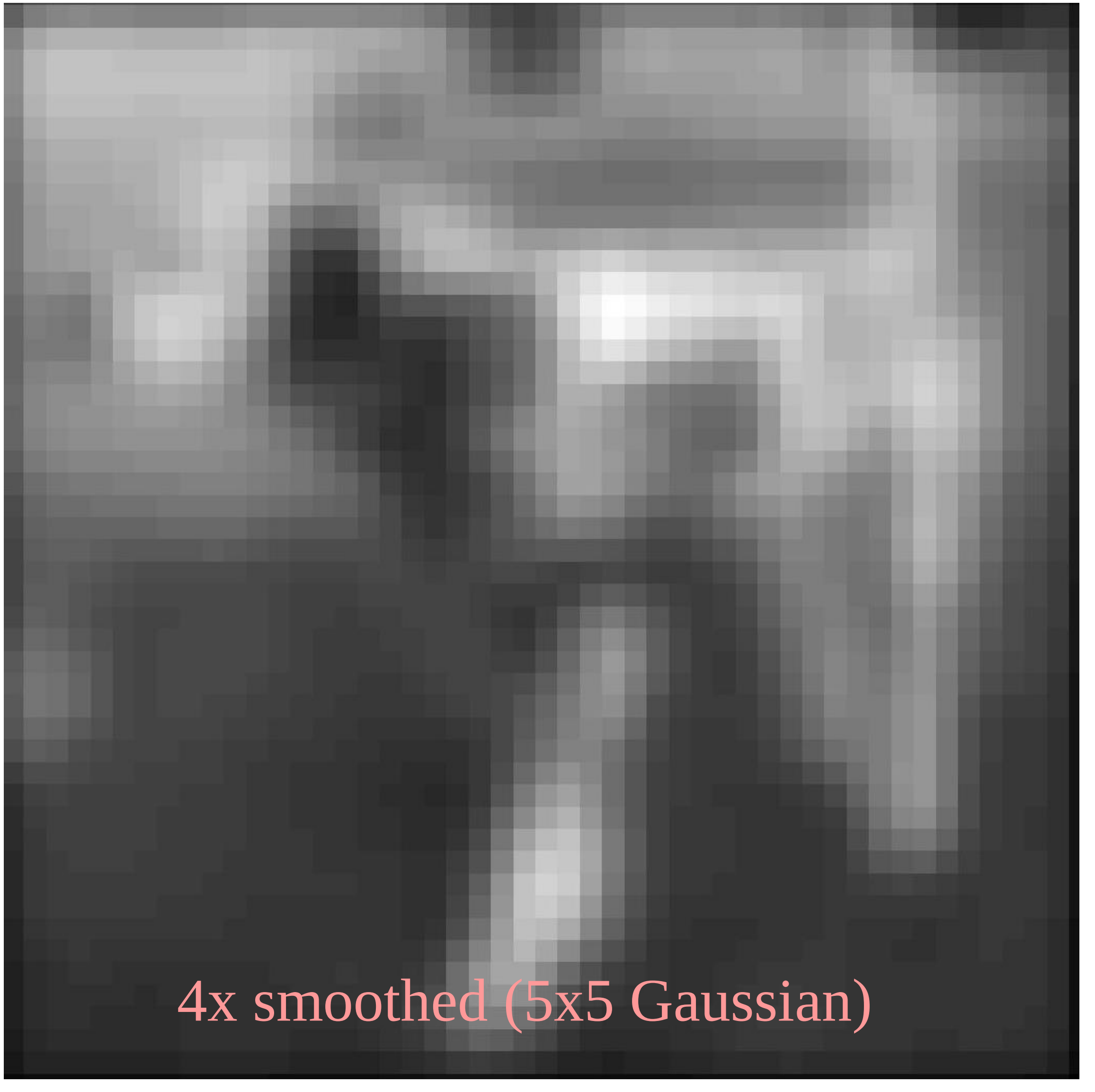
What does blurring take away?

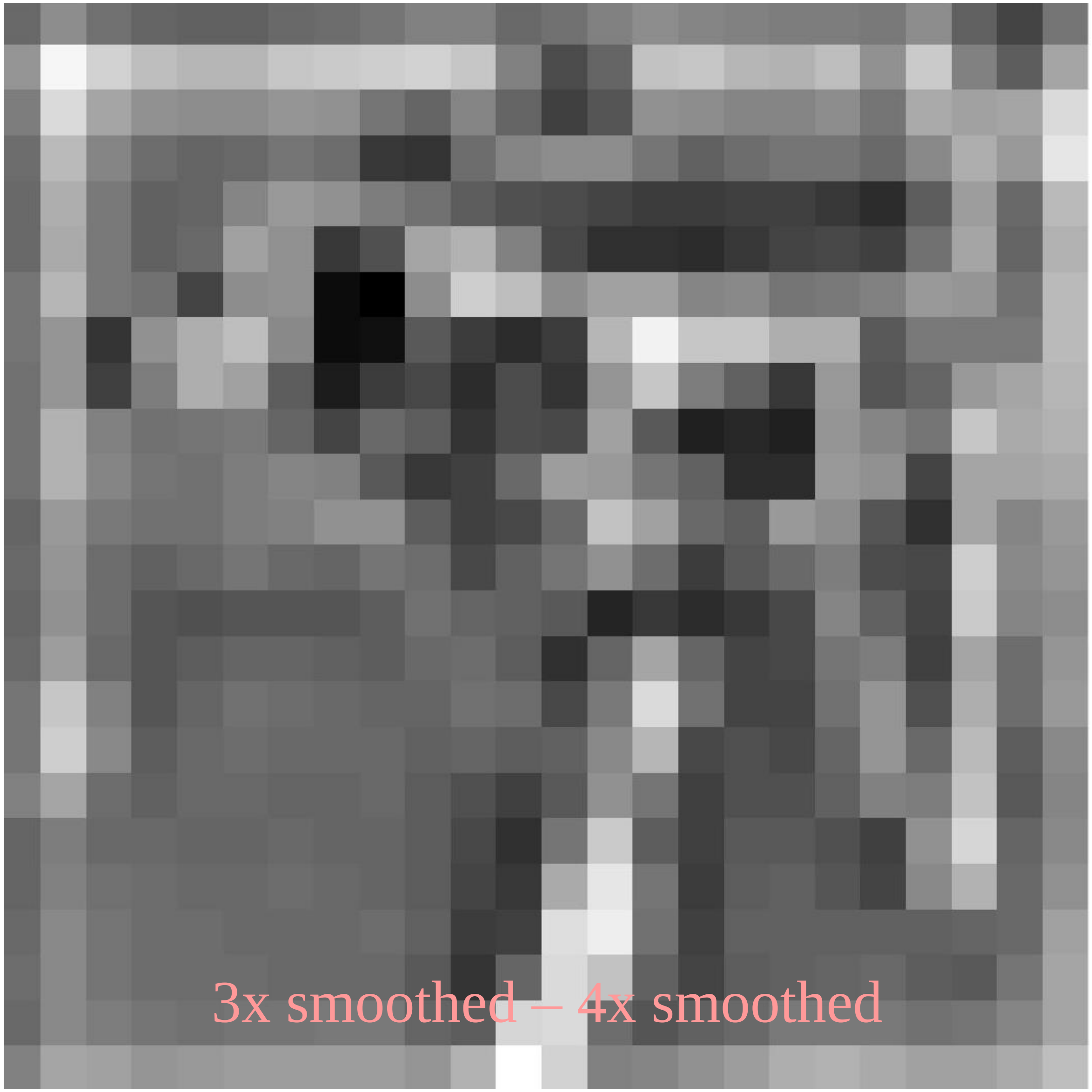



3x smoothed (5x5 Gaussian)

Difference of Gaussian


3x smoothed (5x5 Gaussian)


4x smoothed (5x5 Gaussian)


3x smoothed – 4x smoothed

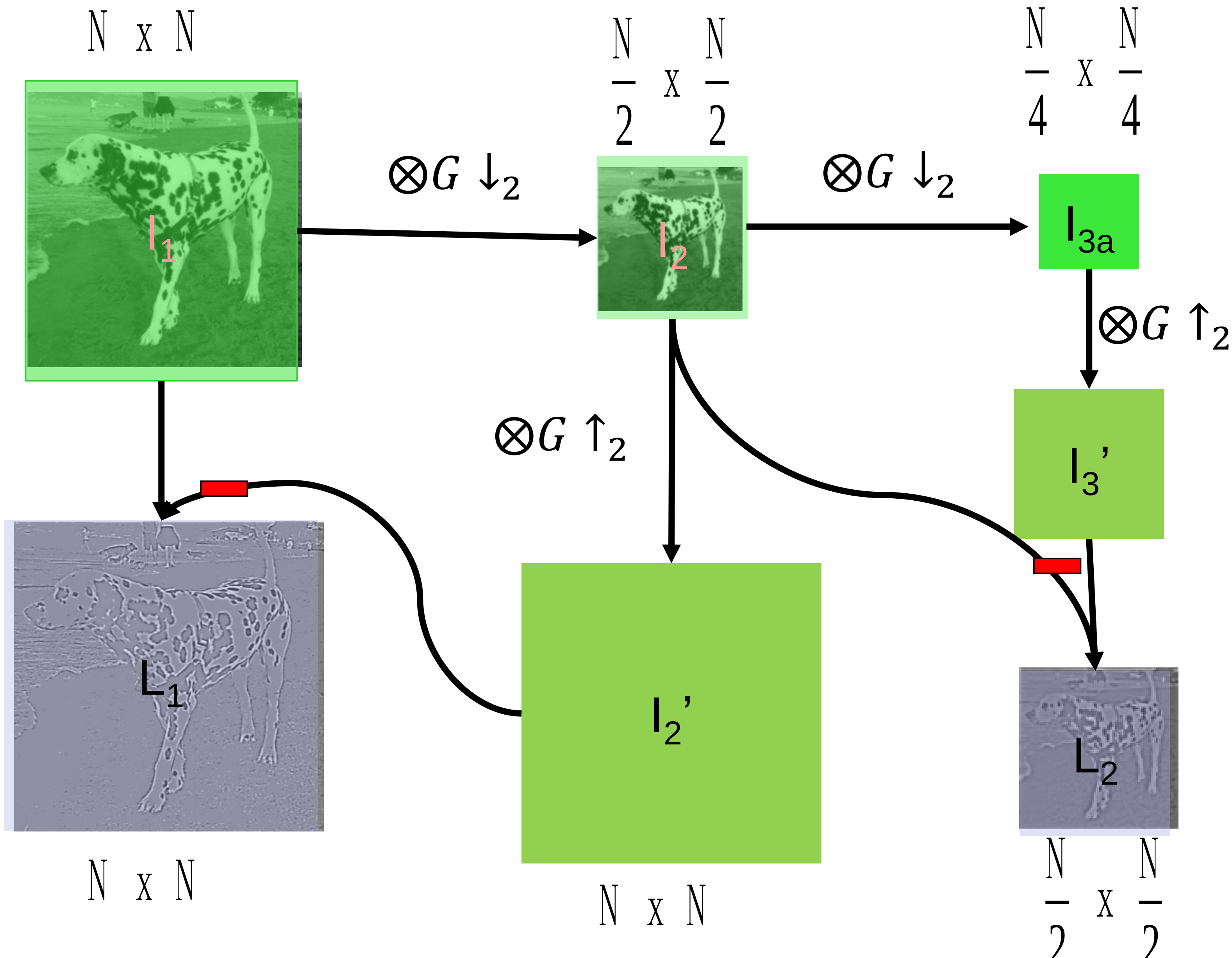
+

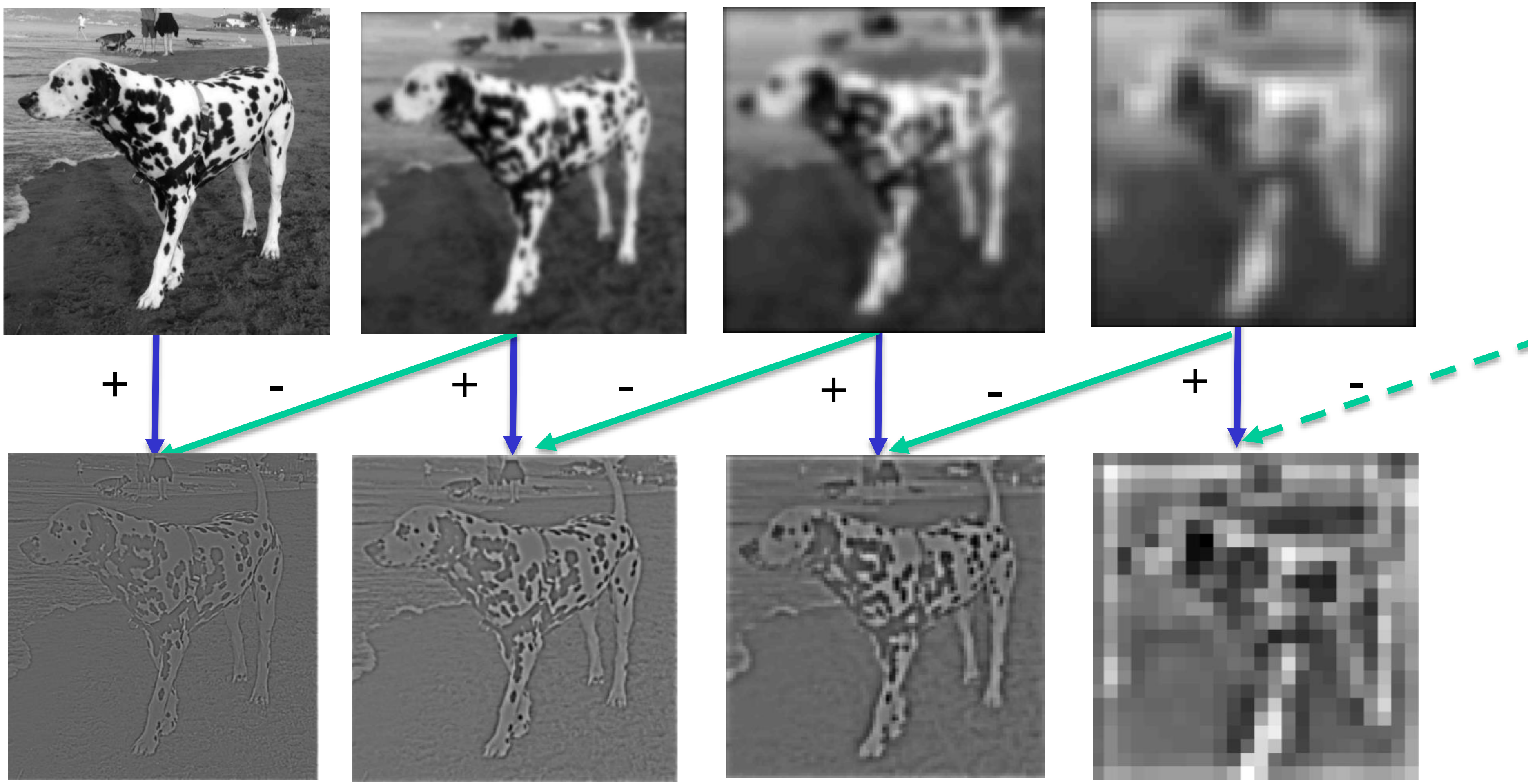
-

Laplacian Image

$$L_l = g_l - \text{EXPAND}[g_{l+1}]$$

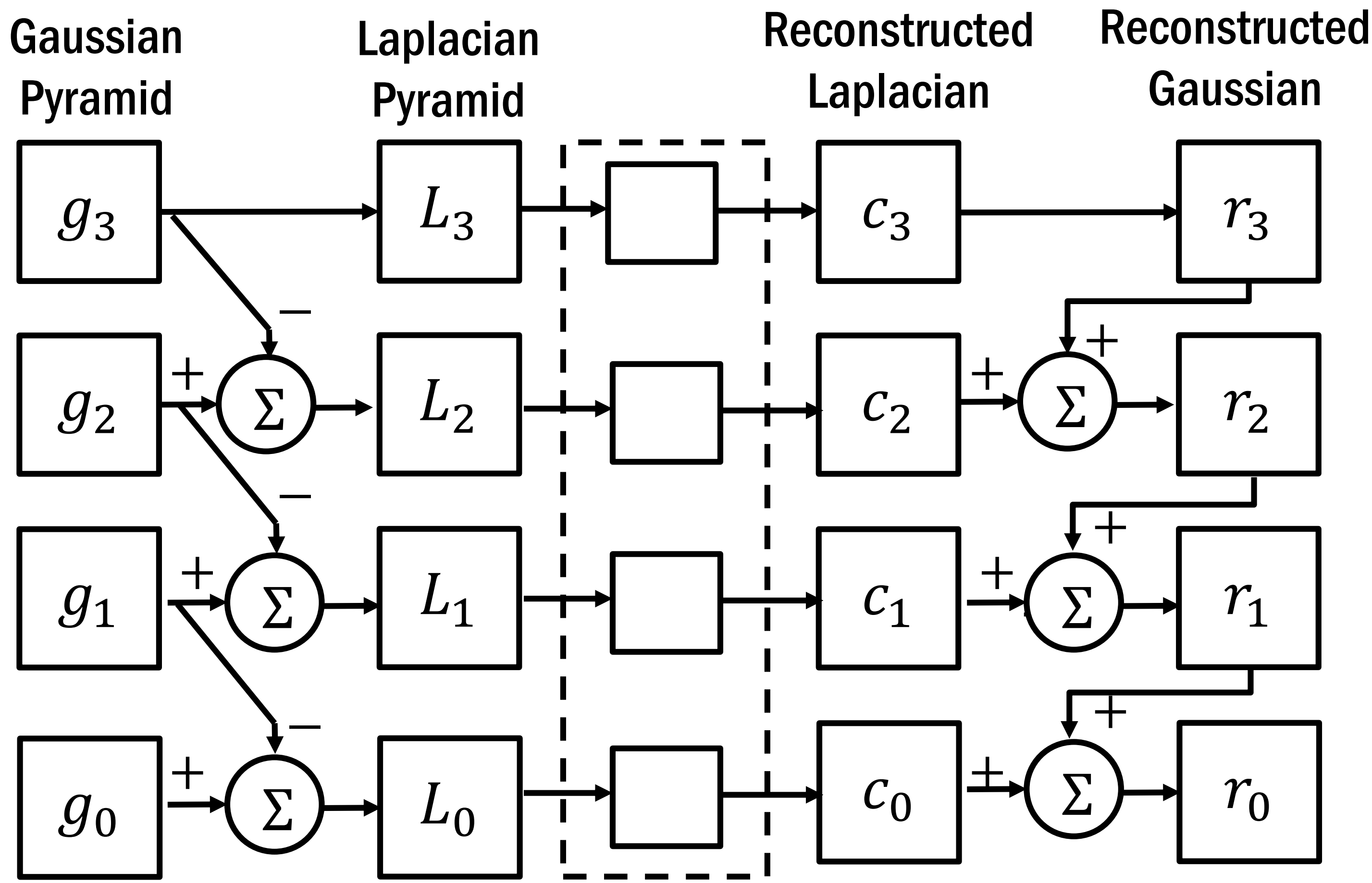
Extraction of Laplacian





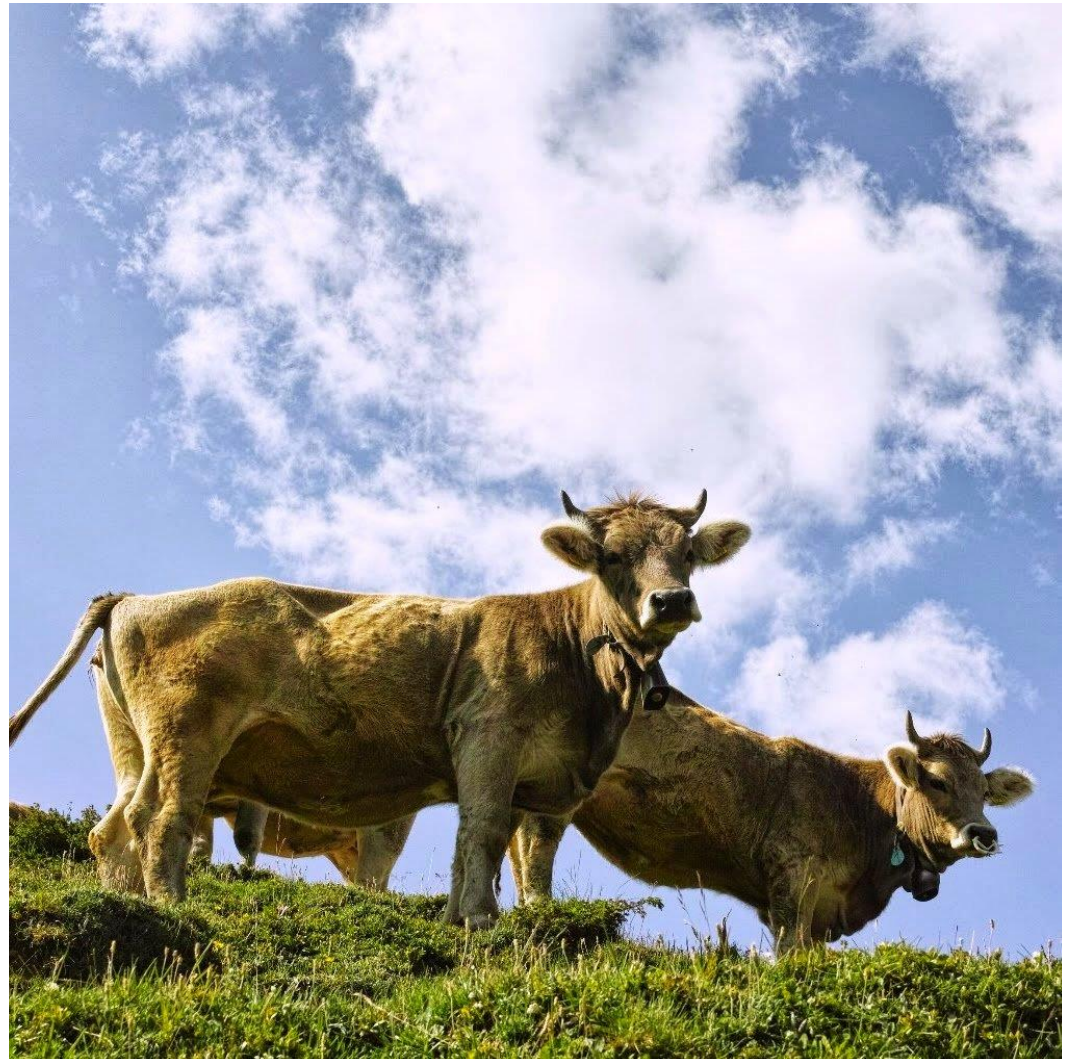
Gaussian pyramid is smooth=> can be subsampled

Laplacian pyramid has narrow band of frequency=> compressed

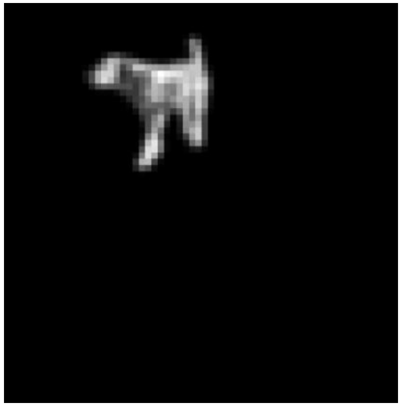


$$g_N = L_N \qquad g_l = L_l + \mathrm{EXPAND}[g_{l+1}]$$

Pyramid Blending

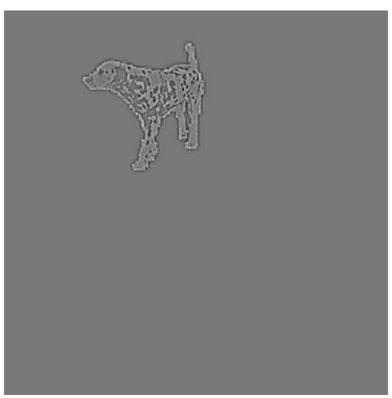


laplacian
level
4



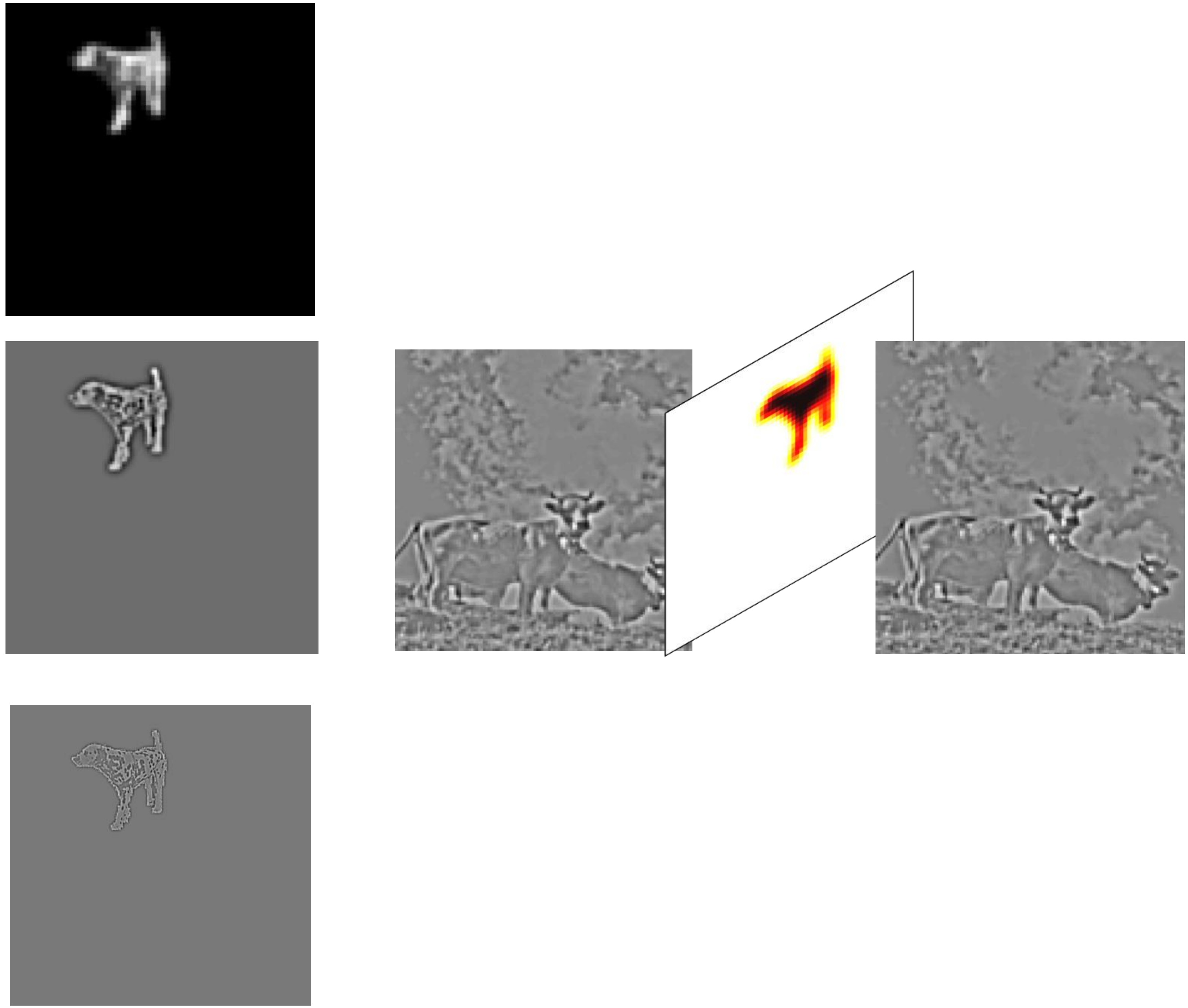
laplacian
level
2

laplacian
level
0



top pyramid

bottom pyramid



laplacian level 4

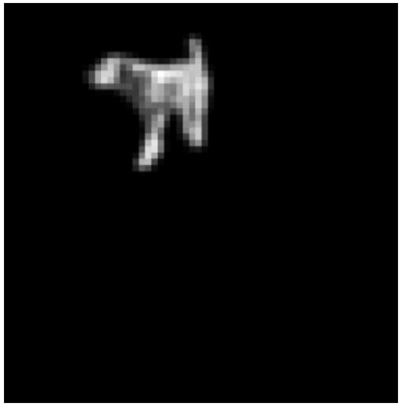
laplacian level 2

laplacian level 0

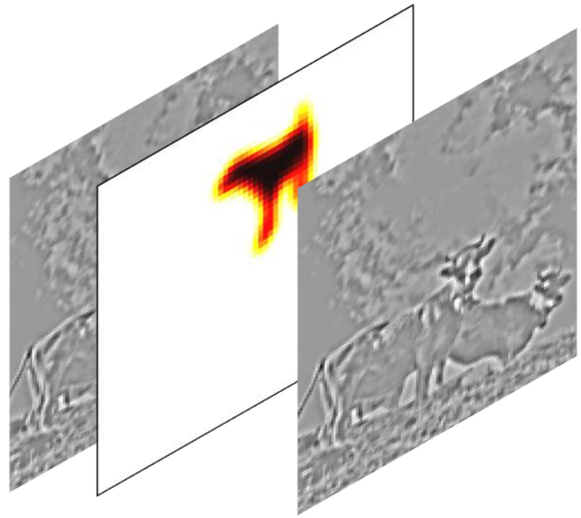
top pyramid

bottom pyramid

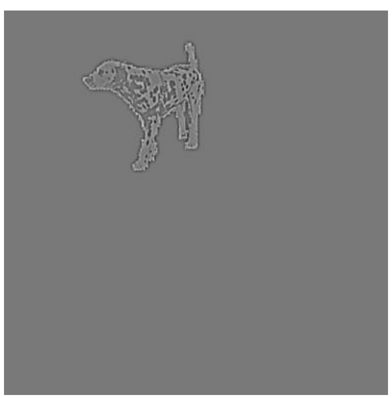
laplacian level 4



laplacian level 2



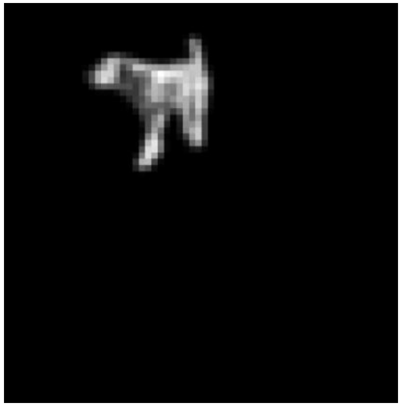
laplacian level 0



top pyramid

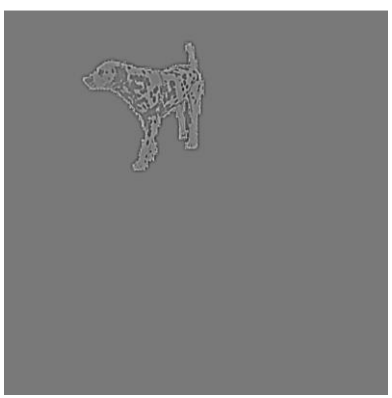
bottom pyramid

laplacian level 4



laplacian level 2

laplacian level 0



top pyramid

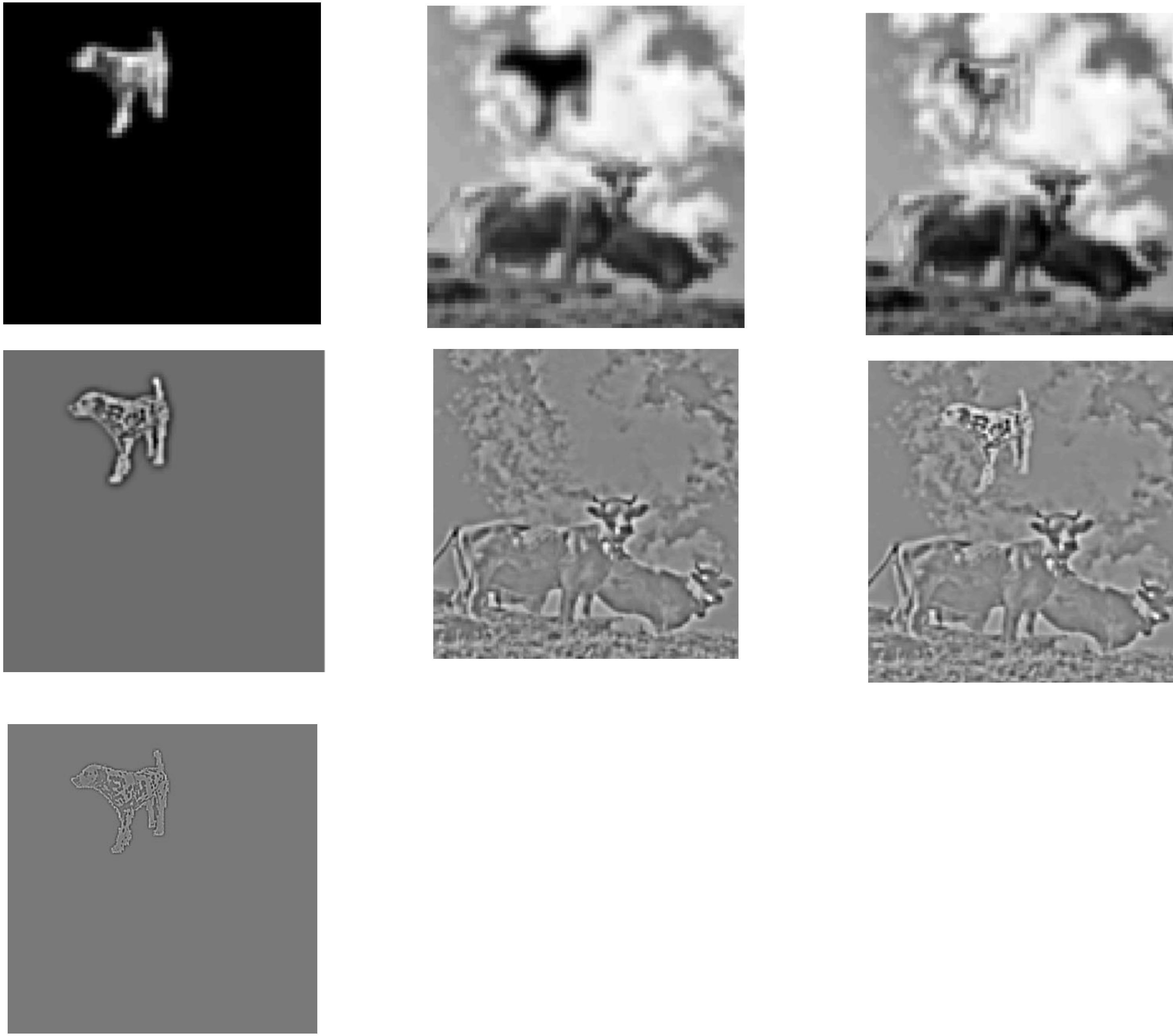
bottom pyramid

blended pyramid

laplacian level 4

laplacian level 2

laplacian level 0



top pyramid

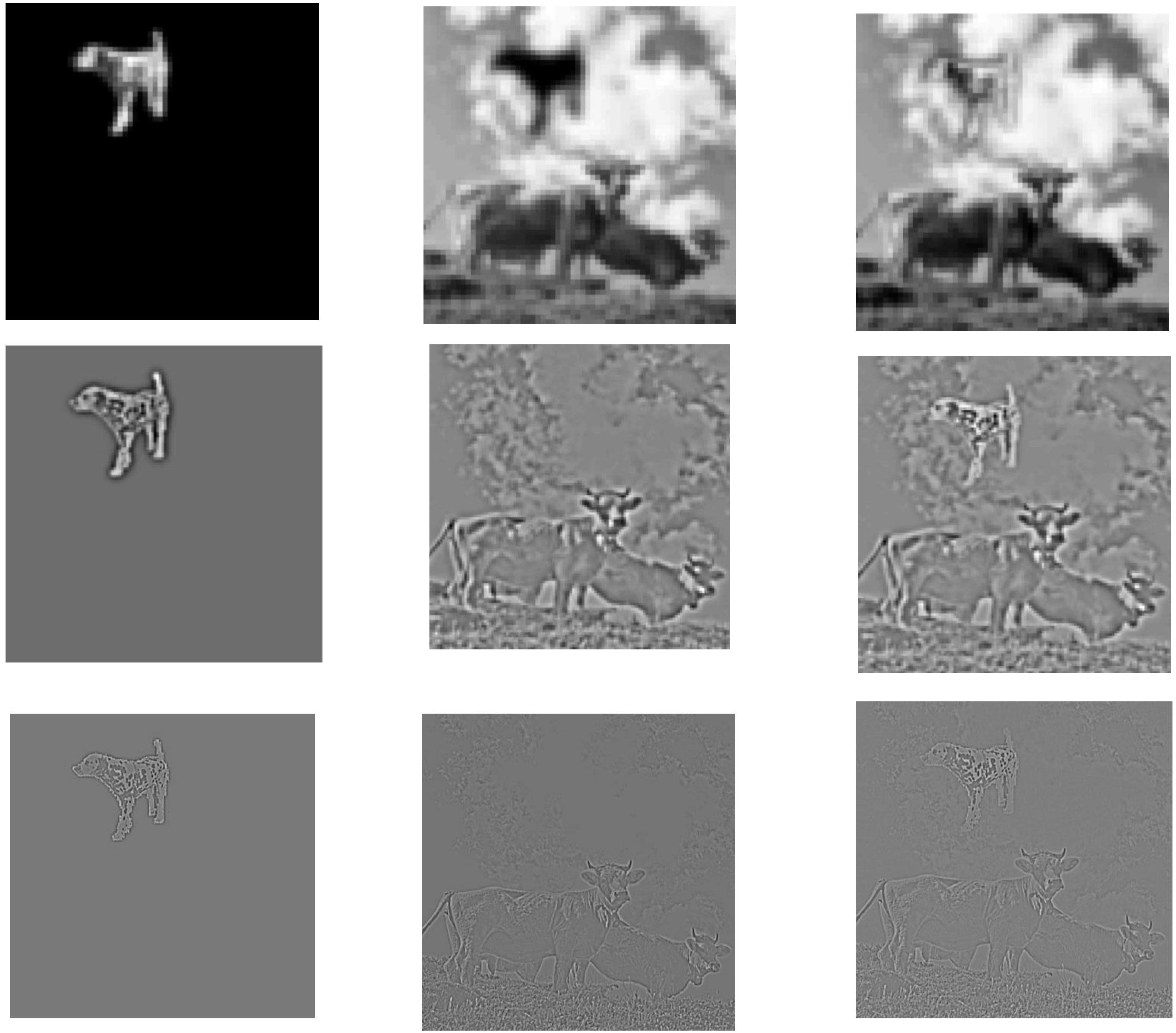
bottom pyramid

blended pyramid

laplacian level 4

laplacian level 2

laplacian level 0



top pyramid

bottom pyramid

blended pyramid

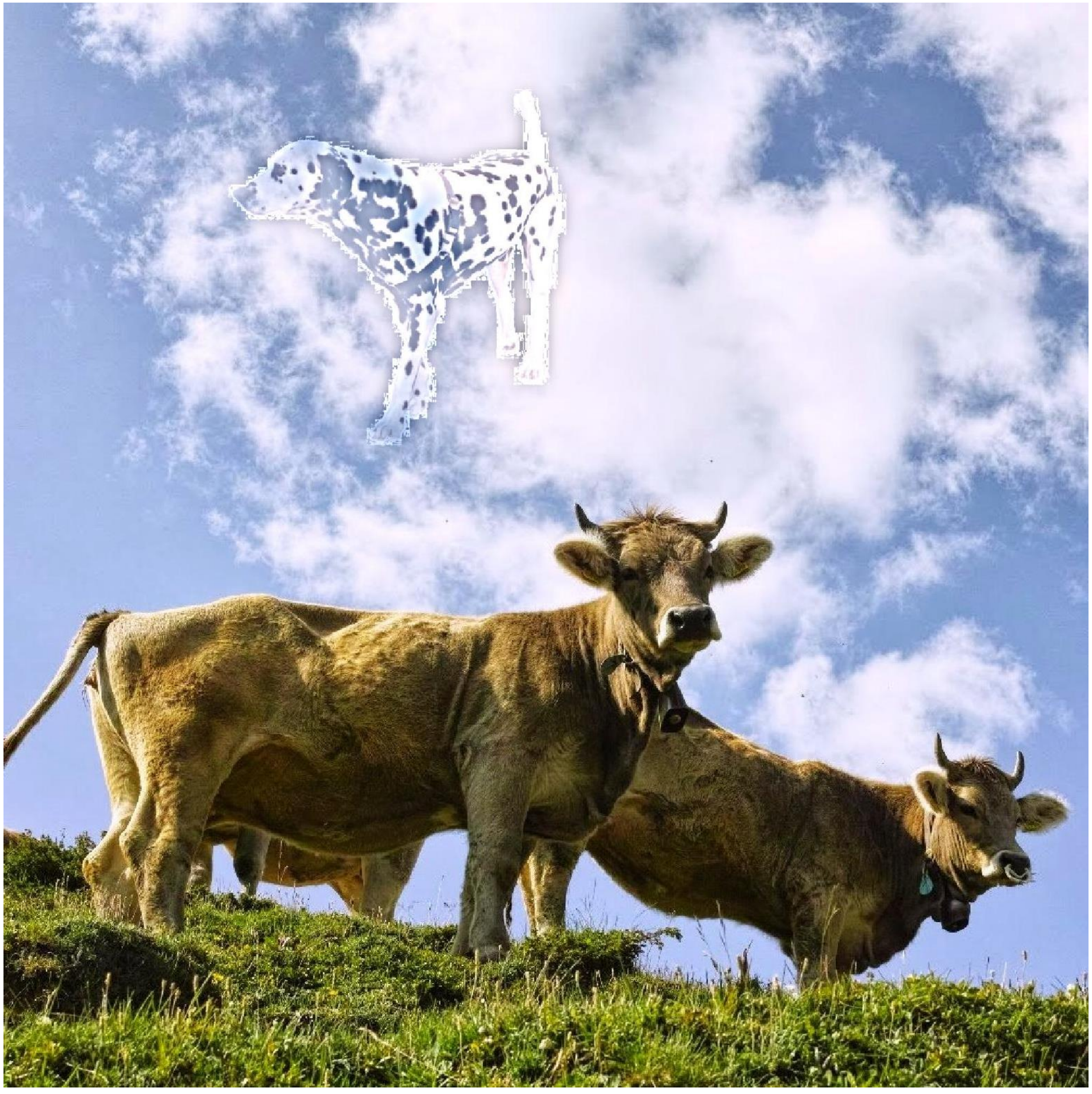
Laplacian Pyramid: Blending

General Approach:

1. Build Laplacian pyramids LA and LB from images A and B
2. Build a Gaussian pyramid $MASK$ from selected region R
3. Form a combined pyramid LS from LA and LB using nodes of GR as weights:

   $$LS(i,j) \ = MASK(i,j,) * LA(i,j) \ + \ (1 - MASK(i,j)) * LB(i,j)$$

4. Collapse the LS pyramid to get the final blended image



Horror Photo