Code Vectorization

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Don’t waste life on LOOP !

Perform pixel-level operation on image.

The loop in matlab or python code is less efficient than C code.

Use the built-in function to replace loop.

MATLAB

```
img = zeros(4000,4000);

for i = 1:size(img,1)
	for j = 1:size(img,2)
		img(i,j) = img(i,j) + 1;
	end
end
```

Run Time:0.57s

```
img = img + 1;
```

Run Time:0.06s

PYTHON

```
Img = np.zeros((4000,4000))

for i in range(img.shape[0]):
	for j in range(img.shape[1]):
		Img[i,j] += 1
```

Run Time:6.47s

```
Img = Img +  1
```

Time:0.029s

PYTHON for loop with range is incredible slow !!!
But numpy built in function is efficient

```
Img = np.zeros((4000,4000))

Img = Img + 1
```

Scalar value 1 broadcasting to matrix(4000,4000)

Broadcasting

In some application, e.g deep learning, it prefered data with mean value 0.
So we need to preprocess the image by extracting the mean before putting into neural network.
The mean value just 1x3 vector for RGB channels.

Suppose you have 1000 image and each image has equal size 500*300*3
And you want to extract mean value from all the image

```
size(img)
    1000, 500 ,300 ,3
       N      H     W    C

size(mean_val)
         3
         C
```

```
For n = 1:N
    For h = 1:H
       For w = 1:W
          For c = 1:C
                 img(n,h,w,c) = img(n,h,w,c) - mean_val(c)
          End
       End
    End
end
```

In some application, e.g deep learning, it prefered data with mean value 0.
So we need to preprocess the image by extracting the mean before putting into neural network.
The mean value just 1x3 vector for RGB channels.

Suppose you have 1000 image and each image has equal size 500*300*3
And you want to extract mean value from all the image

```
size(img)
   1000, 500 ,300 ,3
```

```
Reshaped_mean_val = reshape(c, 1,1,1,3);
Img = img -   Reshaped_mean_val
```

```
size(Reshaped_mean_val)
      1 ,   1 ,  1  , 3
```

size(img)

1000, 500 ,300 ,3

N H W C

size(mean_val)

3

C

img = img - reshape(c, 1,1,1,3);

img	1000	500	300	3
	1000 vs 1 broadcasting Check pass	500 vs 1 broadcasting Check pass	300 vs 1 broadcasting Check pass	3 vs 3 Do nothing Check pass
mean_val	1	1	1	3

Broadcasting will check each dimension
- For each dimension:
  - If have same size in this dimension:
    - Do nothing
  - else:
    - If one of them has size 1:
      - Repeat element to match the other size
    - Else:
      - Raise error

MATLAB

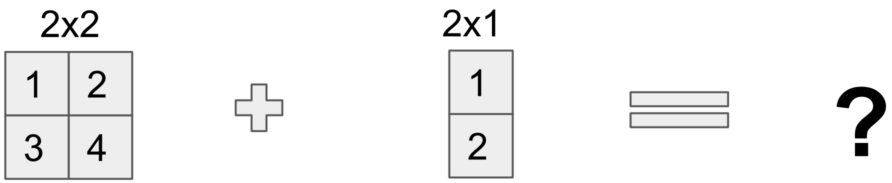
Suppose we have 4-D variable img for all image data

```
%size(img) = 1000,500,500,3
%First we compute mean value for each channel.
R_img = img(:,:,:,1);
G_img = img(:,:,:,2);
B_img = img(:,:.:,:3);
Img_mean = [mean(R_img(:)),mean(G_img(:)),mean(B_img(:))];
%size(img_mean) = 1,3
%reshape img for broadcasting
Img_mean = reshape(img_mean,1,1,1,3);
%size(img_mean) = 1,1,1,3

Img_extracted_mean = img - img_mean;
```

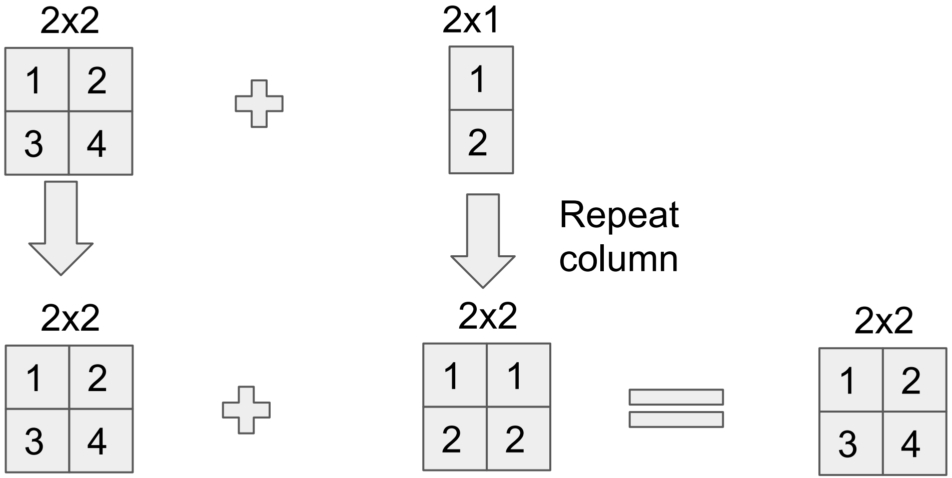
PYTHON

```
#img.shape  1000,500,300,3
#compute mean for all channel
R_mean = np.mean(img[:,:,:,0])
G_mean = np.mean(img[:,:,:,1])
B_mean = np.mean(img[:,:,:,2])
Mean_val = np.array([R_mean,G_mean,B_mean])

reshaped_mean_val = np.reshape(Mean_val,[1,1,1,3])

Img_extracted_mean = img - new_mean_val
```

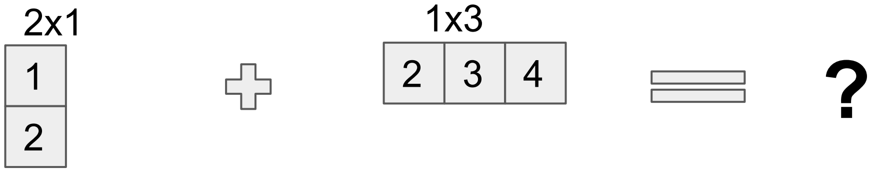
Implicit broadcasting by repeating elements



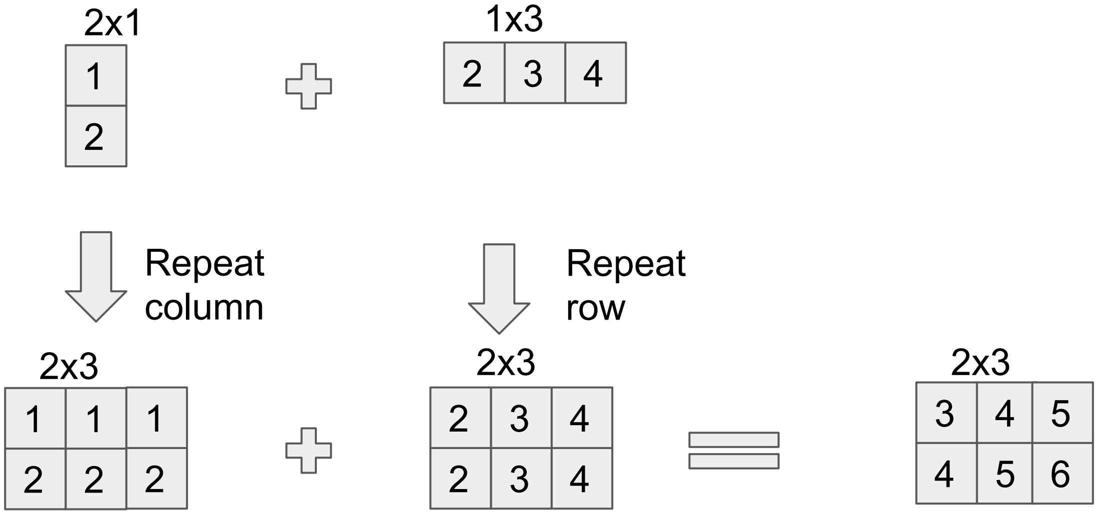
Implicit broadcasting by repeating elements



Implicit broadcasting by repeating elements



Implicit broadcasting by repeating elements



Matlab and python will try expand dim if necessary.

A has size (4,2,3)
B has size(4,1)
C has size(1,3)

compute A-B

In **Matlab**, the dimension checking will be conducted from **left to right**.

Checking direction

A	4	2	3
	4 vs 4 Do nothing Check pass	2 vs 1 broadcasting Check pass	3 vs None expand dim broadcasting Check pass
B	4	1	
B	4	1	1
B	4	2	3

Implicit
Expand dims

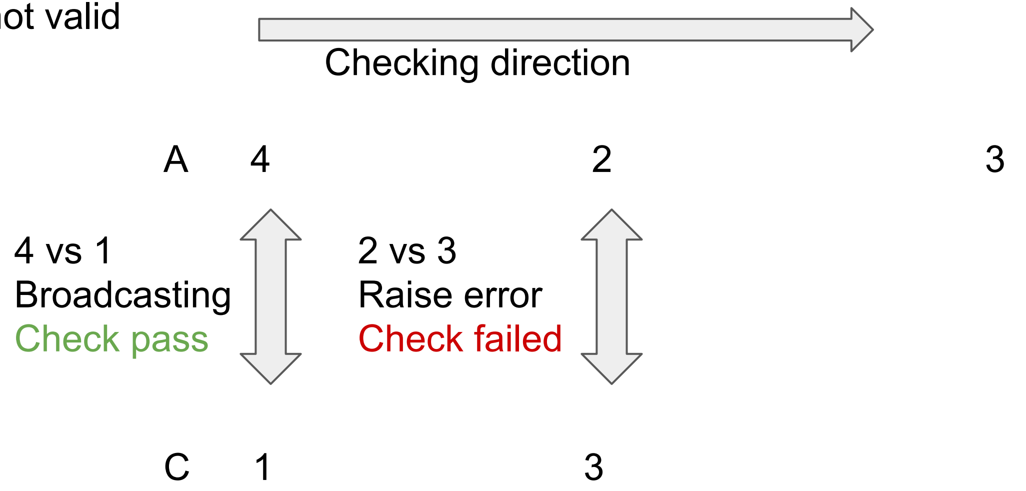
Broadcasting

Matlab and python will try expand dim if necessary.

A has size (4,2,3)
B has size(4,1)
C has size(1,3)

compute A-C

In **Matlab**, the dimension checking will be conducted from **left to right**.
So that A - C is not valid



Matlab and python will try expand dim if necessary.

A has size (4,2,3)
B has size(4,1)
C has size(1,3)

compute A-C

In **PYTHON**, the dimension checking will be conducted from **Right to left**.
So that A - C is valid

Checking direction

A	4	2	3
	4 vs None Expand dims braocasting Check pass	2 vs 1 broadcasting Check pass	3 vs 3 Do nothing Check pass
C		1	3
C	1	1	3
C	4	2	3

Implicit
Expand dims

broadcasting

Matlab and python will try expand dim if necessary.

A has size (4,2,3)
B has size(4,1)
C has size(1,3)

In **PYTHON**, the dimension checking will be conducted from **Right to left**.
And that A - B is not valid

Checking direction

A 4 2 3

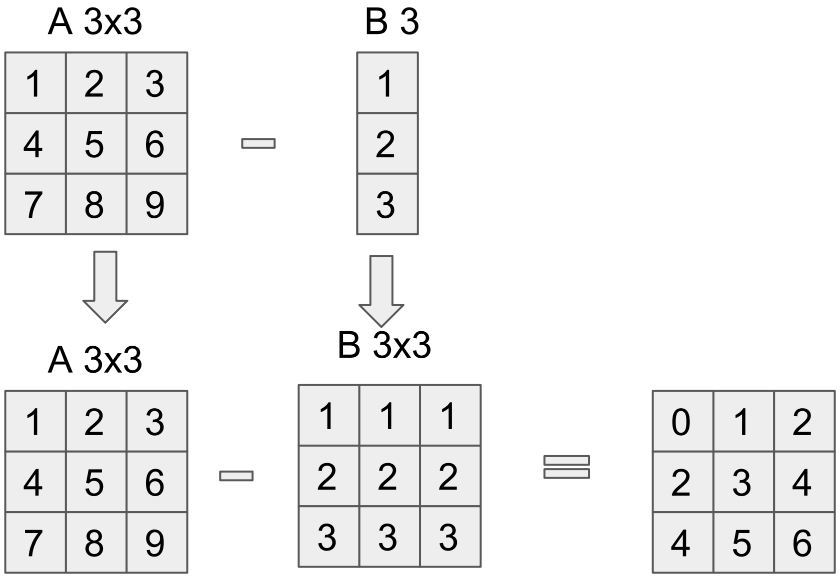
2 vs 4
Raise error
Check failed

3 vs 1
braodcasting
Check pass

B 4 1

Broadcasting could be dangerous

Suppose you have 3x3 matrix A and a 3 element vector B.
You want to subtract each row of A by the corresponding element in B



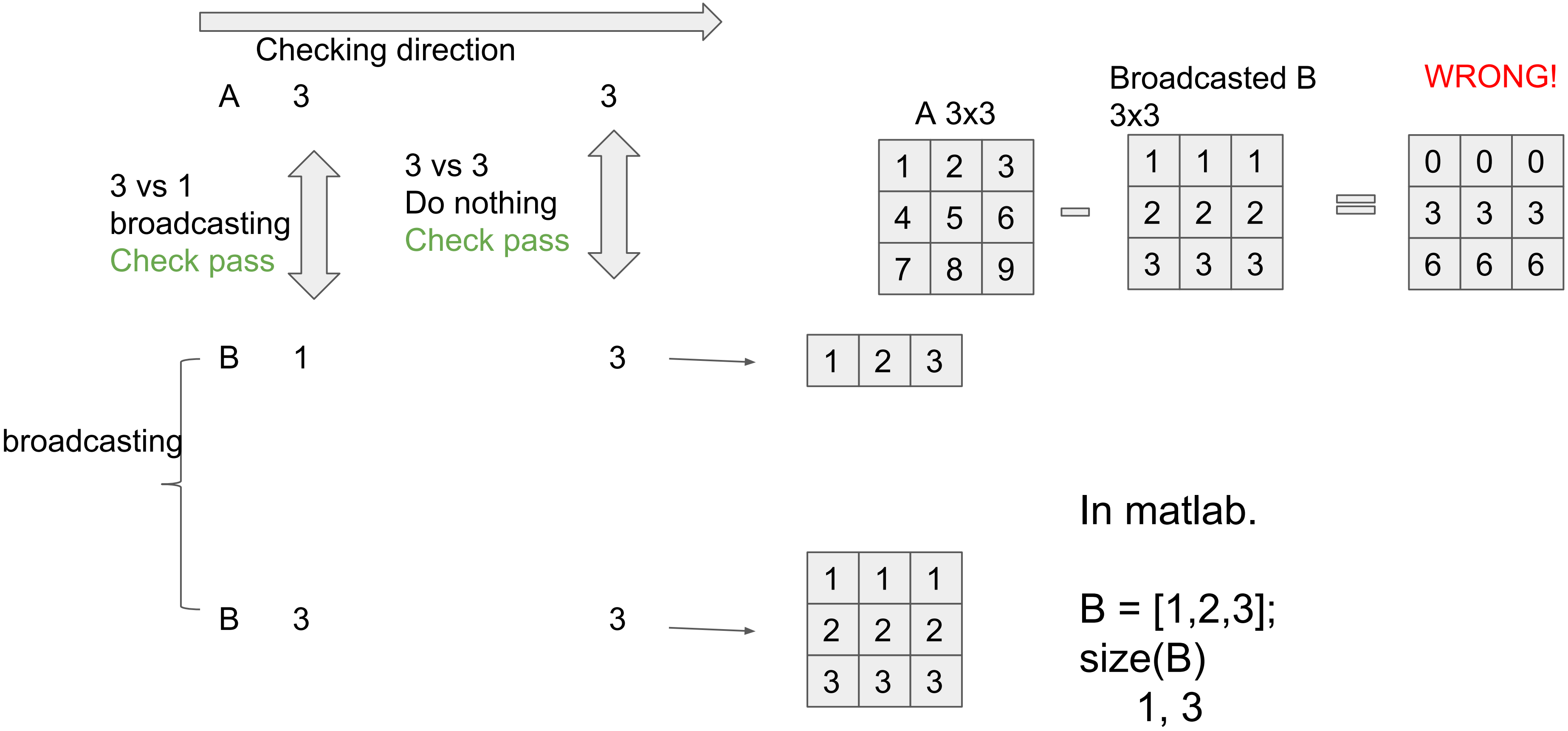
Broadcasting B to 3X3 by repeating each element in row direction

MATLAB code:

```
A = [1,2,3;4,5,6;7,8,9];
B = [1,2,3];

A - B
     0 0 0
     3 3 3
     6 6 6
```

$\neq$

0	1	2
2	3	4
4	5	6



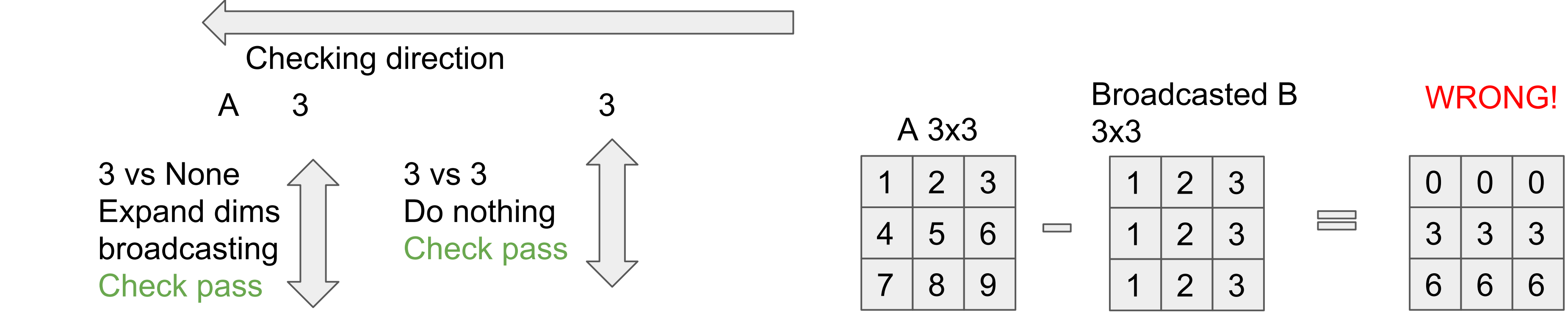
Checking direction
A 3 3
3 vs 1
broadcasting
Check pass
3 vs 3
Do nothing
Check pass
B 1 3
broadcasting
B 3 3
1 2 3
1 1 1
2 2 2
3 3 3
A 3x3
1 2 3
4 5 6
7 8 9
Broadcasted B
3x3
1 1 1
2 2 2
3 3 3
WRONG!
0 0 0
3 3 3
6 6 6
In matlab.
B = [1,2,3];
size(B)
1, 3

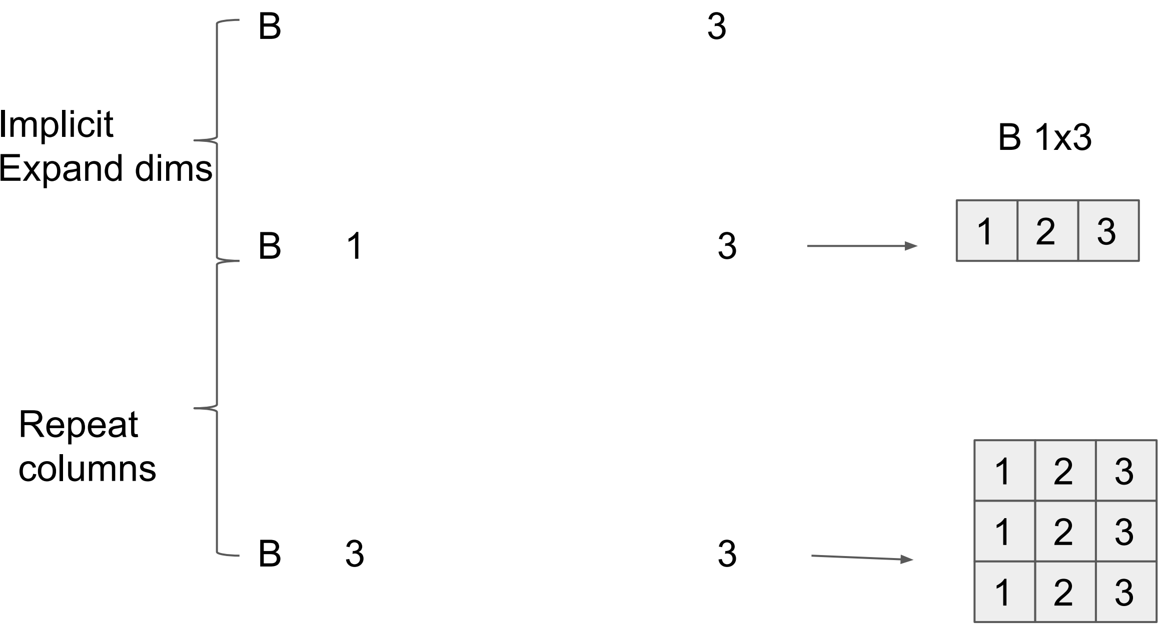
Python code

```
A = (np.arange(9)+1).reshape(3,3)
A
    array([[1, 2, 3],
           [4, 5, 6],
           [7, 8, 9]])
B = np.array([1,2,3])
C = A-B
C
    array([[0, 0, 0],
           [3, 3, 3],
           [6, 6, 6]])

B.shape
    (3,)
```

$\neq$

0	1	2
2	3	4
4	5	6





In python, single dimension vector have only 1 dimension by default

```
B = np.array([1,2,3])
B.shape
    (3,)
```

But the unexpected broadcasting will not pop out an error.
To avoid confusion and control the broadcasting direction.
We recommend:
Always check the shape before using broadcasting
Explicitly reshaping the array or matrix to the desired shape and then using implicit broadcasting.

MATLAB	PYTHON
A - reshape(B,3,1)	A - np.reshape(B,[3,1])
0 1 2	0 1 2
2 3 4	2 3 4
4 5 6	4 5 6

Matlab support scalar broadcasting in the very early version
but it just introduced the implicit broadcasting feature since R2016b.

```
Img = zeros(500,300,3);
V = zeros(1,1,3);
```

So in matlab R2016a or early version, you can’t do:

```
Img + V
Error using +
Matrix dimensions must agree.
```

But it’s now valid for R2016b and its later version.
For early version or explicit broadcasting,
Use built-in function **bsxfun** or **repmat** instead

Example:

Compute gaussian distribution

Compute gaussian distribution
For the range that x = [0,1] and y = [0,1]

$$f(x,y) = \frac{1}{2\sigma\pi}\exp\left(-\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma^2}\right)$$

μ_x = 0.5

μ_y = 0.5

σ = 0.2

Meshgrid

$$f(x,y) = \frac{1}{2\sigma\pi}\exp(-\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma^2})$$

X_line = [1,2,3];
Y_line = [4,5,6];
[mesh_x, mesh_y] = meshgrid(x_line,y_line)
(**x_line is the line in width and y is the line in height**)

mesh_x

1	2	3
1	2	3
1	2	3

mesh_y

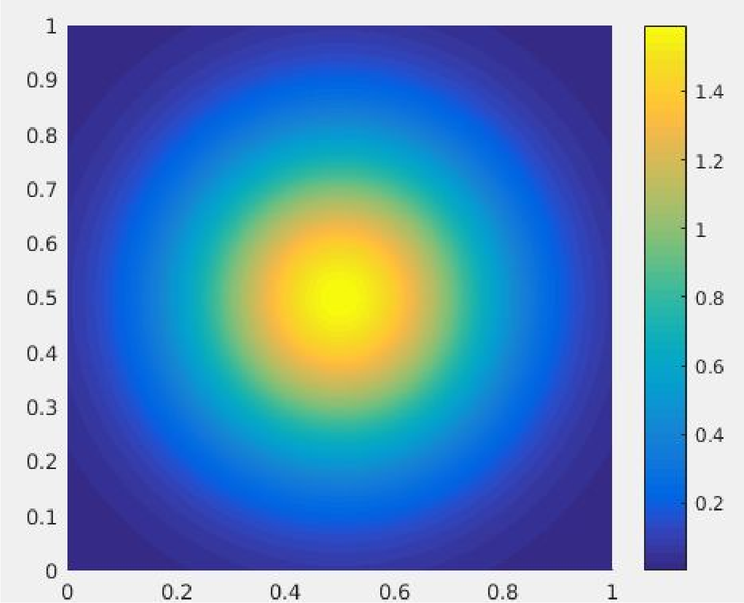
4	4	4
5	5	5
6	6	6

1. Build meshgrid
2. Shift meshgrid by μ_x and μ_y
3. Compute gaussian function

$$f(x,y) = \frac{1}{2\sigma\pi}\exp(-\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma^2})$$

MATLAB

```
X_line = 0:0.001:1;
Y_line = 0:0.001:1;
[mesh_x, mesh_y] = meshgrid(X_line,Y_line);
Constant_term = 1/(0.2*pi);
val = exp(-((mesh_x-0.5).^2+(mesh_y-0.5).^2)/(2*0.2^2));
val = Constant_term * val;
imagesc('XData',[0,1],'YData',[0,1],'CData',val);
axis image
colorbar;
```



PYTHON

```
x = np.arange(0,1,0.001)
y = np.arange(0,1,0.001)
mesh_x, mesh_y = np.meshgrid(x,y)
constant_term = 1/(0.2*np.pi)
val = np.exp(-1*((mesh_x - 0.5)**2 + (mesh_y - 0.5)**2)/(2*(0.2)**2))
val  = val * constant_term
```

Example:

Compute multi-channel convolution

Multi channel convolution computation

Image patch
M1

Convolution kernel
F1

1	2	1
2	1	2
1	2	1

M1(:,:,1)

1	2	-1
2	1	2
1	3	1

M1(:,:,2)

2	2	1
2	1	2
1	0	1

M1(:,:,3)

0	1	0
0	1	0
0	0	0

F1(:,:,1)

0	1	0
0	0	0
1	0	0

F1(:,:,2)

0	1	0
0	0	0
0	0	1

F1(:,:,3)

Multi channel convolution computation

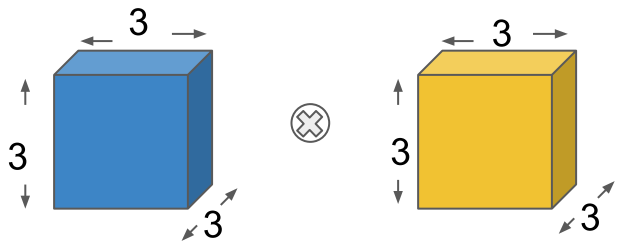


Image patch Convolution kernel

Compute center value for each channel
Sum up the result from all channel
Just flip the kernel on H and W space

M1

F1

$$= \left(\underset{M1(:,:,1)}{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}} \otimes \underset{F1(:,:,1)}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \right) + \left(\underset{M1(:,:,2)}{\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}} \otimes \underset{F1(:,:,2)}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}} \right) + \left(\underset{M1(:,:,3)}{\begin{bmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}} \otimes \underset{F1(:,:,3)}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \right)$$

$$= 3 + 2 + 2 = 7$$

MATLAB

```
img_patch = zeros(3,3,3);
img_patch(:,:,1) = [1,2,1;2,1,2;1,2,1];
img_patch(:,:,2) = [1,2,-1;2,1,2;1,3,1];
img_patch(:,:,3) = [2,2,1;2,1,2;1,0,1];

kernel = zeros(3,3,3);
kernel(:,:,1) = [0,1,0;0,1,0;0,0,0];
kernel(:,:,2) = [0,1,0;0,0,0;1,0,0];
kernel(:,:,3) = [0,1,0;0,0,0;0,0,1];

Flipped_kernel = flipud(fliplr(kernel));
val _matrix = img_patch.*Flipped_kernel;
Result = sum(val_matrix(:));

Result
    7.0
```

PYTHON

```
Img_patch = np.zeros((3,3,3))
img[:,:,0] = np.array([[1,2,1],[2,1,2],[1,2,1]])
img[:,:,1] = np.array([[1,2,-1],[2,1,2],[1,3,1]])
img[:,:,2] = np.array([[2,2,1],[2,1,2],[1,0,1]])

kernel = np.zeros((3,3,3))
kernel[:,:,0] = np.array([[0,1,0],[0,1,0],[0,0,0]])
kernel[:,:,1] = np.array([[0,1,0],[0,0,0],[1,0,0]])
kernel[:,:,2] = np.array([[0,1,0],[0,0,0],[0,0,1]])

Flipped_kernel = kernel[::-1,:,:][:,::-1,:]
Result = np.sum(Flipped_kernel*img)

Result
       7.0
```

Example:

Compute multi-kernel multi-channel convolution (image to column)

Input: feature map or image

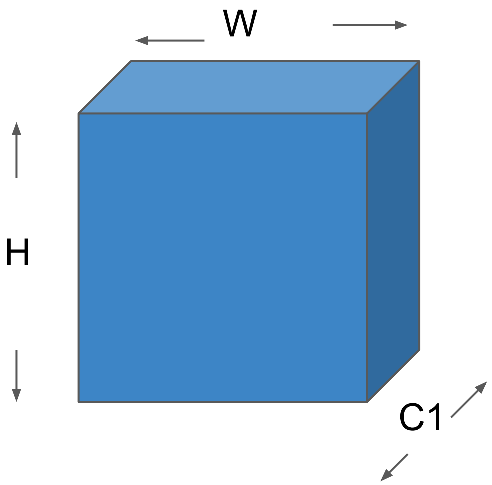
H * W * C1

Convolutional kernel group

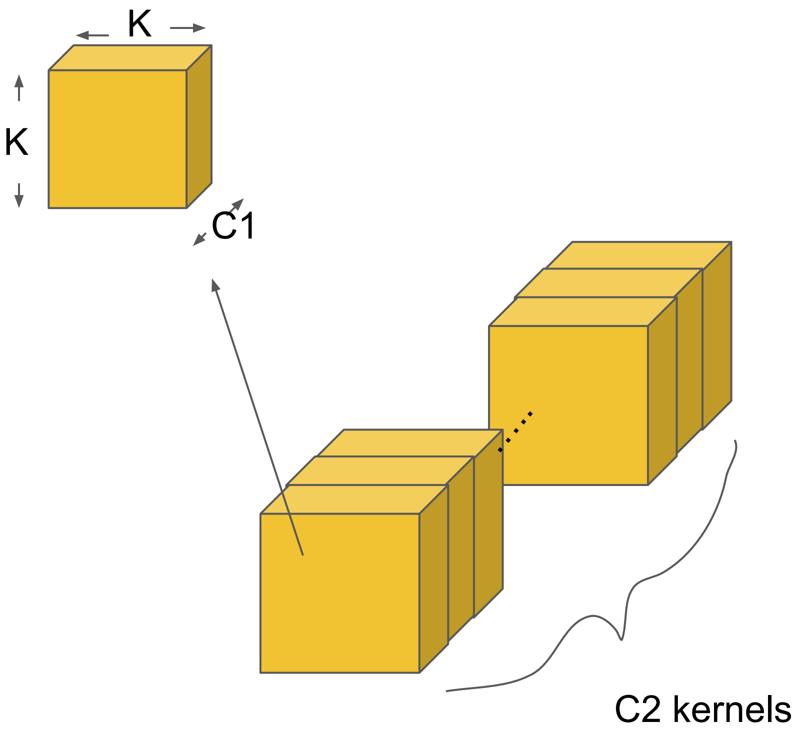
C2 * K * K * C1

Output: feature map

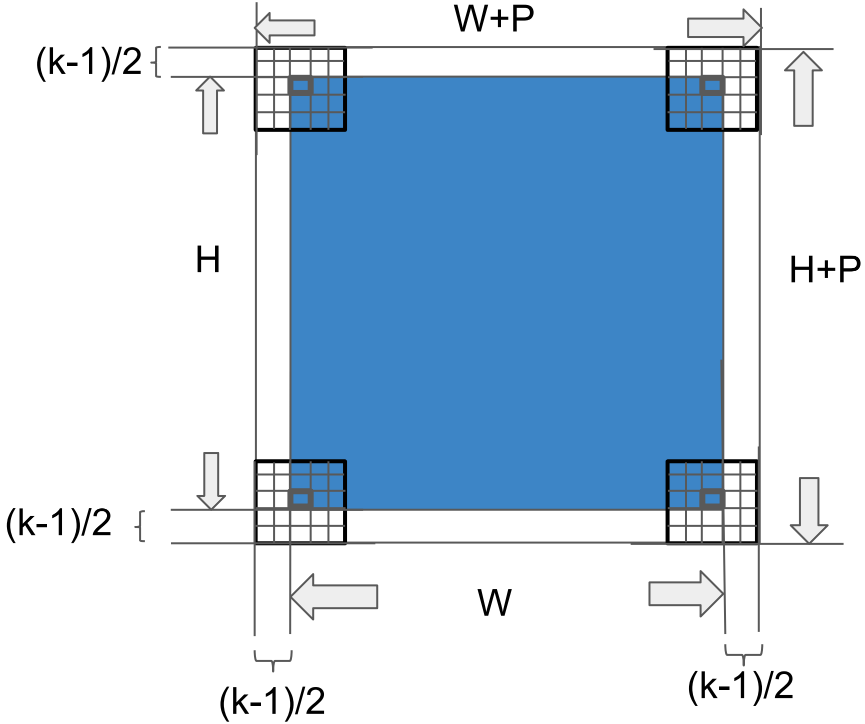
H * W * C2



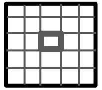
RGB image C1 = 3
Gray image C1 = 1



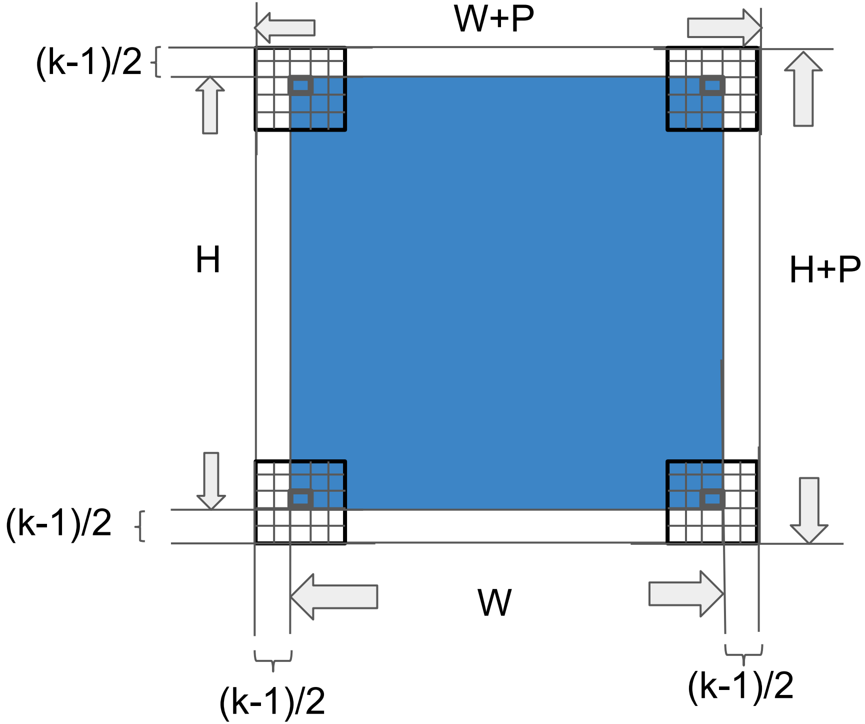
C2 kernels in kernel group
Each kernel has size K*K*C1



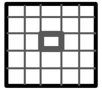
Zero padding input feature map M1
For input kernel size K
P = K-1



Convolution kernel
k*k, k = 5, P = 4



If k*k sampling window move 1 pixel each time,
it will move W times on each row
it will move H times on column



Convolution kernel
k*k, k = 5

```
Naive implementation
For each sampling window in feature map
        Img_patch = get_patch(img, window)
        For kernel in all c2 convolution kernels
            compute_center_val(kernel, img_path)
        End
End
```

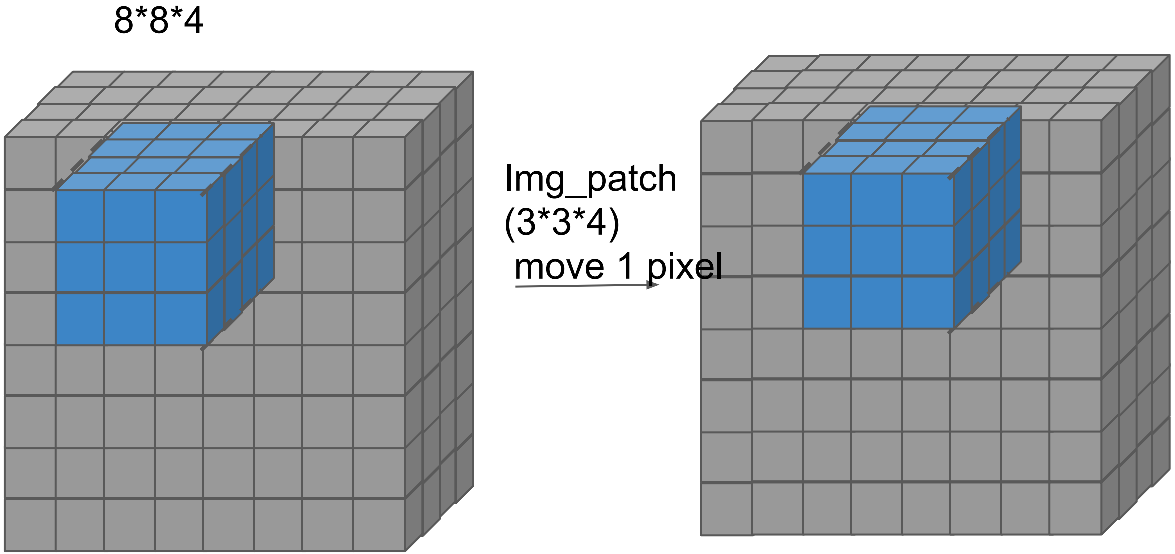
Input feature map **H*W*C1**

We padded it to **(H+P)*(W+P)*C1**

Suppose sampling window move 1 pixel each time

Outer loop executed **H*W** times

Inner loop executes **C2** times



For each sampling window in feature map

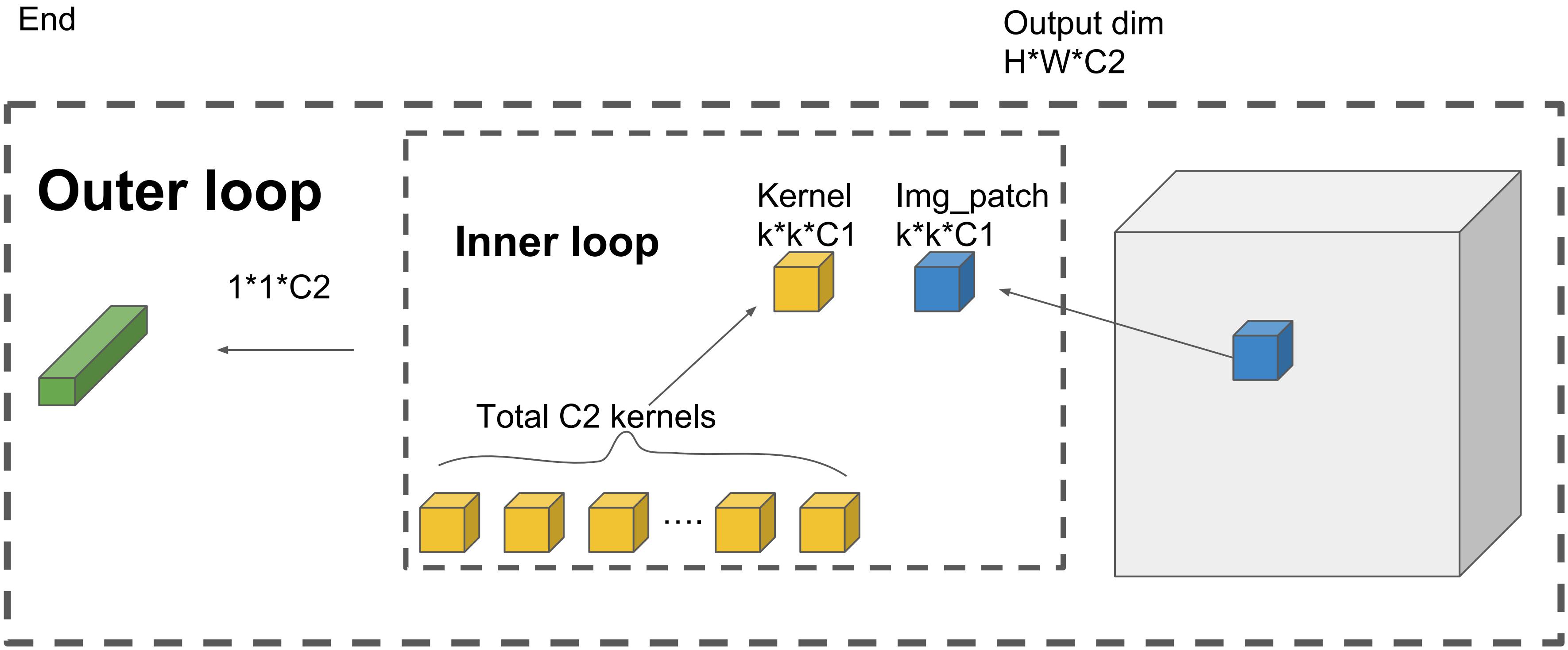
Img_patch = get_patch(img, window)

For kernel in all c2 convolution kernels:

compute_center_val(kernel, img_path)

End

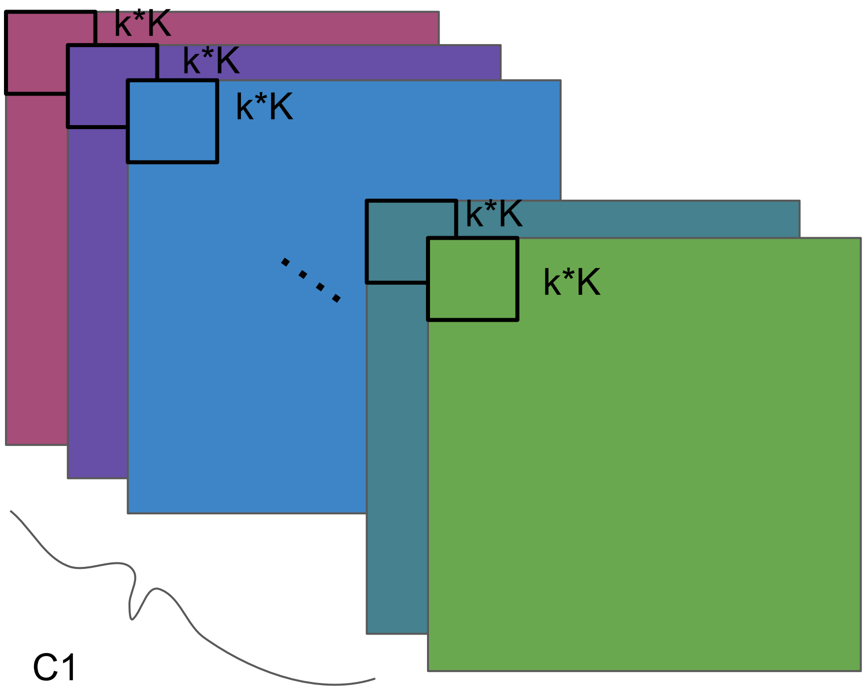
End



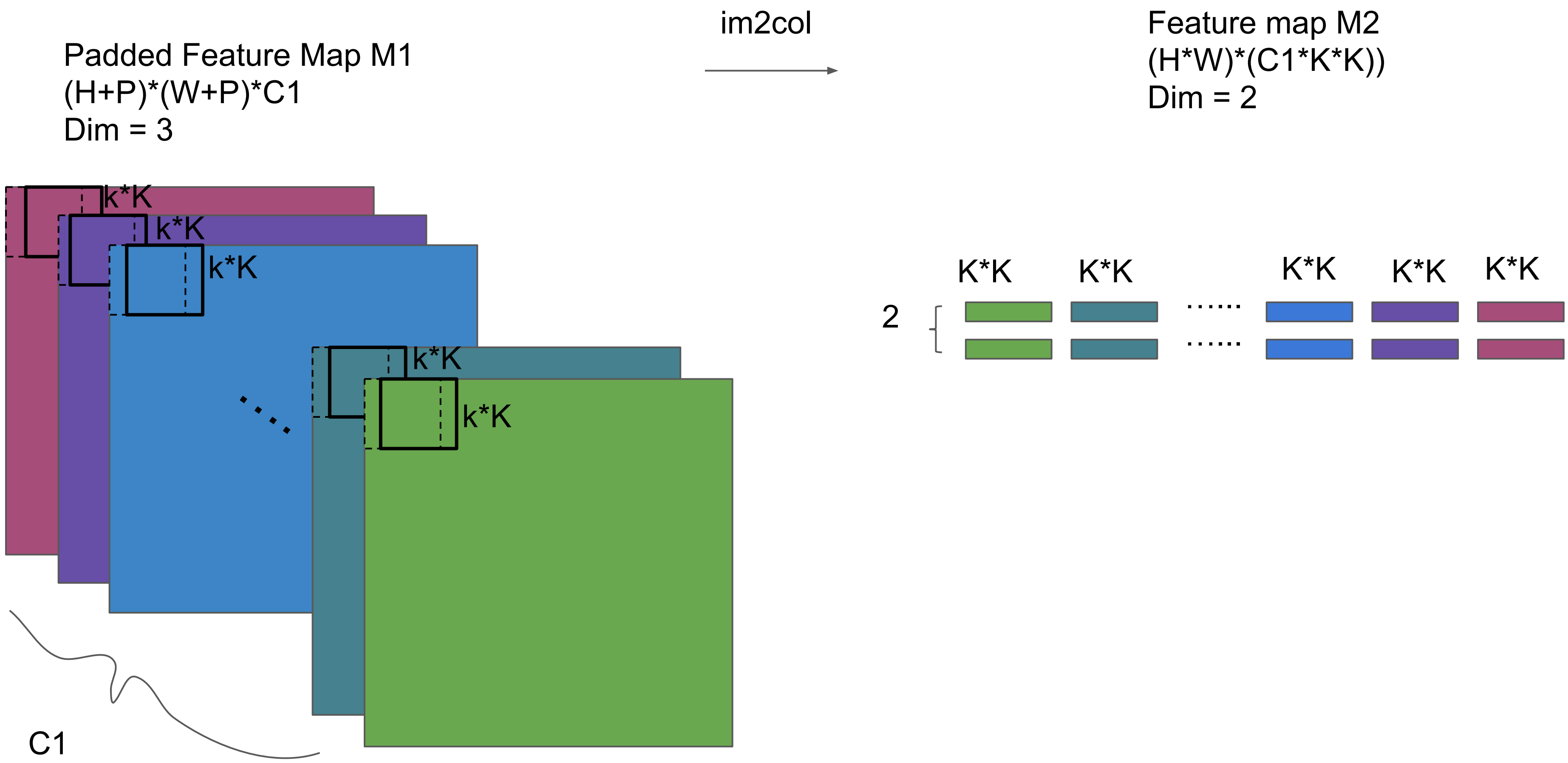
Padded Feature Map M1
(H+P)*(W+P)*C1
Dim = 3

im2col

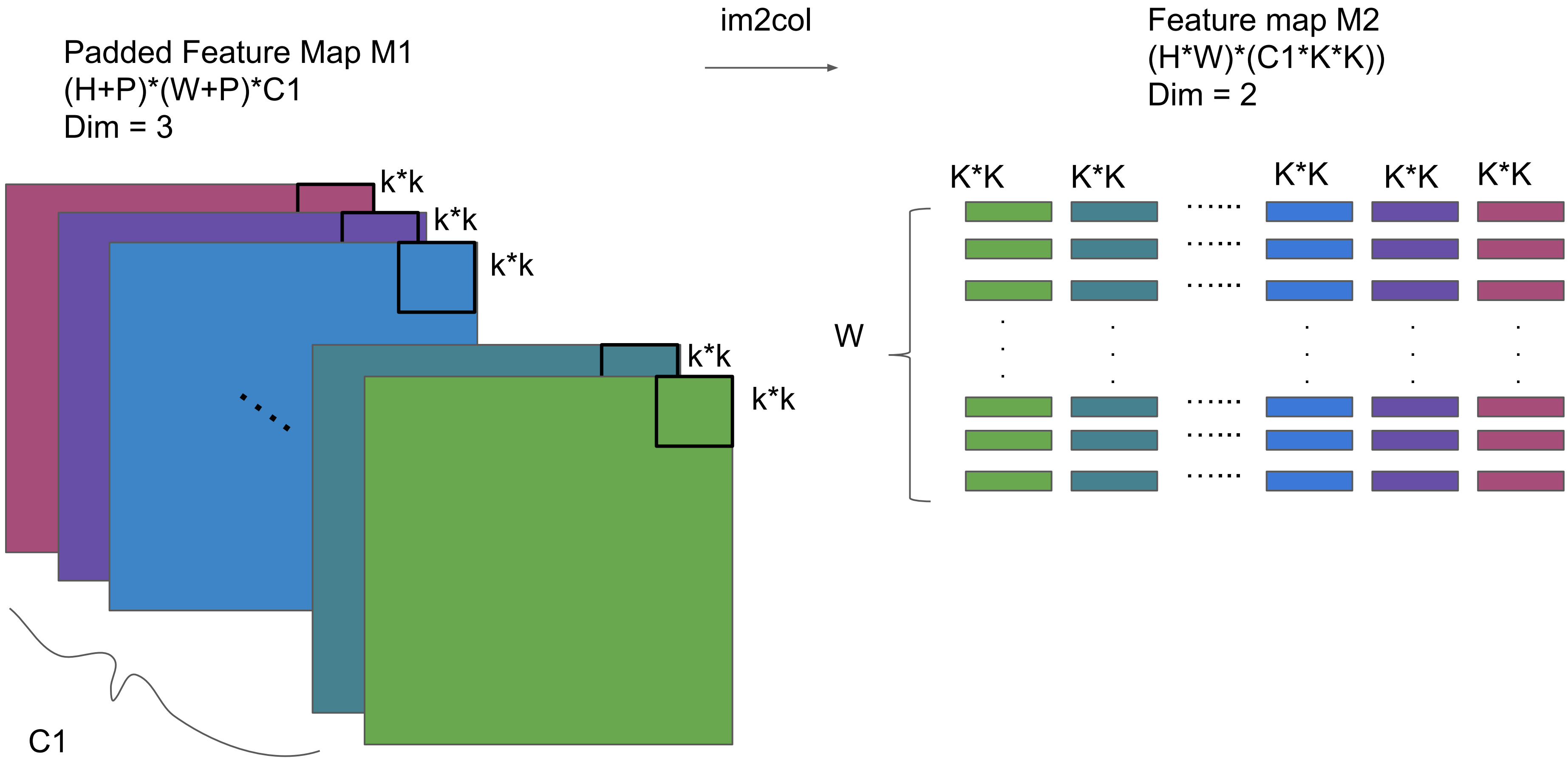
Feature map M2
(H*W)*(C1*K*K))
Dim = 2

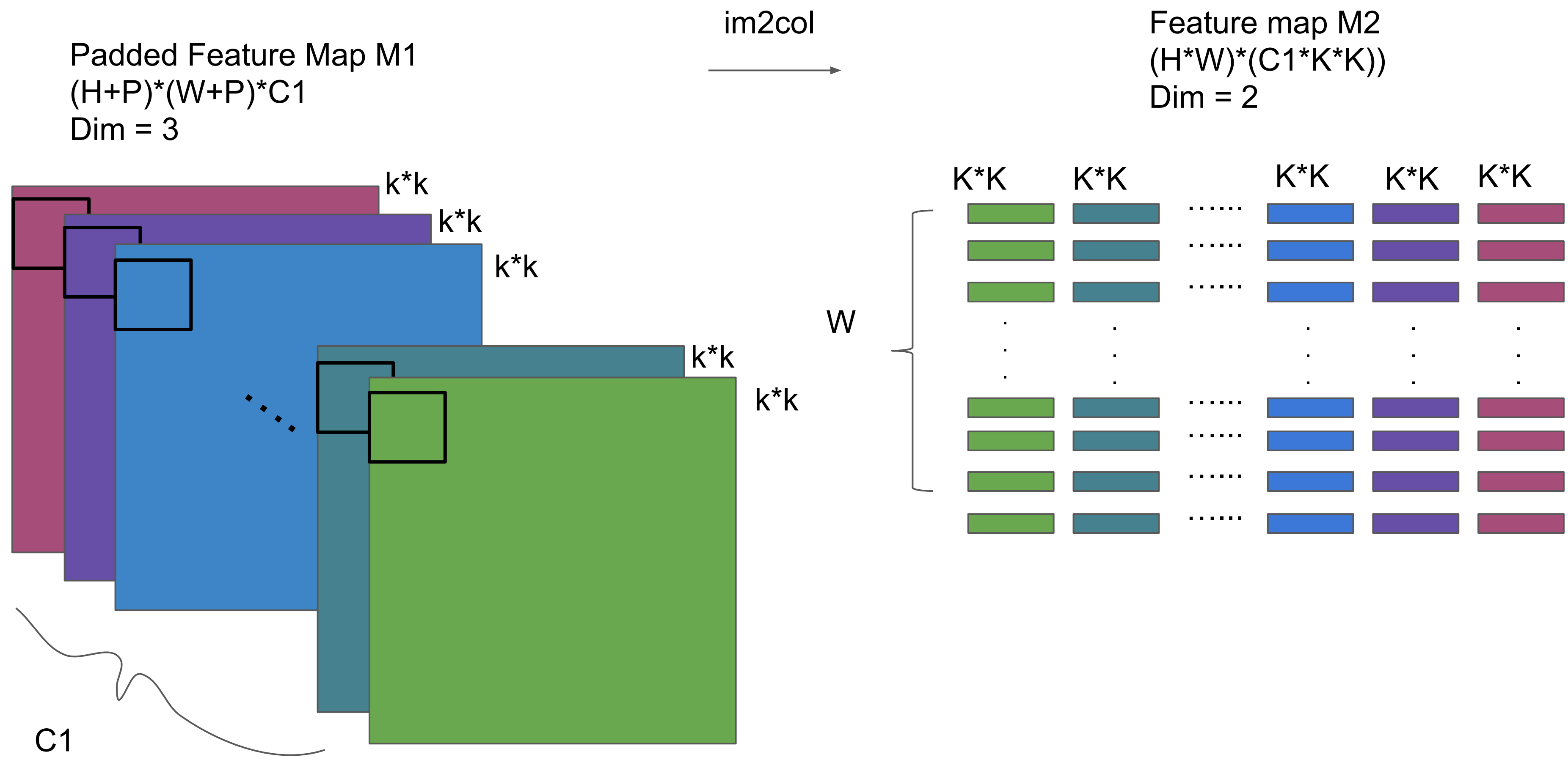


The actual size of feature map
(H+P) * (W+P) * C1
Where P denotes the padding size to
make output has same size of input

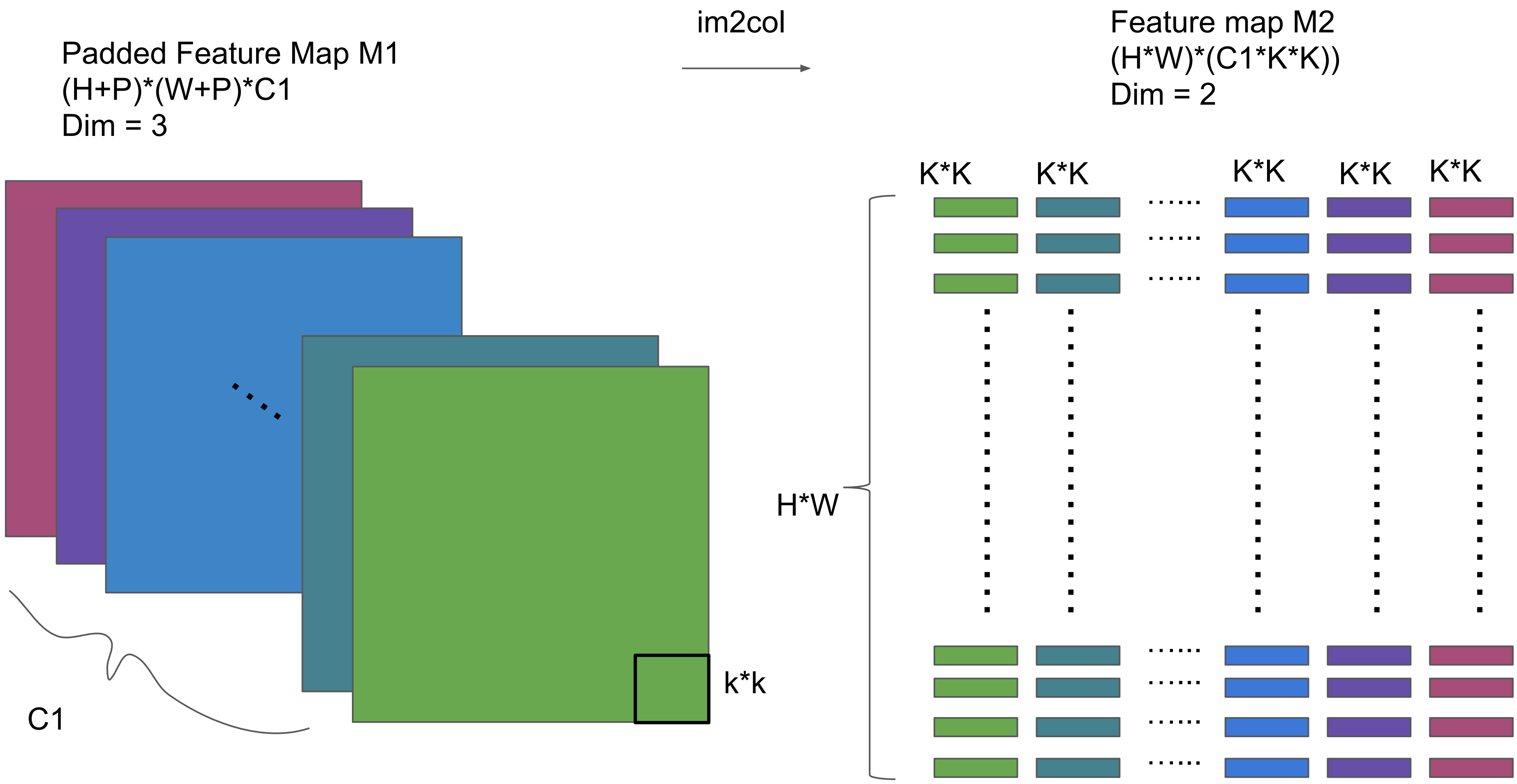


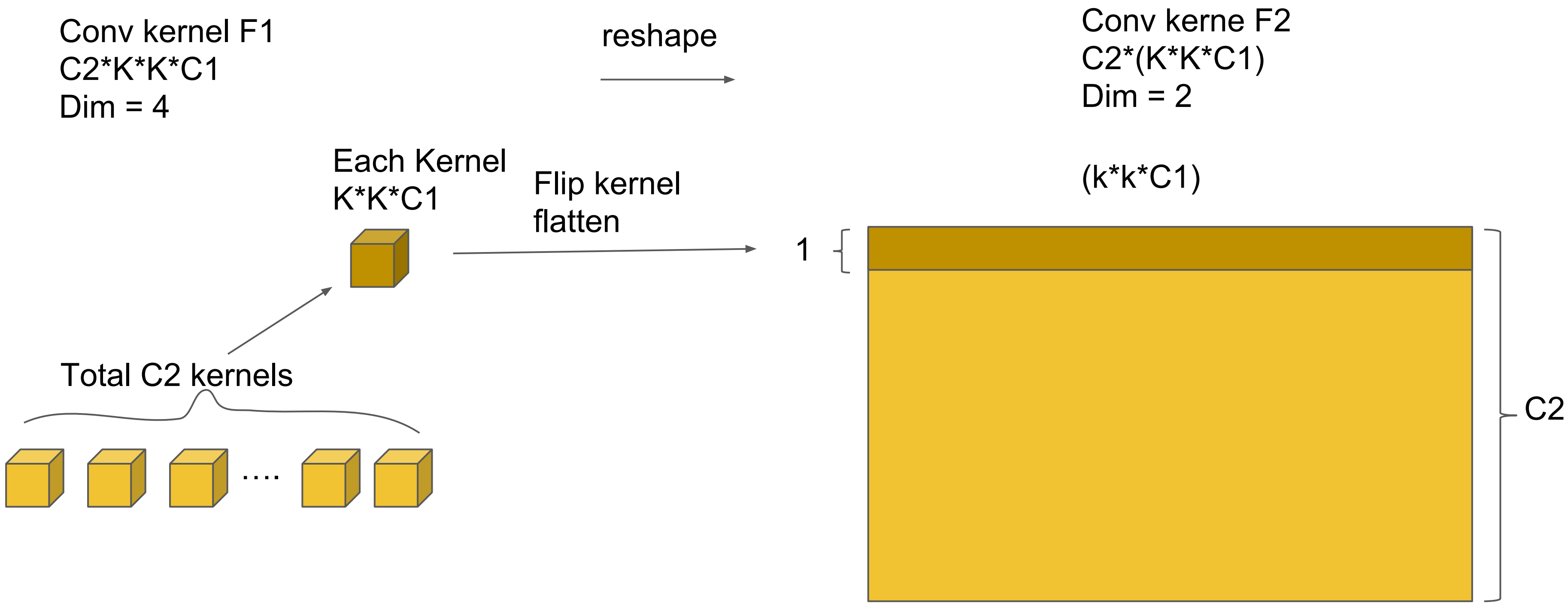
Padded Feature Map M1
(H+P)*(W+P)*C1
Dim = 3
im2col
Feature map M2
(H*W)*(C1*K*K))
Dim = 2
k*K
k*K
k*K
k*K
k*K
C1
K*K
K*K
K*K
K*K
K*K
2
......
......

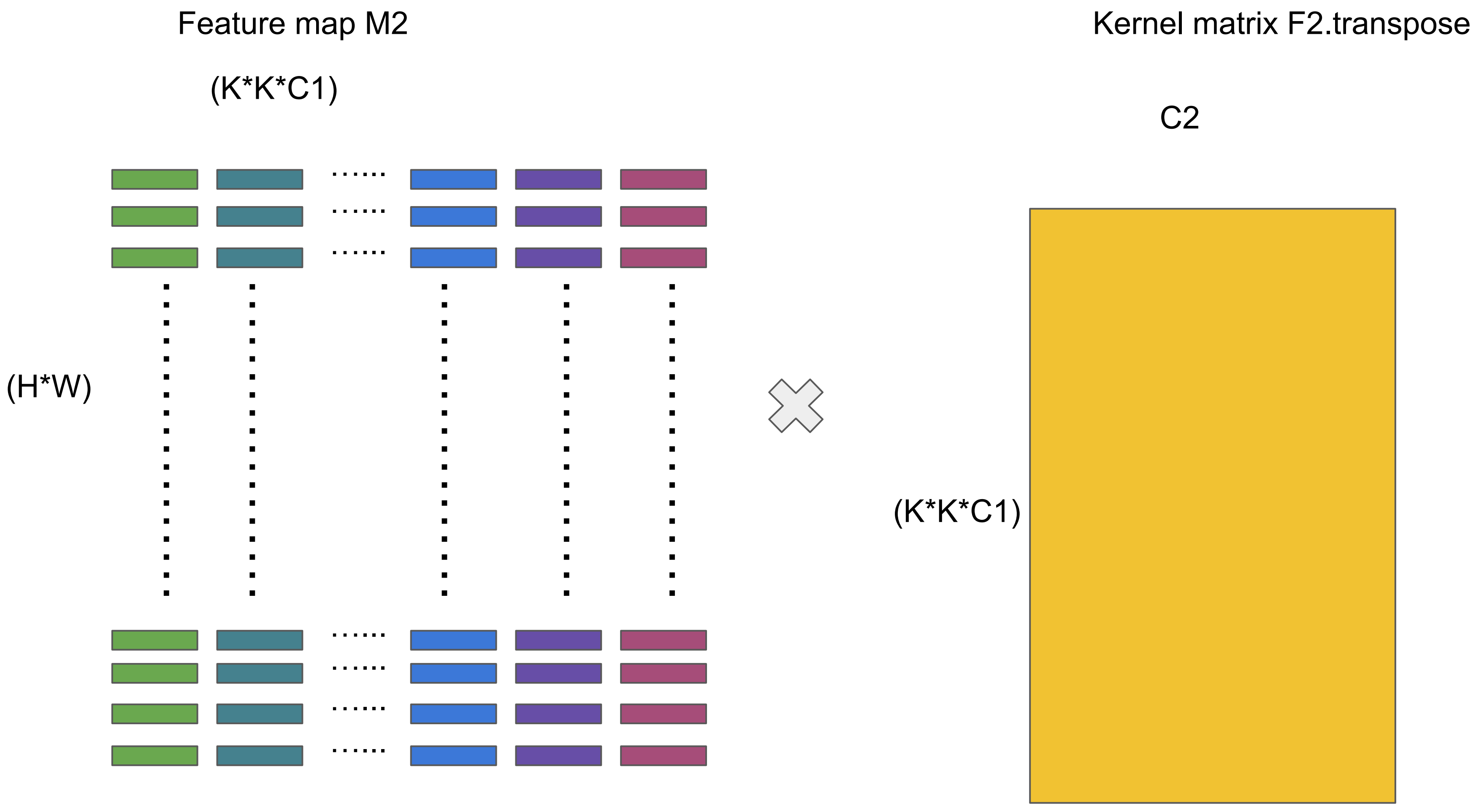

Padded Feature Map M1
(H+P)*(W+P)*C1
Dim = 3
k*k
k*k
k*k
k*k
k*k
C1
im2col
Feature map M2
(H*W)*(C1*K*K))
Dim = 2
K*K
K*K
K*K
K*K
K*K
W

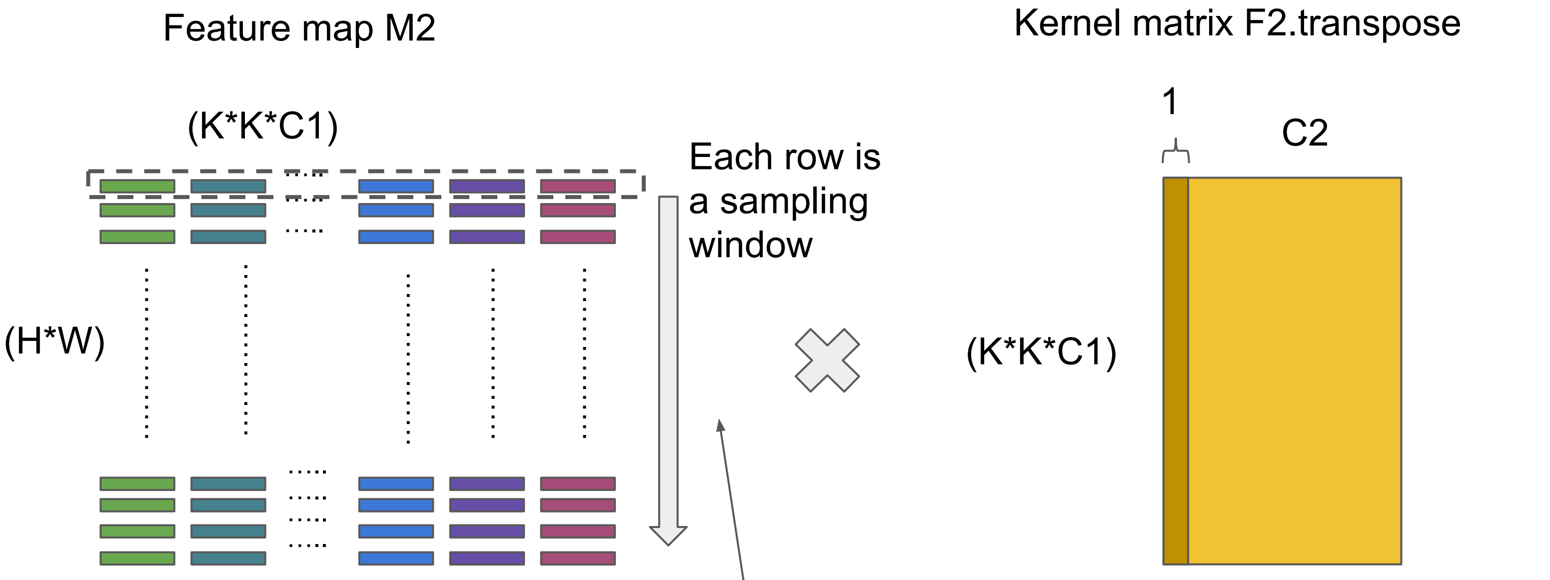


Padded Feature Map M1
(H+P)*(W+P)*C1
Dim = 3
im2col
Feature map M2
(H*W)*(C1*K*K))
Dim = 2
k*k
k*k
k*k
k*k
k*k
C1
K*K
K*K
K*K
K*K
K*K
W


im2col
Padded Feature Map M1
(H+P)*(W+P)*C1
Dim = 3
C1
k*k
Feature map M2
(H*W)*(C1*K*K))
Dim = 2
K*K
K*K
K*K
K*K
K*K
H*W


Conv kernel F1
C2*K*K*C1
Dim = 4
reshape
Conv kerne F2
C2*(K*K*C1)
Dim = 2
Each Kernel
K*K*C1
Flip kernel
flatten
(k*k*C1)
1
Total C2 kernels
....
C2


Feature map M2
(K*K*C1)
(H*W)
Kernel matrix F2.transpose
C2
(K*K*C1)



Each column is a kernel

For each sampling window in feature map:

    Img_patch = get_patch(img, window)
    For kernel in all c2 convolution kernels:
        conv(kernel, img_patch)
    End

End

Img2col

- No loop
- Formulate the convolution operation as matrix multiplication

Conv kernel F1
C2*K*K*C1
Dim = 4

→ Flip kernel reshape →

Conv kerne F2
C2*(K*K*C1)
Dim = 2

→ transpose →

Conv kerne F2.transpose
(K*K*C1)*C1
Dim = 2

Feature Map M1
H*W*C1
Dim = 3

→ Zero padding im2col →

Feature map M2
(H*W)*(K*K*C1))
Dim = 2

→ Matrix multiplication →

Output
(H*W)*C2

→ reshape →

Output
H*W*C2

Feature Map M1
H*W*C1
Dim = 3

im2col $\longrightarrow$

Feature map M2
(H*W)*(C1*K*K))
Dim = 2

M1 has H*W*C1 element
M2 has H*W*(C1*K*K) element

The cost to killing loop:
more storage.

Trade off between space and time