



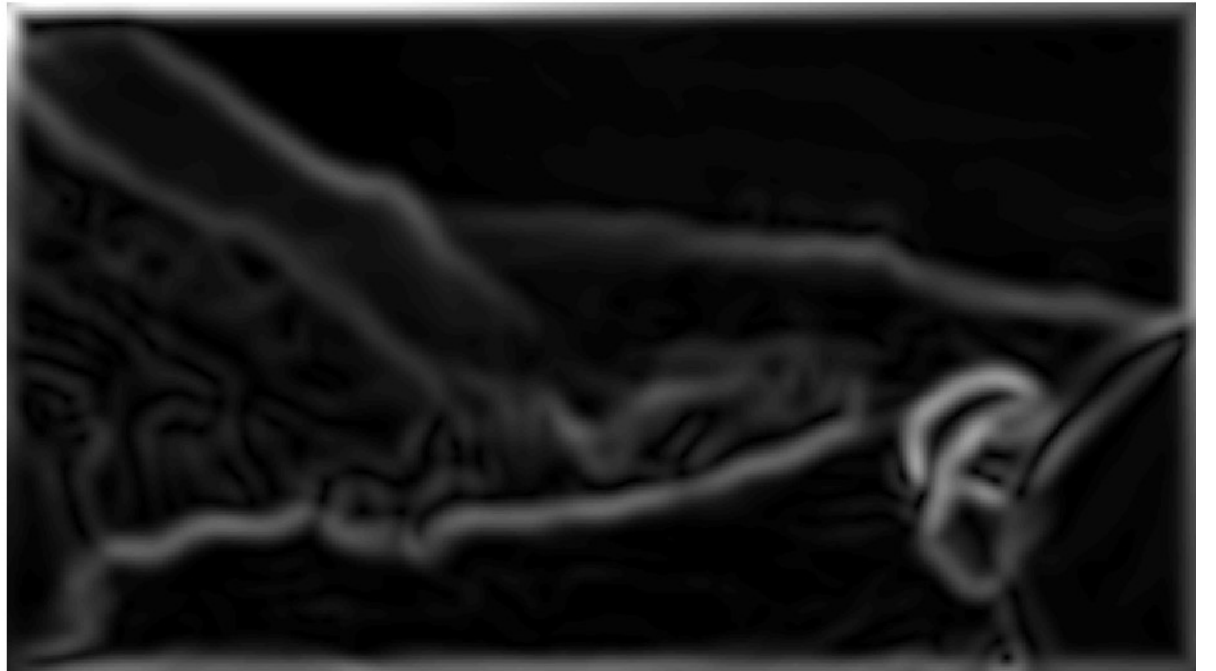
Pyramid Paris 1

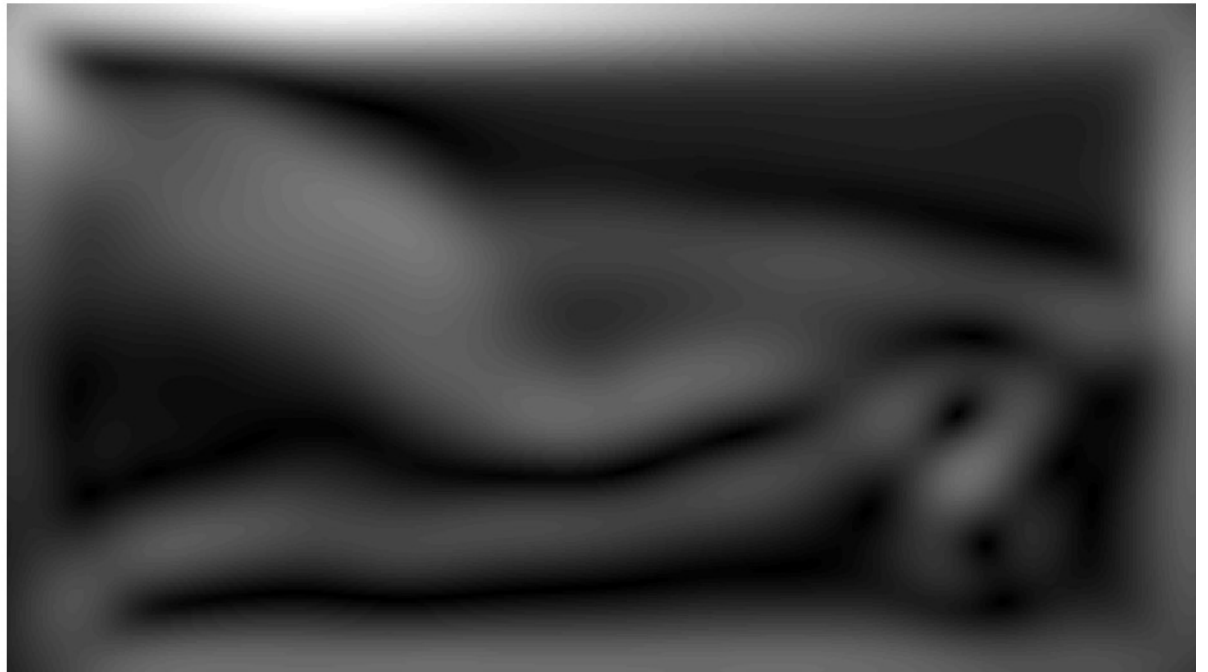
Image Pyramid, CIS581



Image Scale





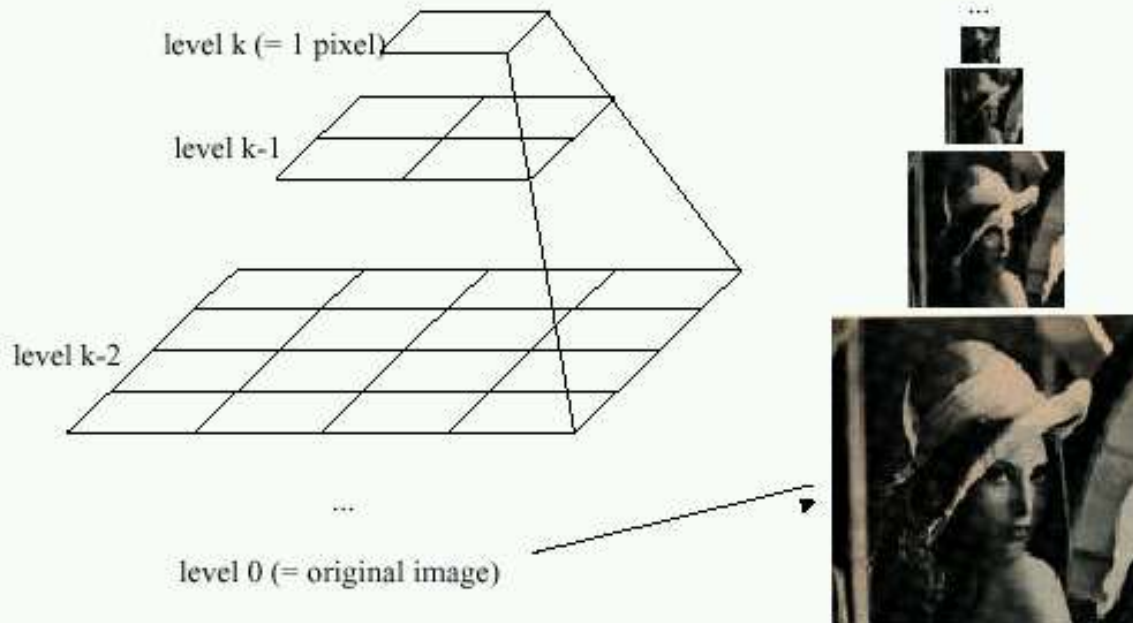




Different scale of image encodes different edge response.

Image Pyramids

Idea: Represent $N \times N$ image as a “pyramid” of $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$ images (assuming $N = 2^k$)



Known as a Gaussian Pyramid [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*



Authors, left to right: Bergen, Anderson, Adelson, Burt.

A detour through scale space

Image encoding-decoding

- 1) Image statistics: pixel in neighborhood are correlated, encode per pixel value is redundant
- 2) Predictive Coding: use raster scan, predict based on pass value, and store only the error in prediction. Simple and fast

signal

10	10	20	22	24	24
----	----	----	----	----	----

prediction

10	10	10	15	21	23
----	----	----	----	----	----

error-encoded

0	0	10	7	3	1
---	---	----	---	---	---

Image encoding-decoding (part1)

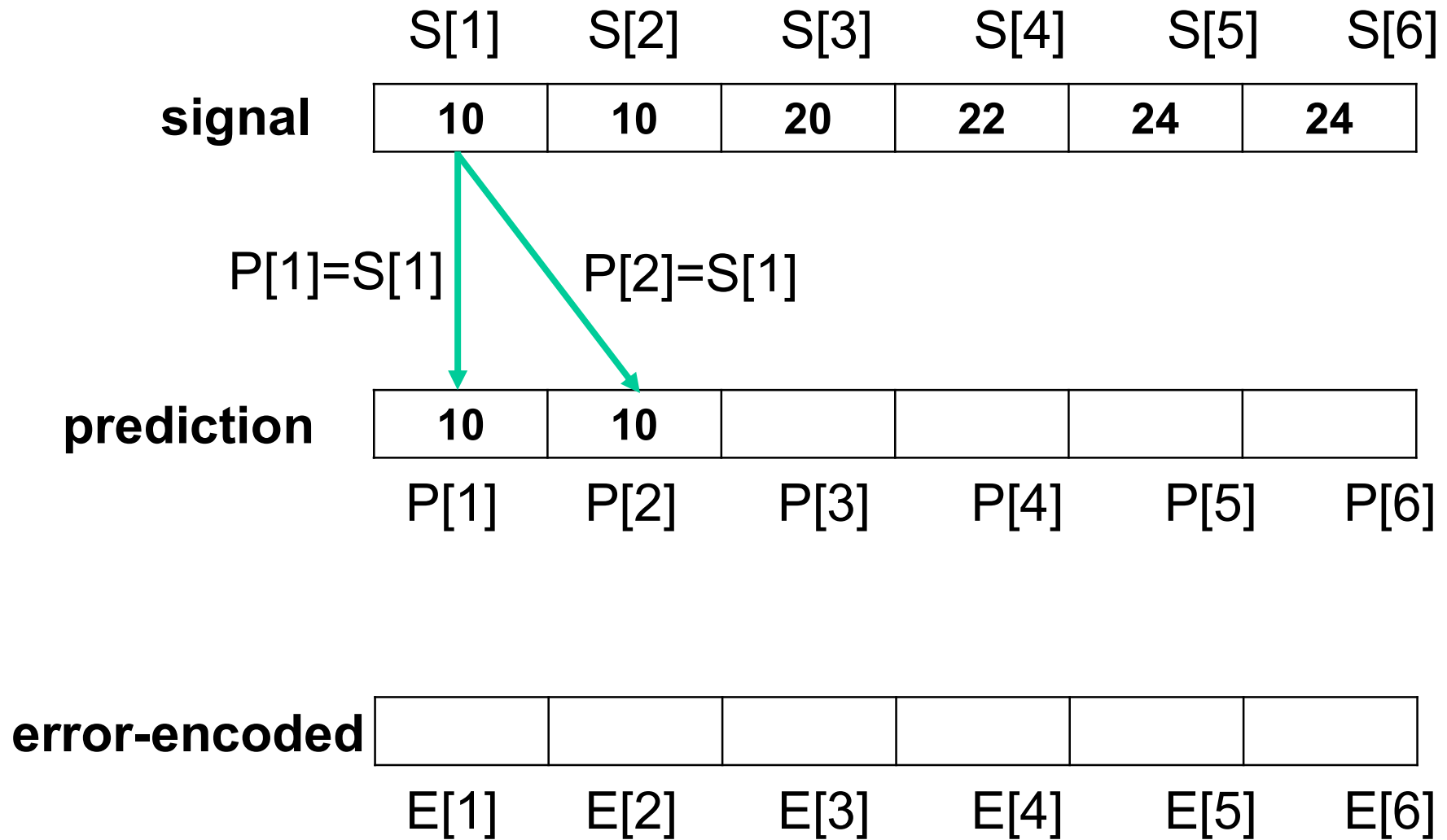


Image encoding-decoding (part2)

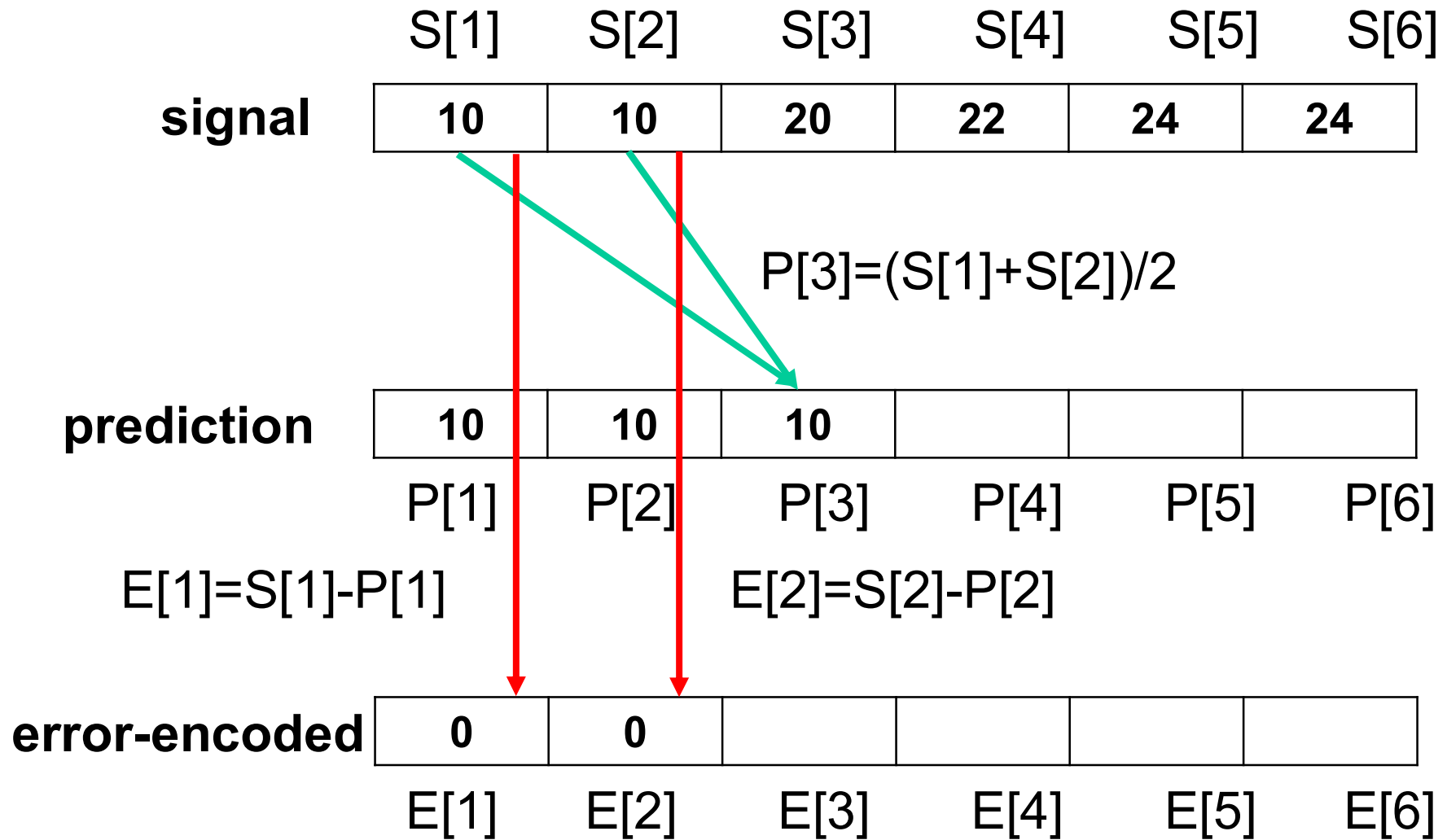


Image encoding-decoding (part2)

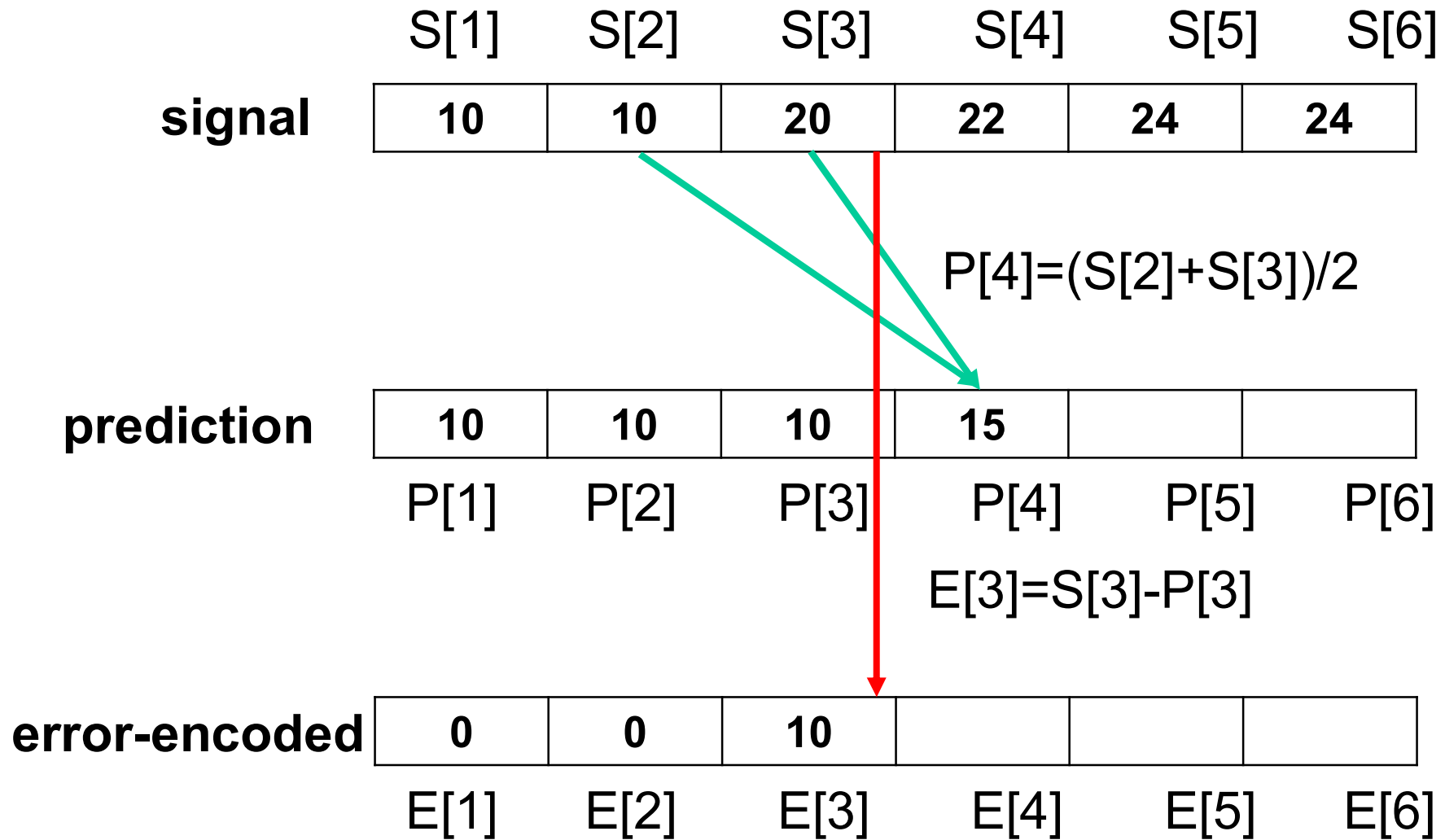
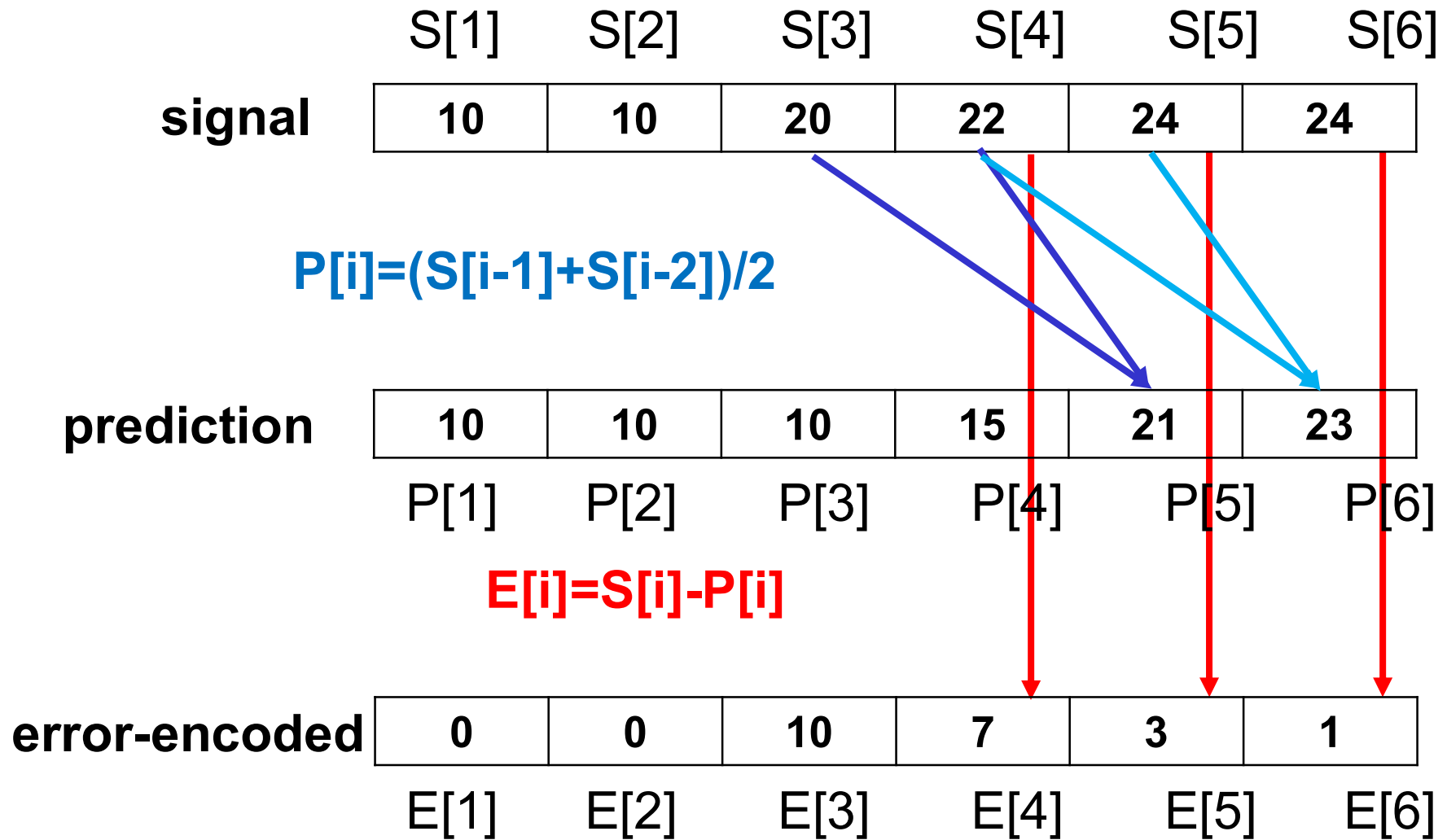
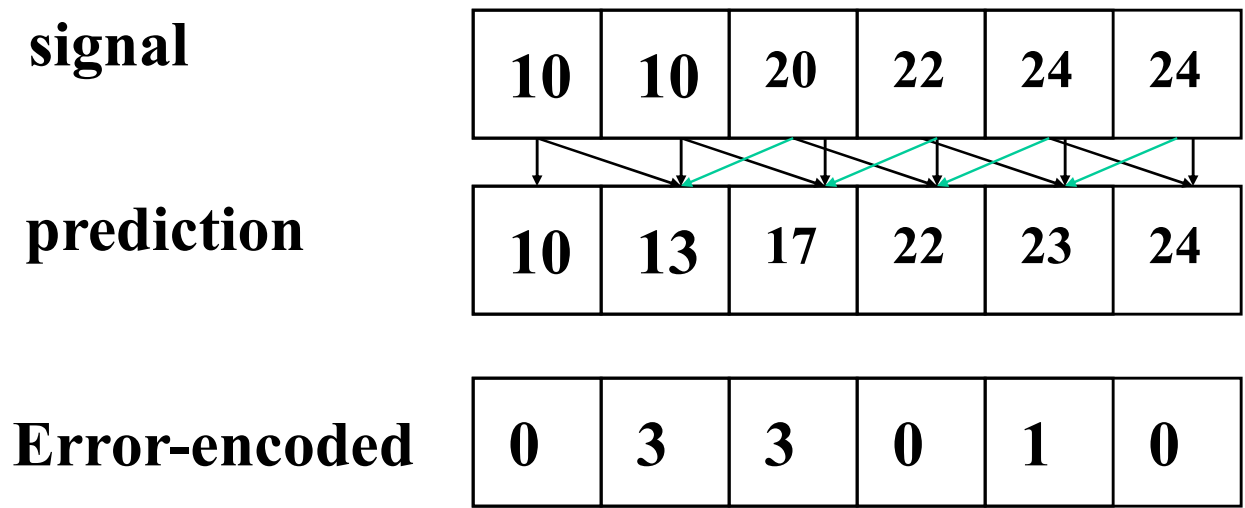


Image encoding-decoding (part3)

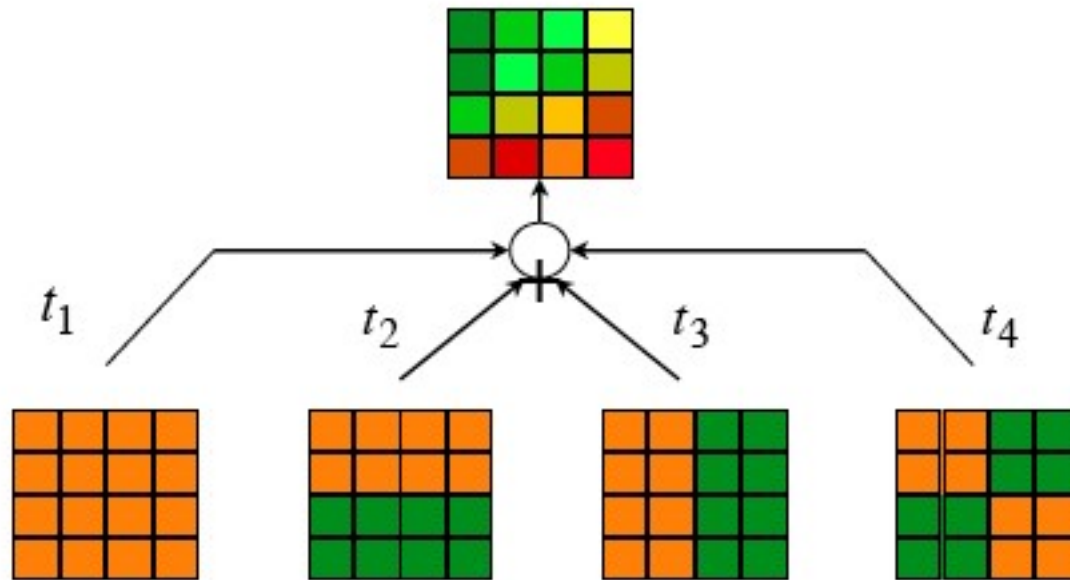


Non-causal prediction

non-causal involves typically transform, or solution to a large sets of equations. Encode block by block. Bigger compression but slower.



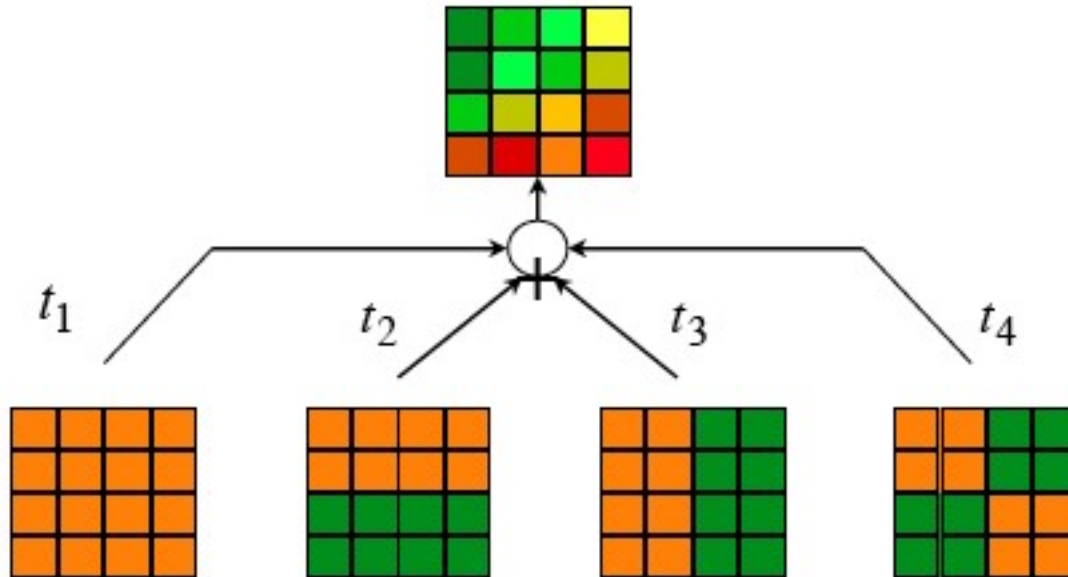
More general building blocks of image



We can have a block of pixels as a building block, “code”.

Think image as made of LEGO, how to take it apart, and put it back.

More general building blocks of image



$$I = T_1 * t_1 + T_2 * t_2 + T_3 * t_3 + T_4 * t_4;$$

How to estimate t_i

Coding book of DCT (discrete Cosine)

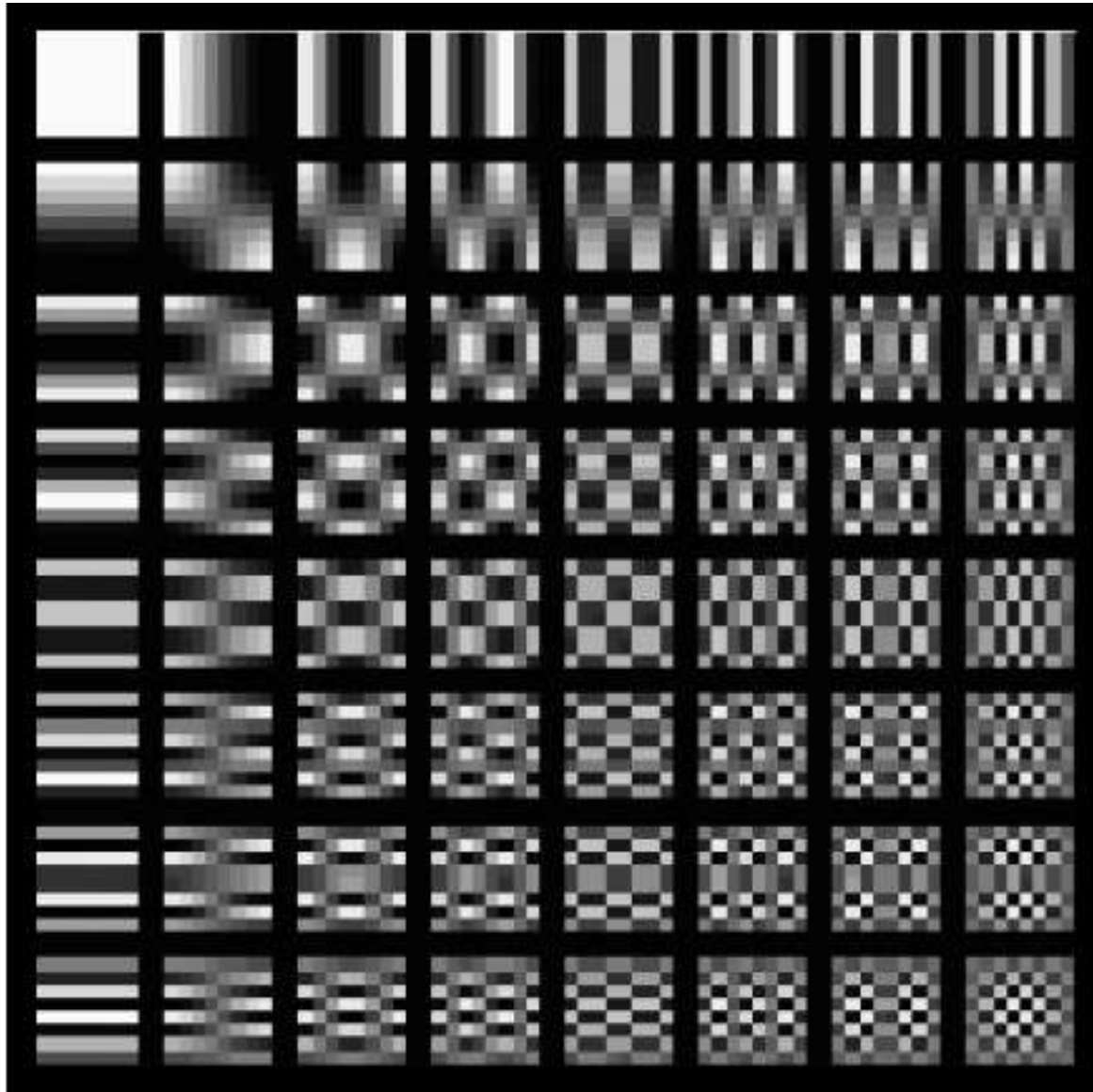


Image coding with DCT

Original



With 16/64
Coefficients



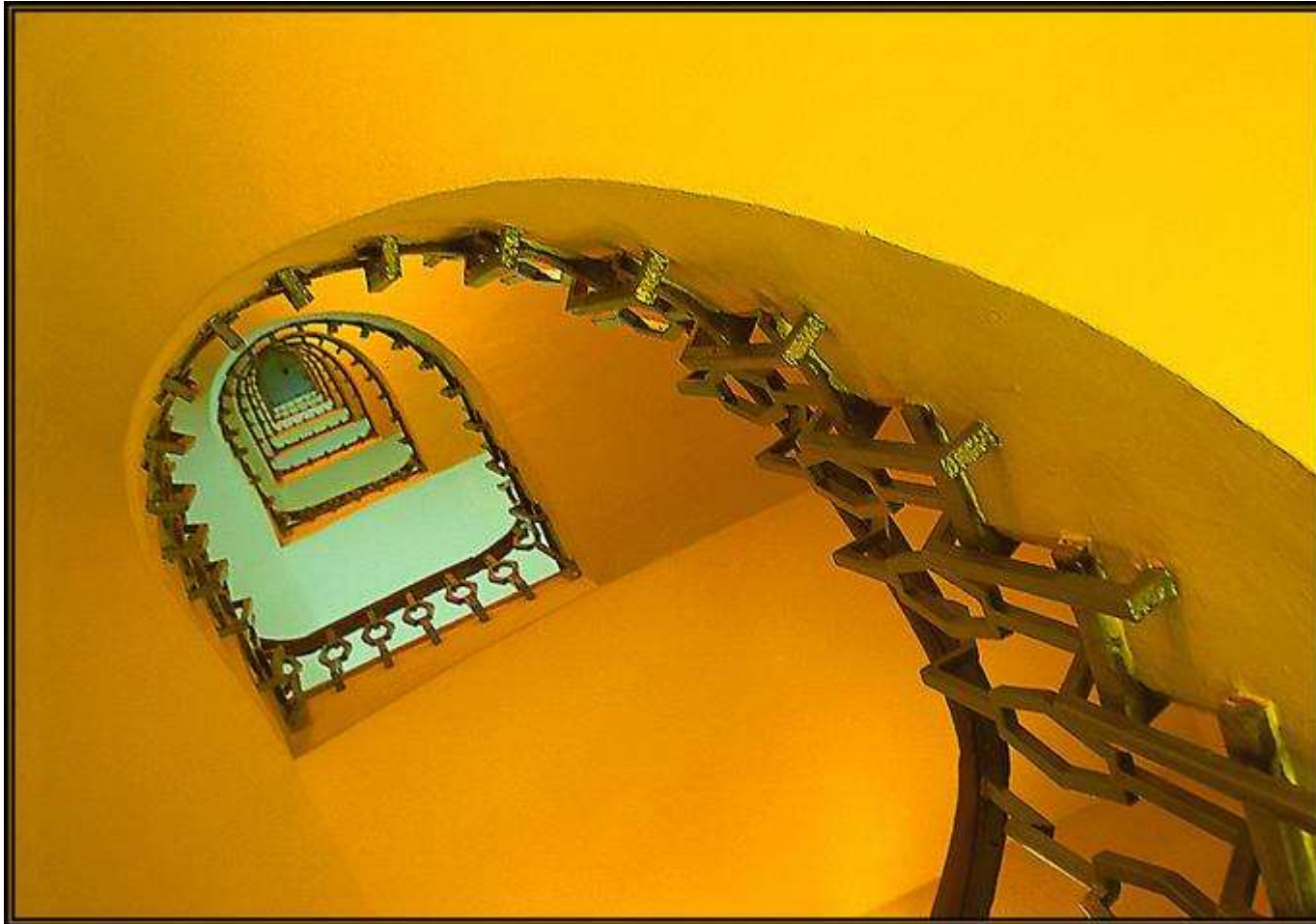
With 8/64
Coefficients



With 4/64
Coefficients



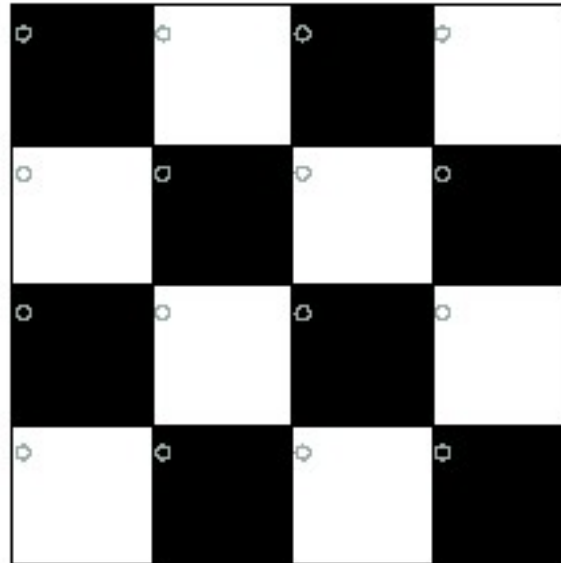
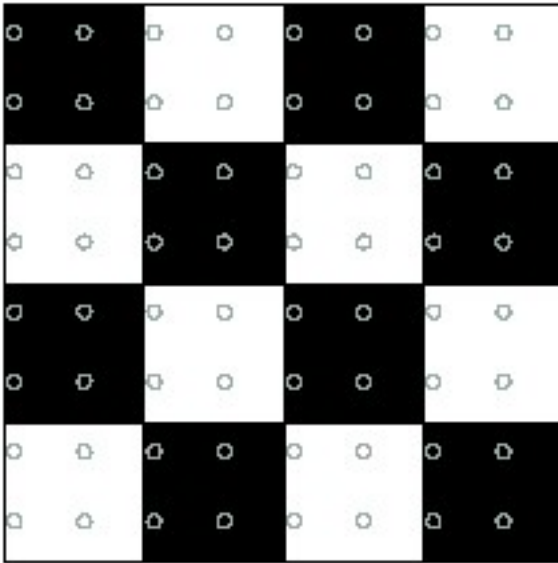
Image Pyramids



© Kenneth Kwan

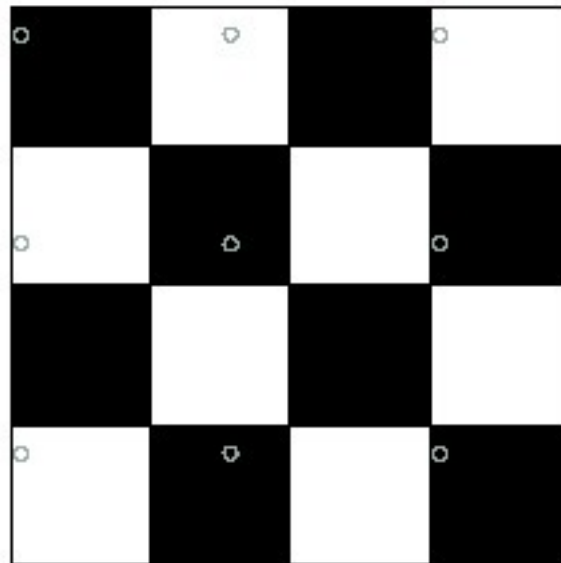
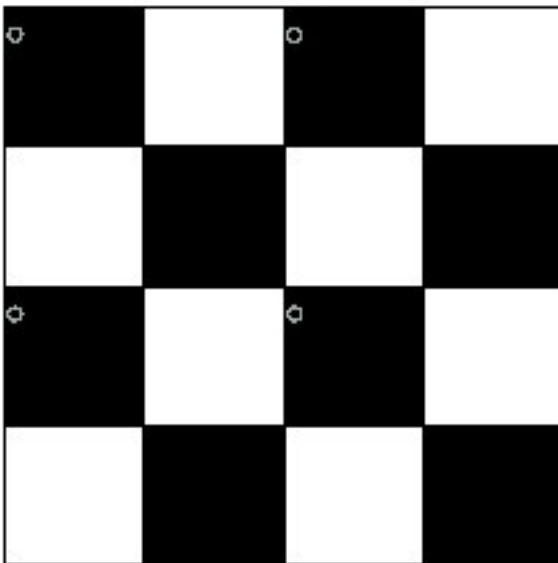
Slides Modified from Alexei Efros, CMU,

Sampling



Good sampling:

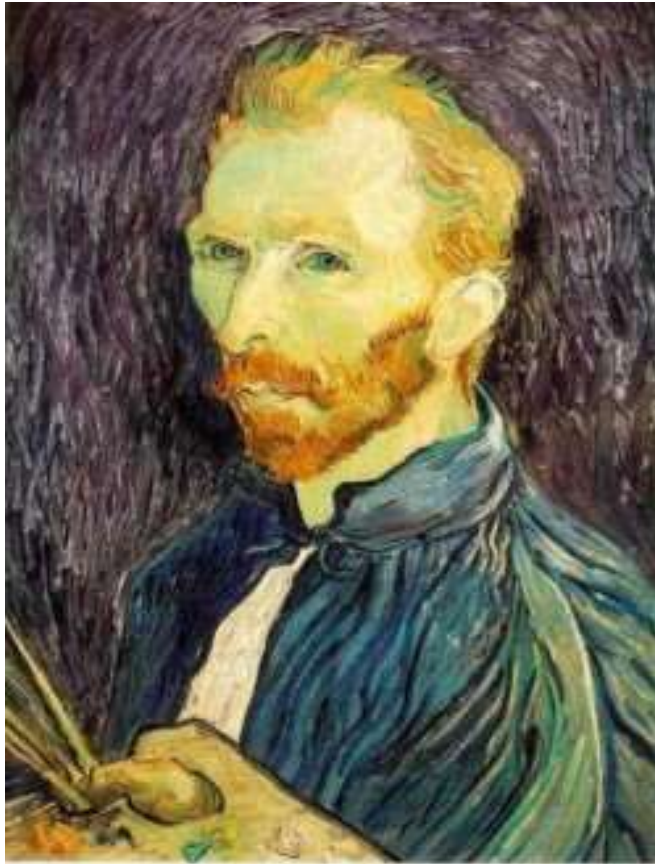
- Sample often or,
- Sample wisely



Bad sampling:

- see aliasing in action!

Gaussian pre-filtering



Gaussian 1/2



G 1/4



G 1/8

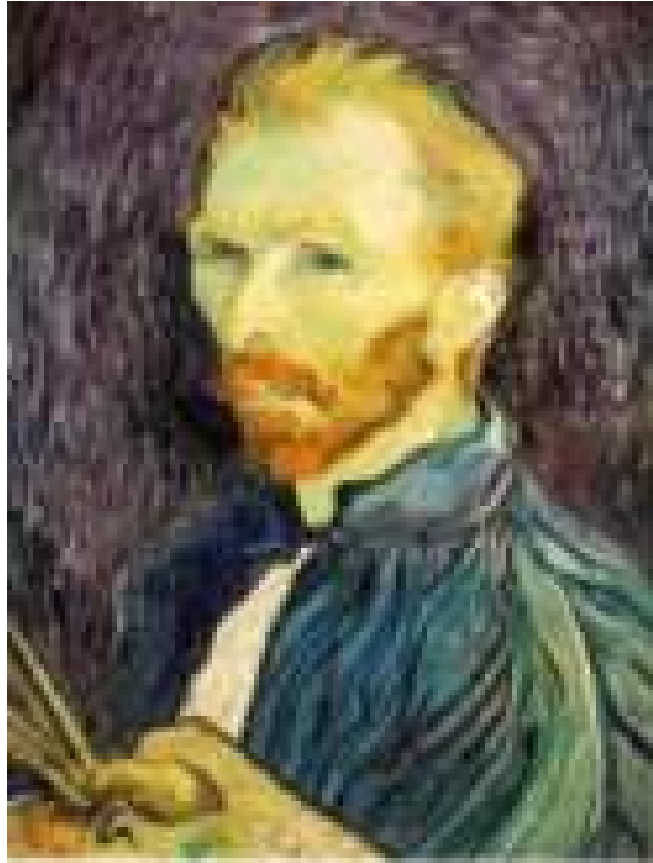
Solution: filter the image, *then* subsample

- Filter size should double for each $\frac{1}{2}$ size reduction. Why?

Subsampling with Gaussian pre-filtering



Gaussian 1/2



G 1/4



G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each $\frac{1}{2}$ size reduction. Why?
- How can we speed this up?

Compare with...



1/2



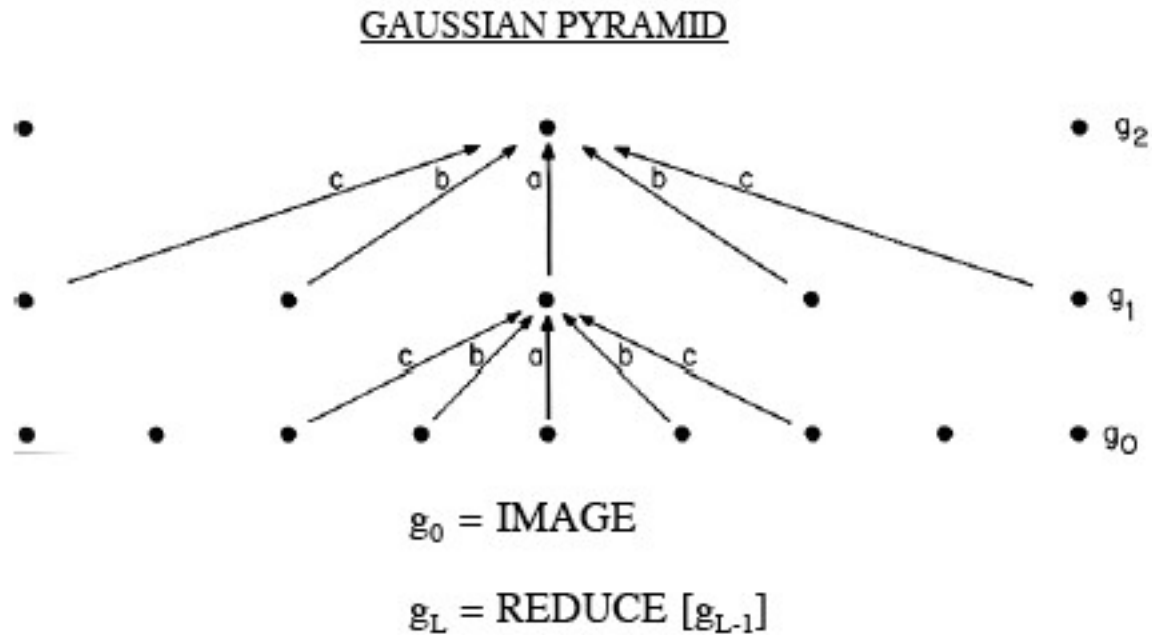
1/4 (2x zoom)



1/8 (4x zoom)

Gaussian Pyramid for encoding

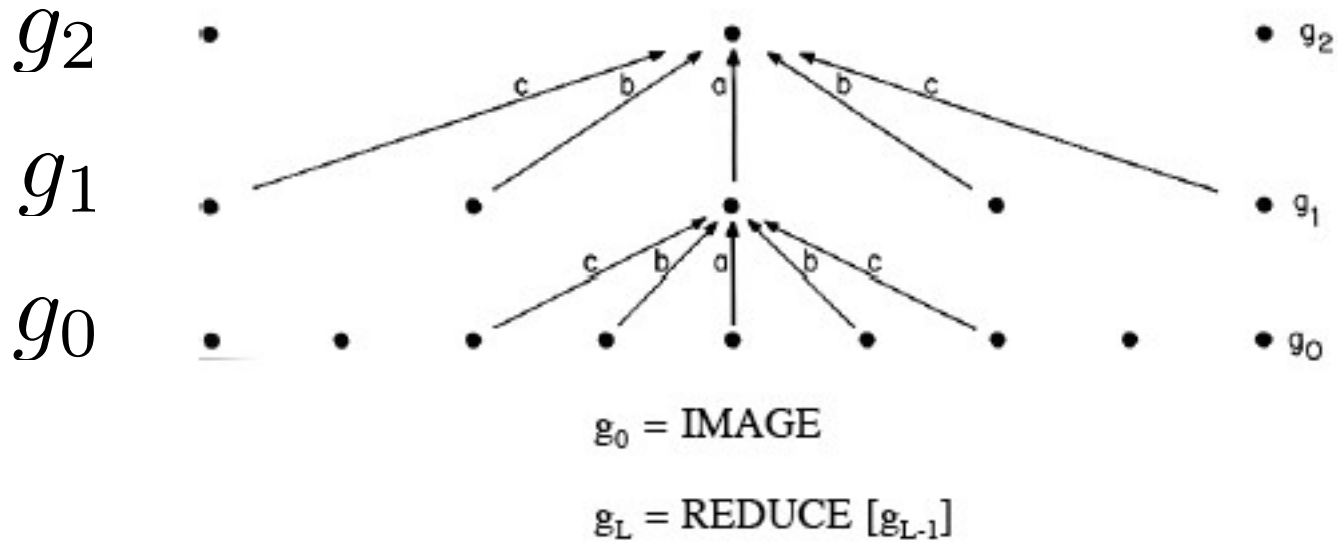
[Burt & Adelson, 1983]



- 1) Prediction using weighted local Gaussian average
- 2) Encode the difference as the Laplacian
- 3) Both Laplacian and the Averaged image is easy to encode

Gaussian pyramid

GAUSSIAN PYRAMID

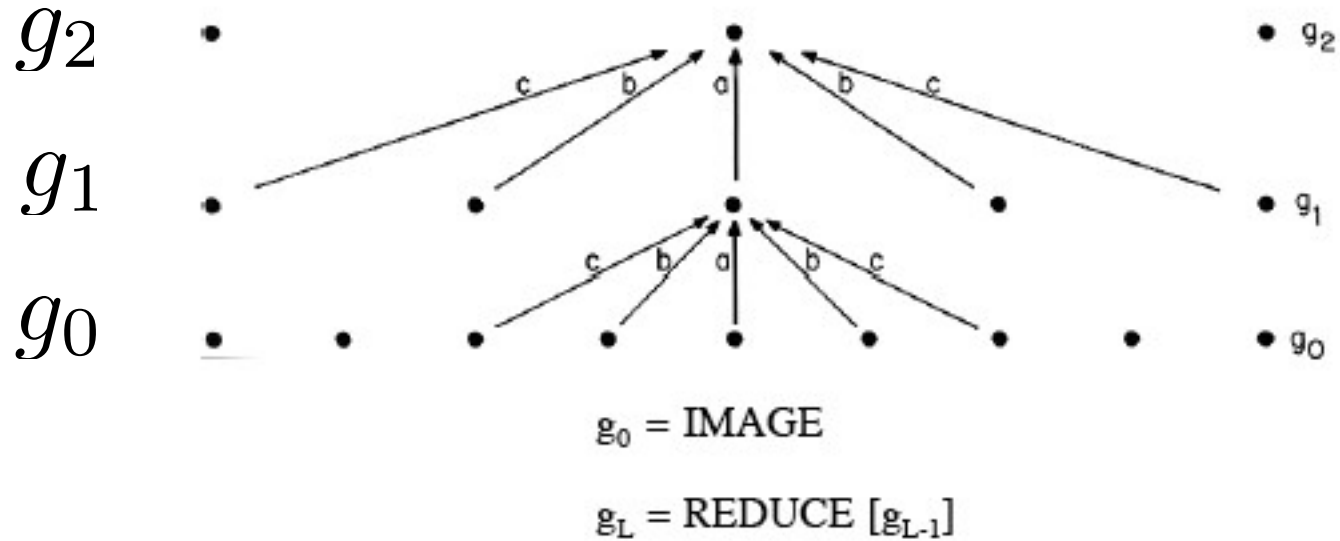


$$g_l(i, j) = \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) g_{l-1}(2i + m, 2j + n).$$

$$w(m, n) = \hat{w}(m) \hat{w}(n), \quad \sum_{m=-2}^2 \hat{w}(m) = 1, \quad \hat{w}(i) = \hat{w}(-i)$$

Gaussian pyramid

GAUSSIAN PYRAMID



$$g_l(i, j) = \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) g_{l-1}(2i + m, 2j + n).$$

$$g_l = [g_{l-1} \otimes w] \downarrow 2$$

Choice in weighting function

$$\hat{w}(0) = a$$

$$\hat{w}(-1) = \hat{w}(1) = 1/4$$

$$\hat{w}(-2) = \hat{w}(2) = 1/4 - a/2.$$

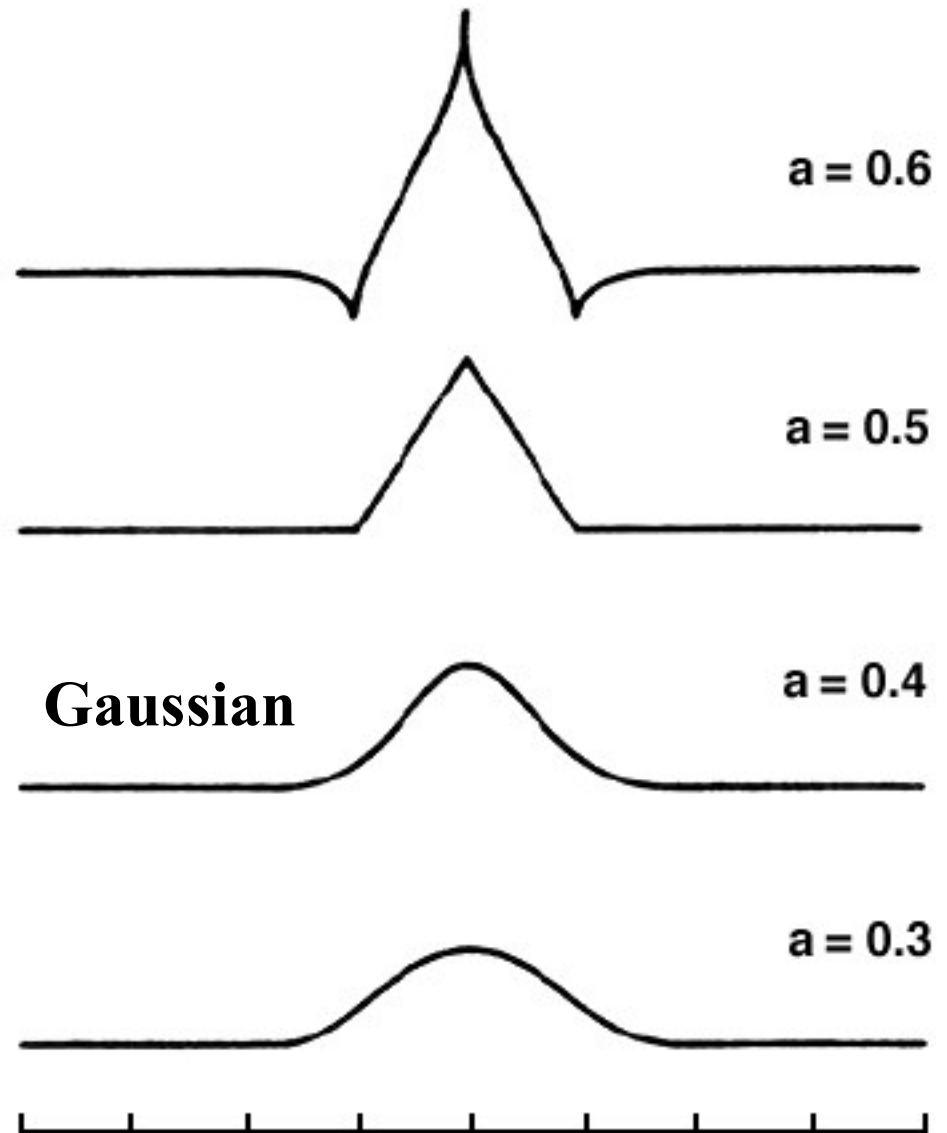
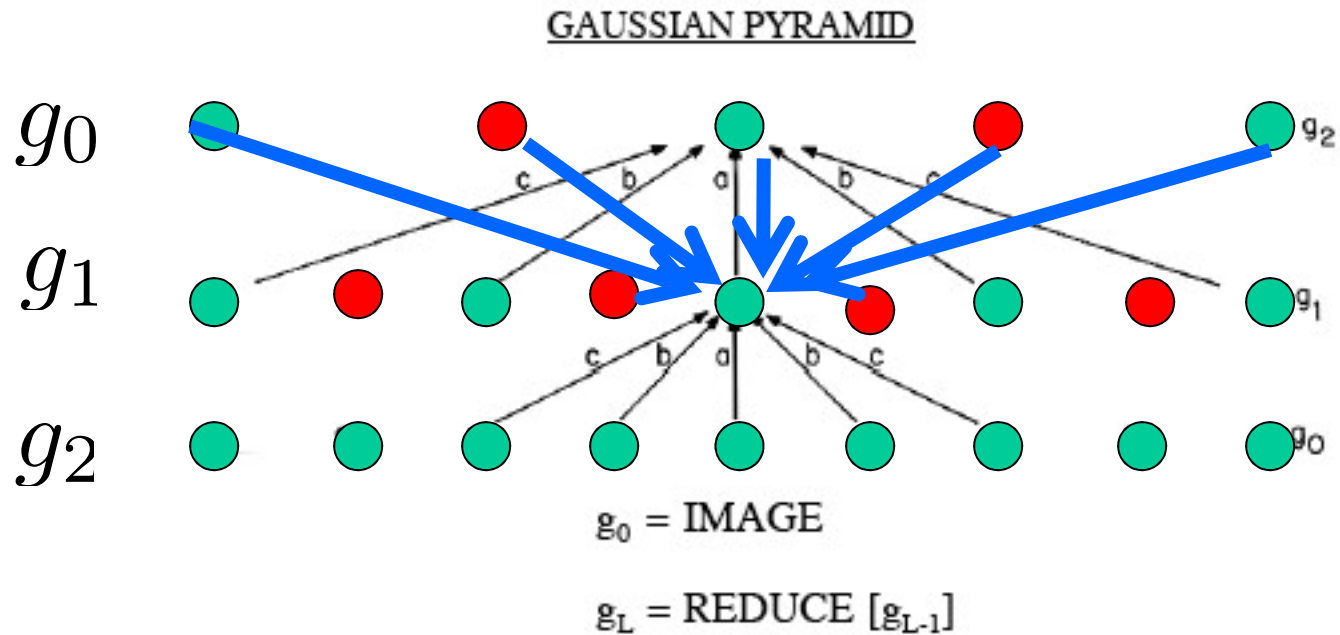


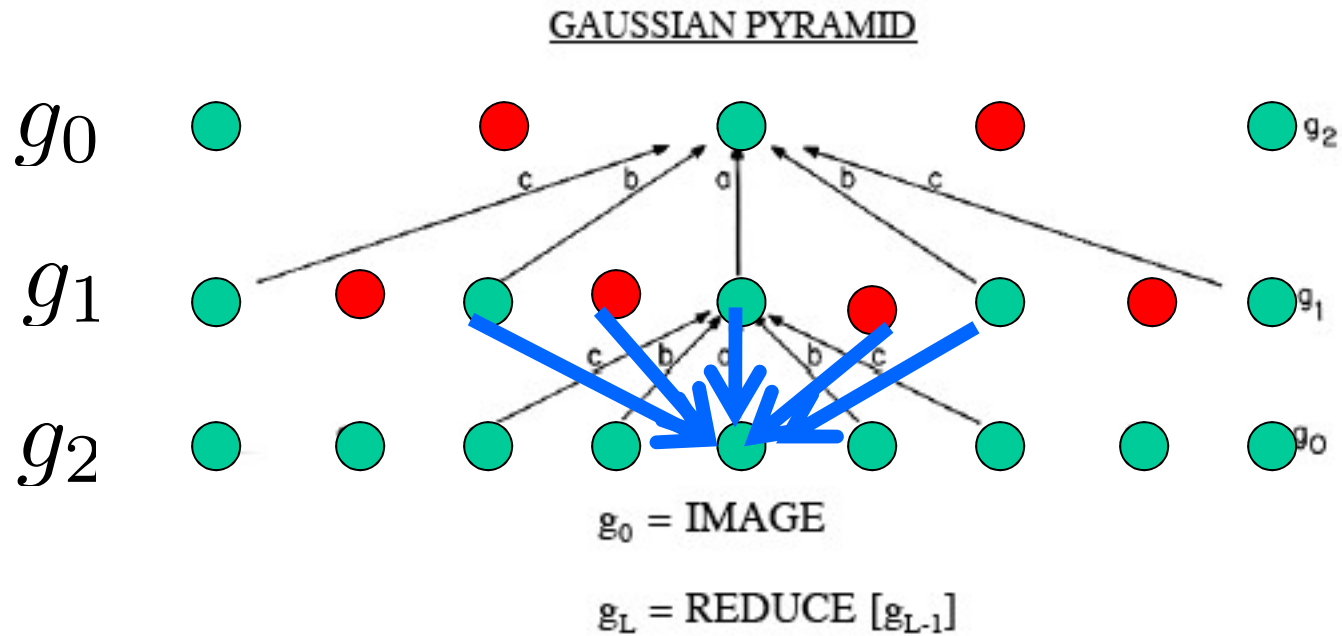
Image Expansion



$$g_{l,n}(ij) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m,n) \cdot g_{l,n-1} \left(\frac{i-m}{2}, \frac{j-n}{2} \right).$$

$(i-m)/2$ and $(j-n)/2$ are integers

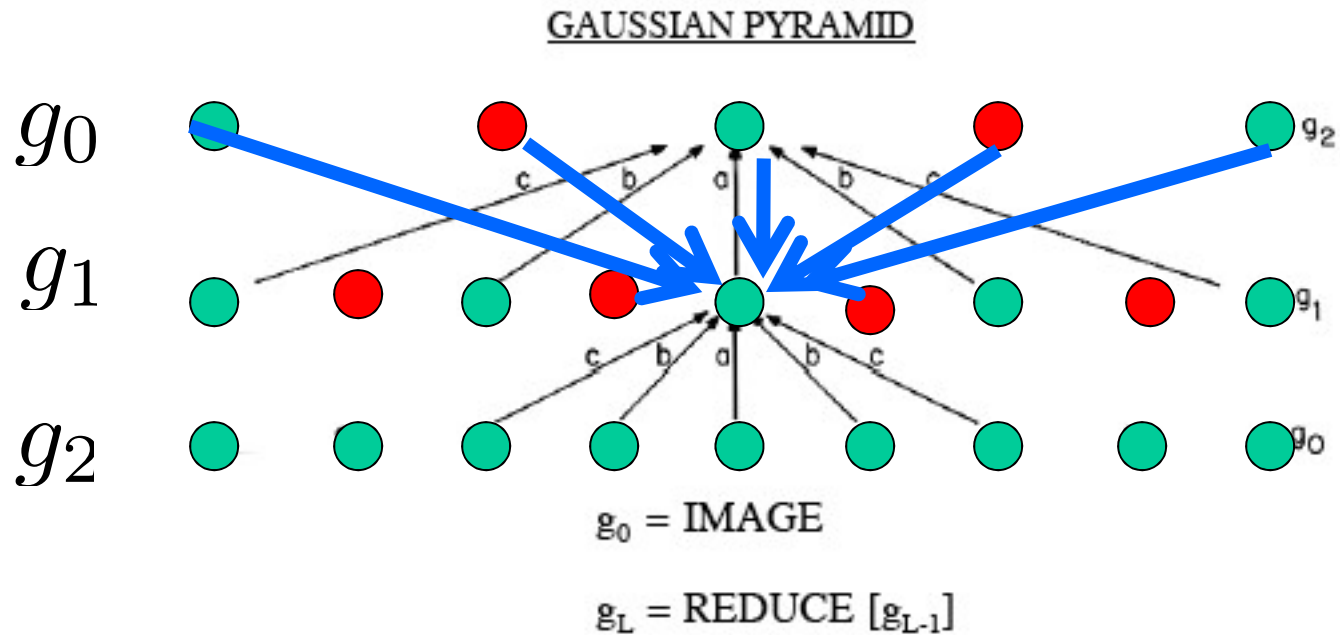
Image Expansion



$$g_{l,n}(ij) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m,n) \cdot g_{l,n-1} \left(\frac{i-m}{2}, \frac{j-n}{2} \right).$$

$(i-m)/2$ and $(j-n)/2$ are integers

Image Expansion

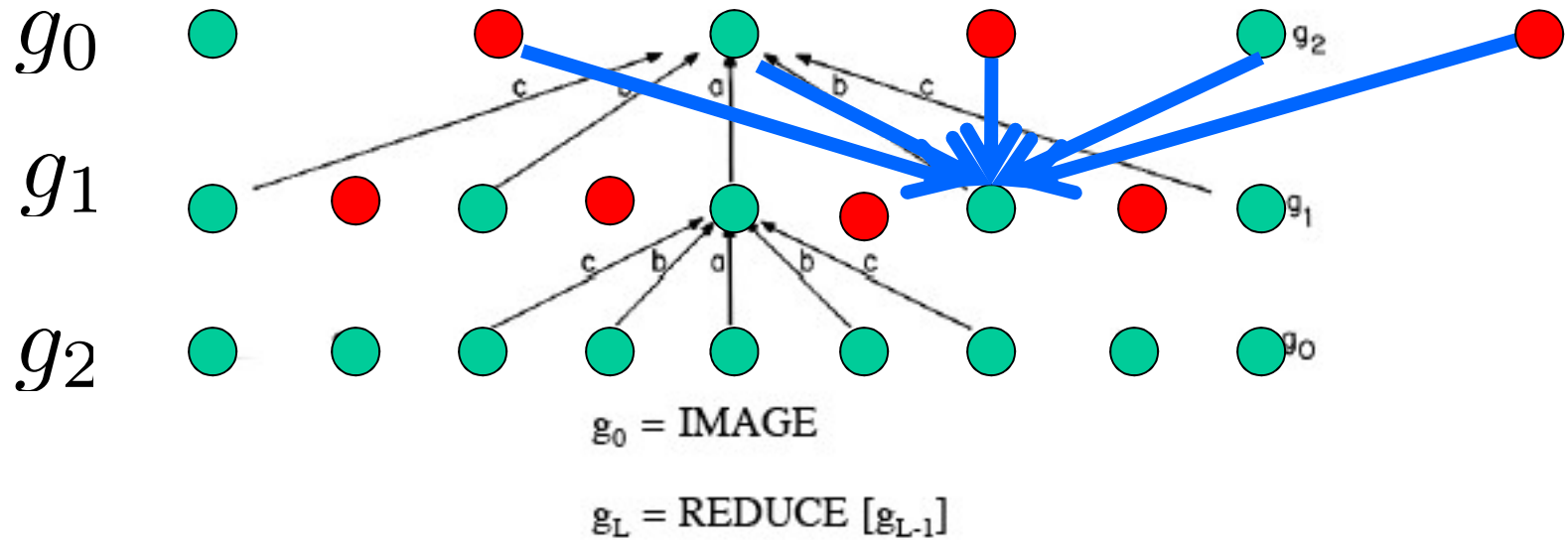


$$g_{l,n}(ij) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m,n) \cdot g_{l,n-1} \left(\frac{i-m}{2}, \frac{j-n}{2} \right).$$

$(i-m)/2$ and $(j-n)/2$ are integers

Image Expansion

GAUSSIAN PYRAMID

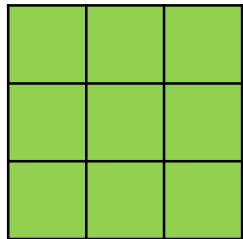


$$g_{l,n}(ij) = 4 \sum_{m=-2}^2 \sum_{n=-2}^2 w(m,n) \cdot g_{l,n-1} \left(\frac{i-m}{2}, \frac{j-n}{2} \right).$$

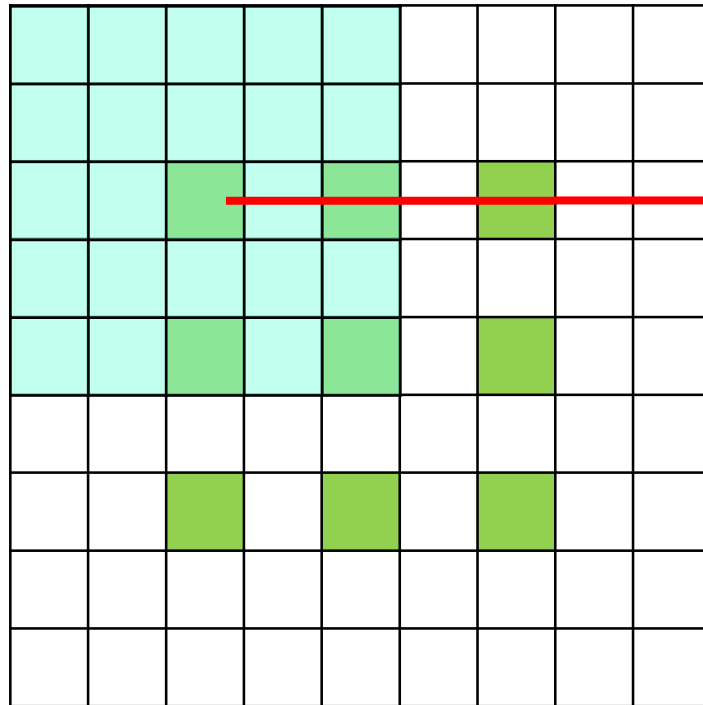
$(i-m)/2$ and $(j-n)/2$ are integers

2D Image Expansion (part1)

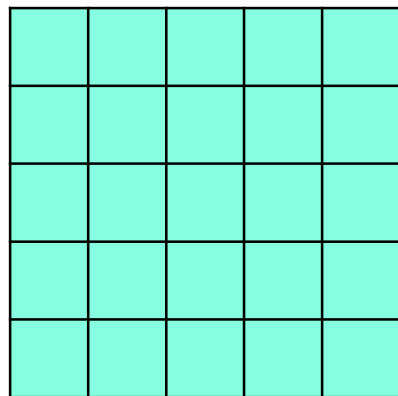
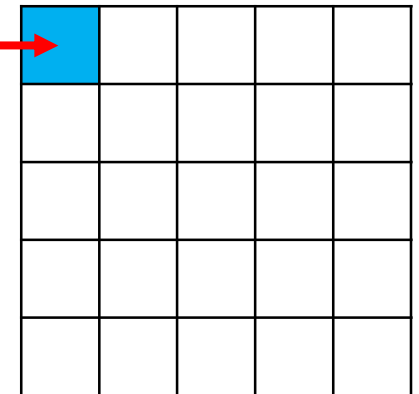
g_0



g_0 padded with 0s



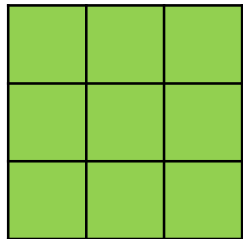
g_1



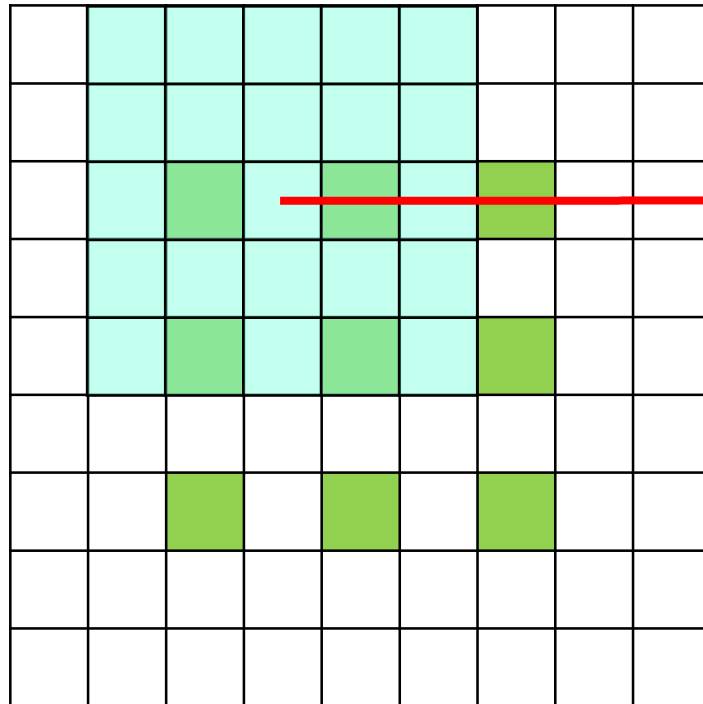
2D Gaussian kernel

2D Image Expansion (part2)

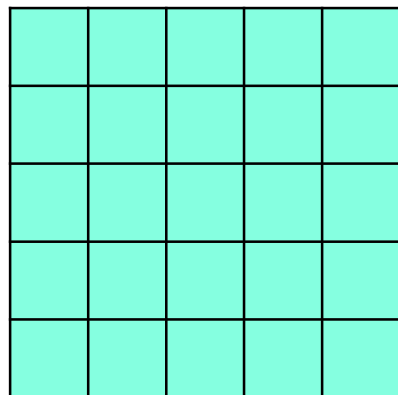
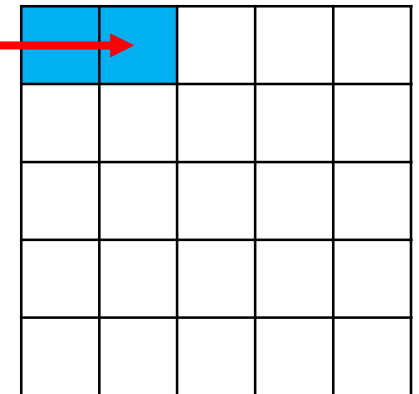
g_0



g_0 padded with 0s



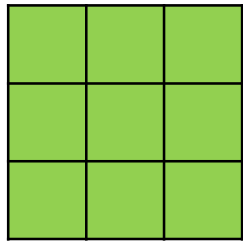
g_1



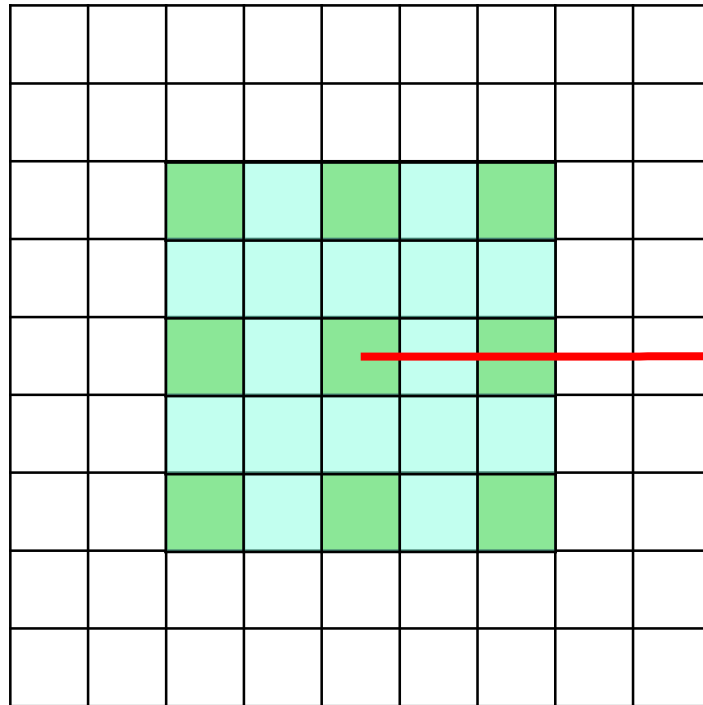
2D Gaussian kernel

2D Image Expansion (part3)

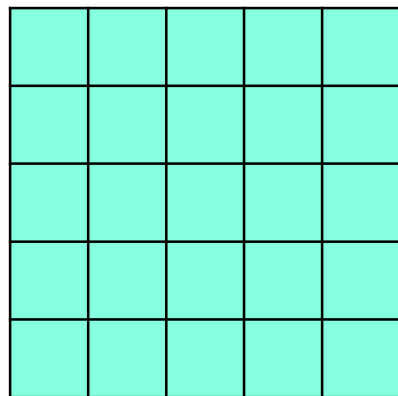
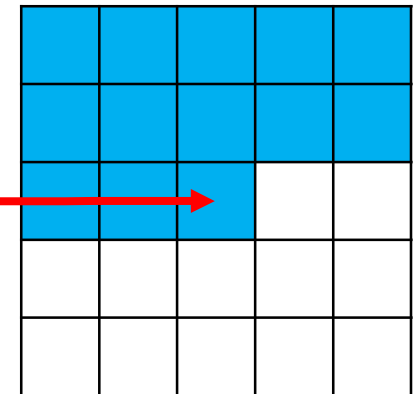
g_0



g_0 padded with 0s



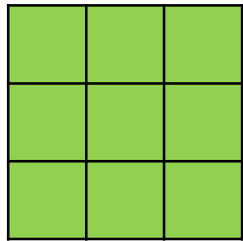
g_1



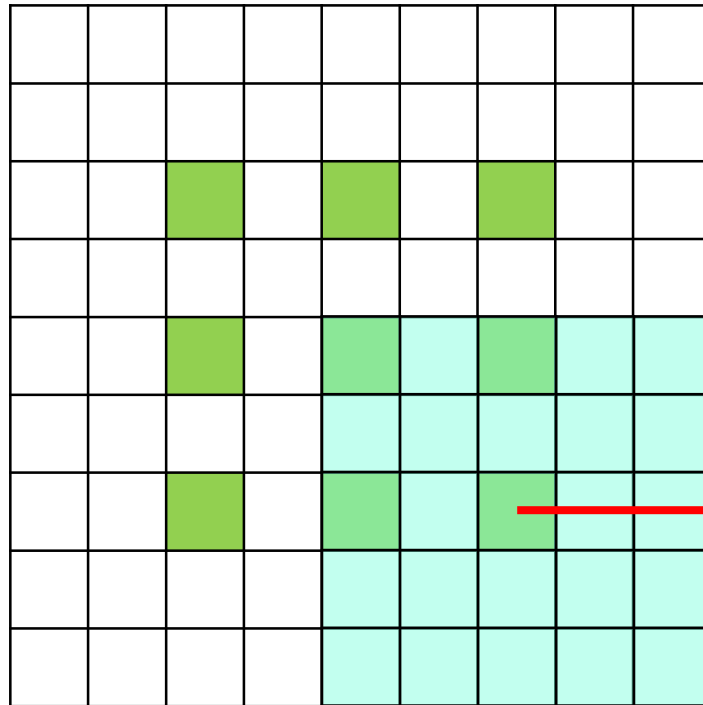
$2D$ Gaussian kernel

2D Image Expansion (part4)

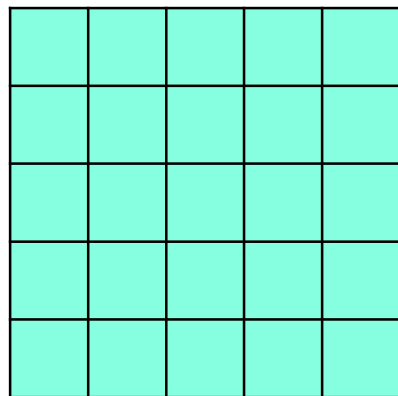
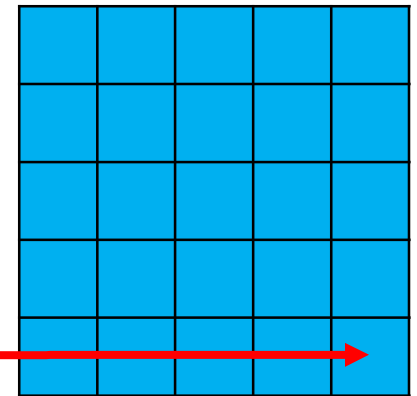
g_0



g_0 padded with 0s



g_1



$2D$ Gaussian kernel

What does blurring take away?



original

What does blurring take away?



smoothed (5x5 Gaussian)

High-Pass filter



smoothed – original



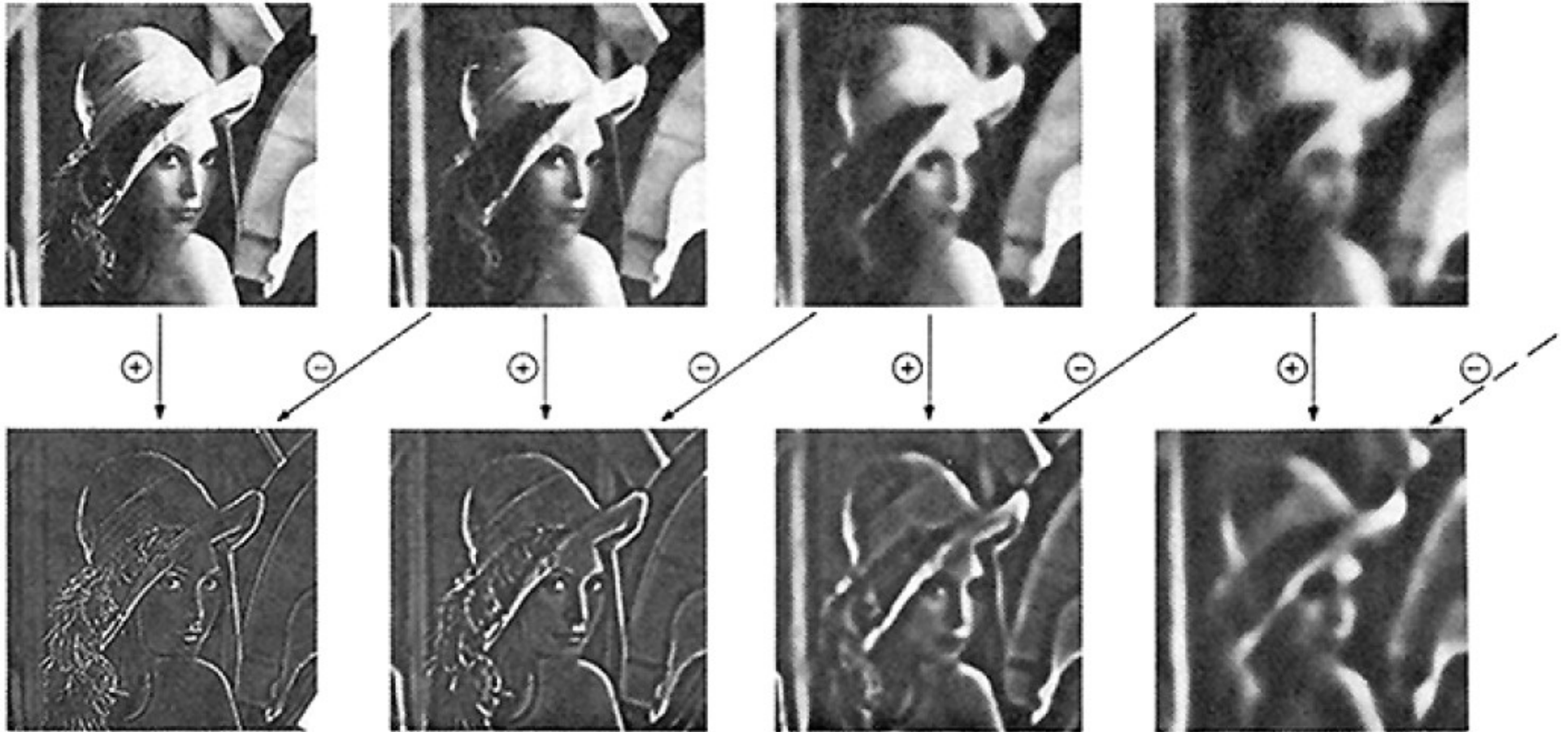
+ ↓

-



Laplacian Image

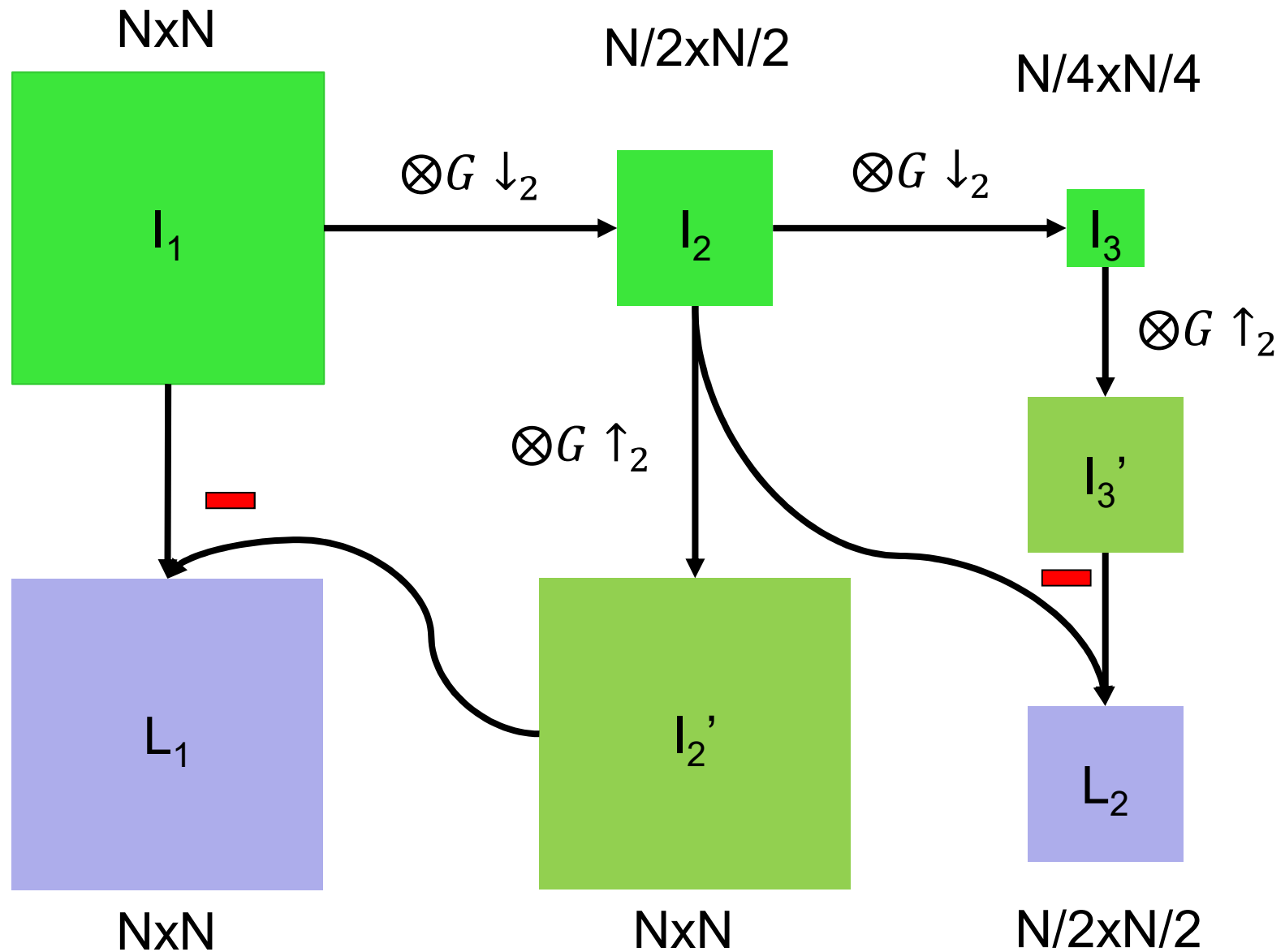
$$L_l = g_l - \text{EXPAND}(g_{l+1})$$

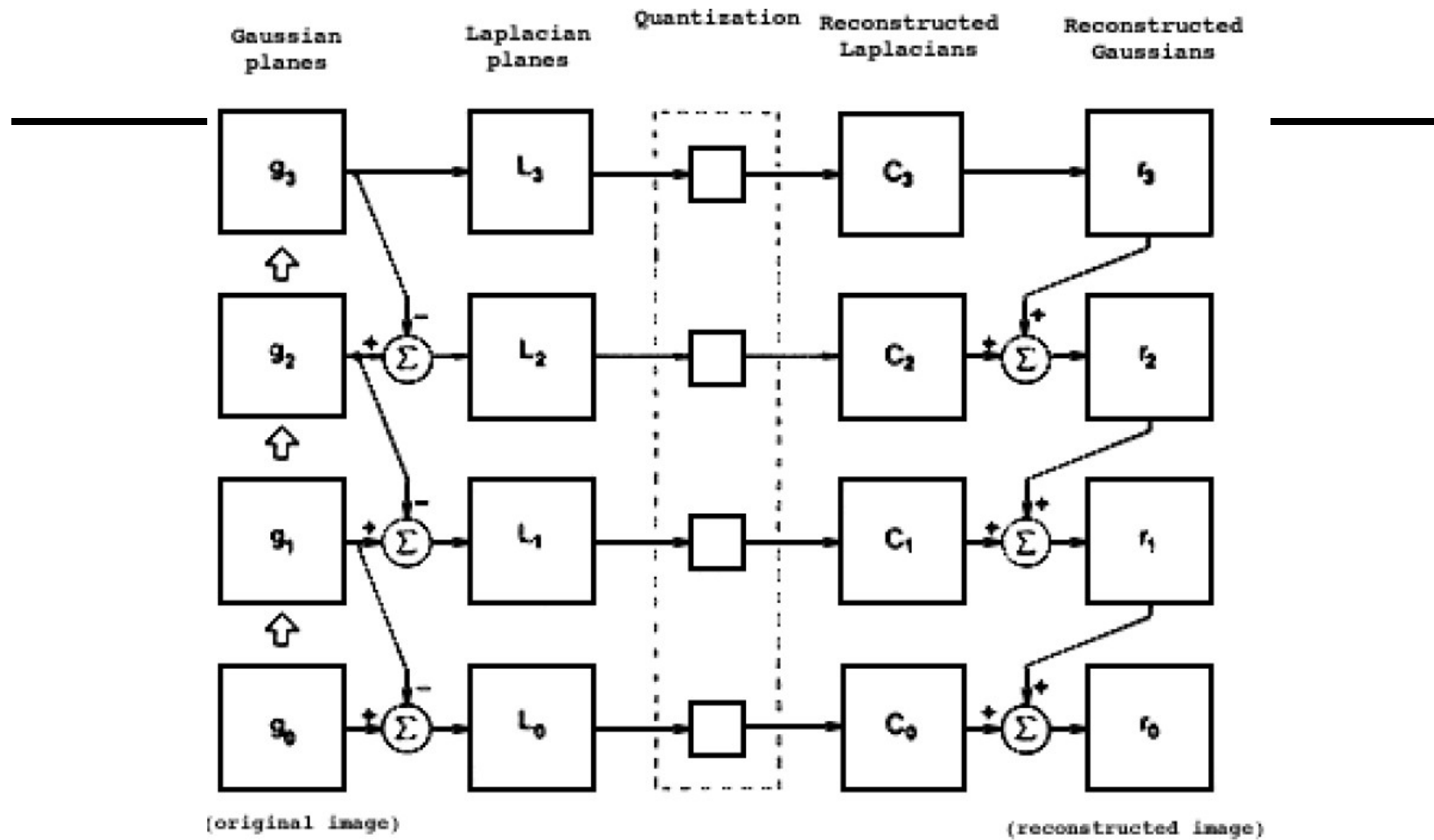


Gaussian pyramid is smooth=> can be subsampled

Laplacian pyramid has narrow band of frequency=> compressed

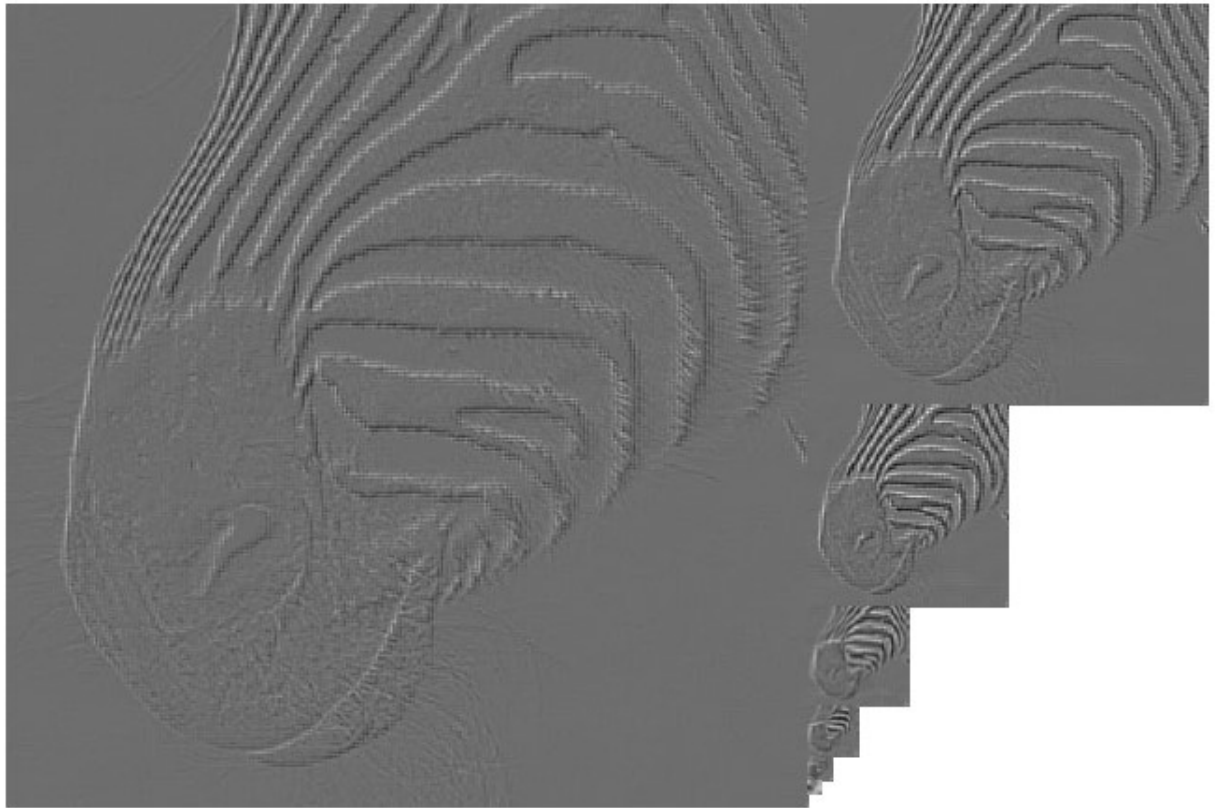
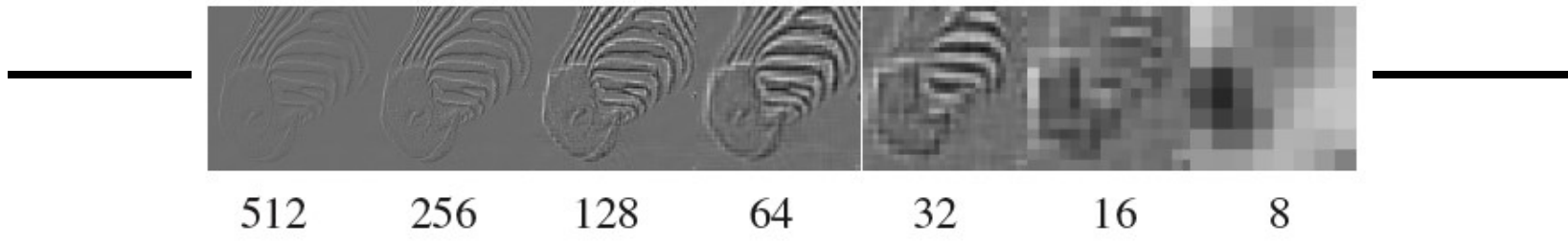
Pyramid extraction of Laplacian



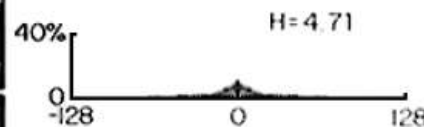
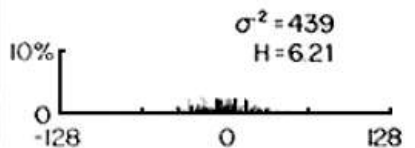
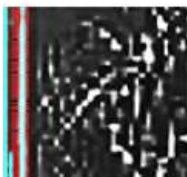
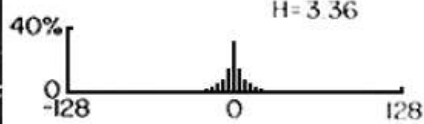
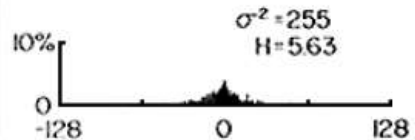
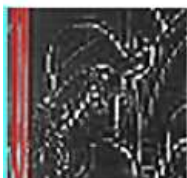
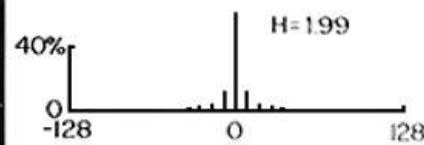
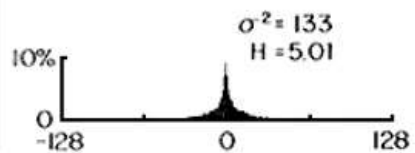
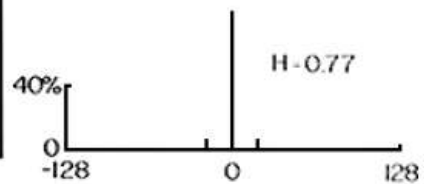
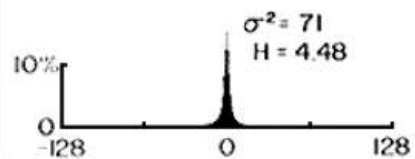
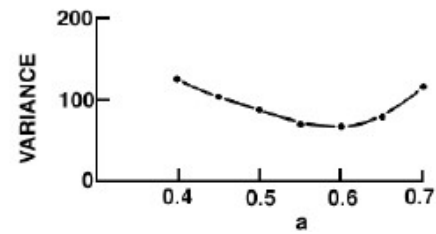
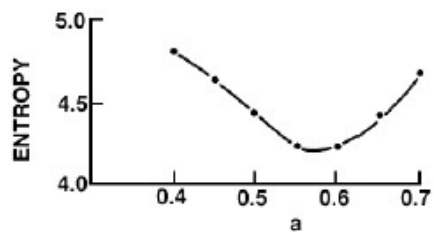
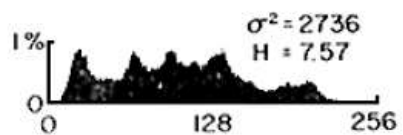


$$g_N = L_N$$

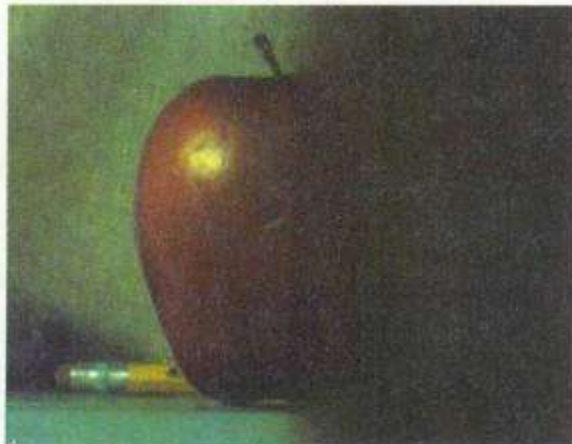
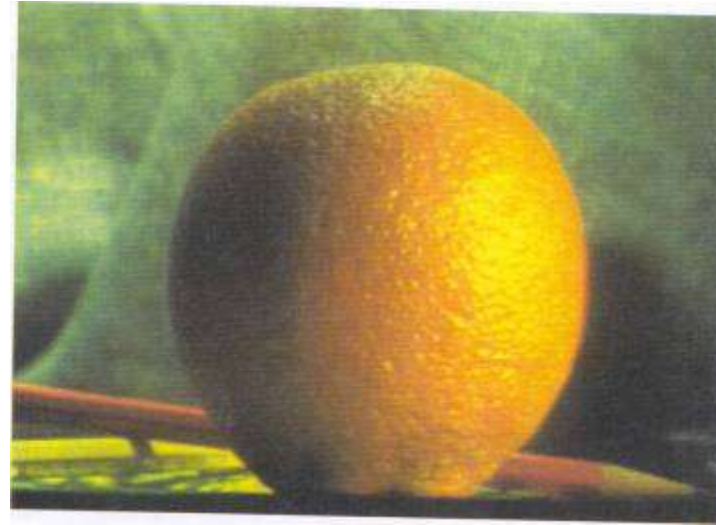
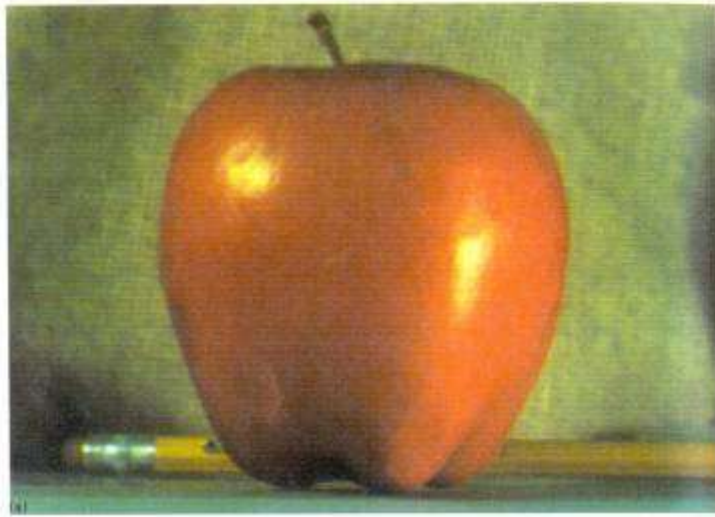
$$g_l = L_l + \text{EXPAND}(g_{l+1}).$$



$$L_n = G_n$$



Pyramid Blending



(d)

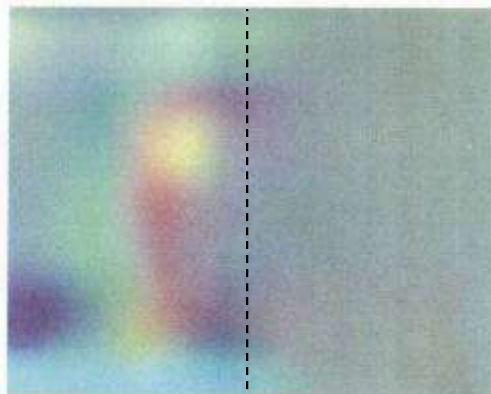


(h)



(l)

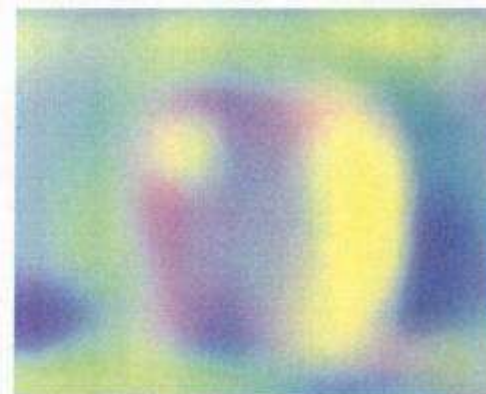
laplacian
level
4



(c)

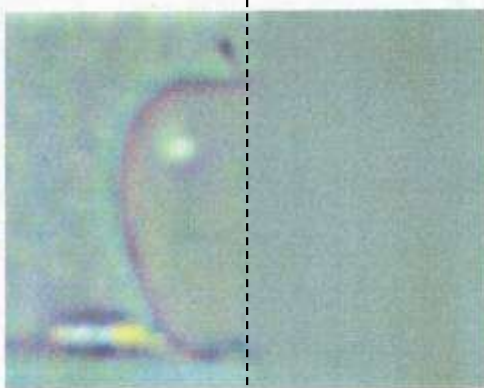


(g)

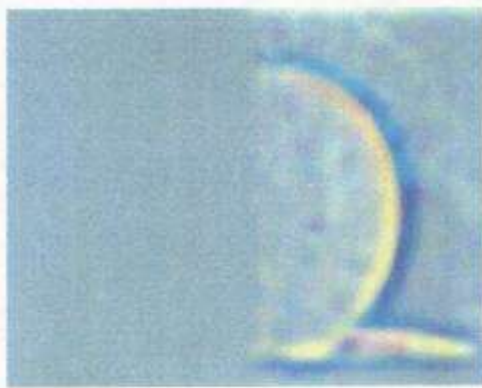


(k)

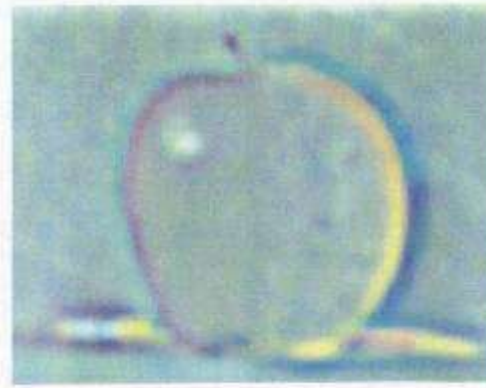
laplacian
level
2



(b)

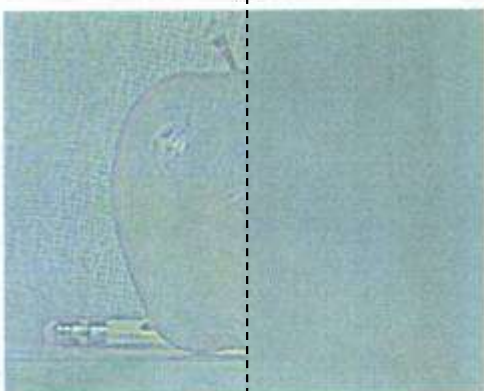


(f)

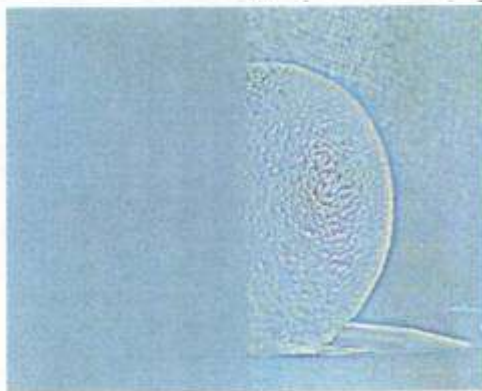


(j)

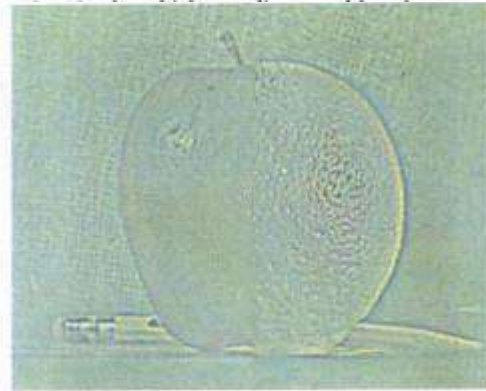
laplacian
level
0



(a)



(e)



(i)

left pyramid

right pyramid

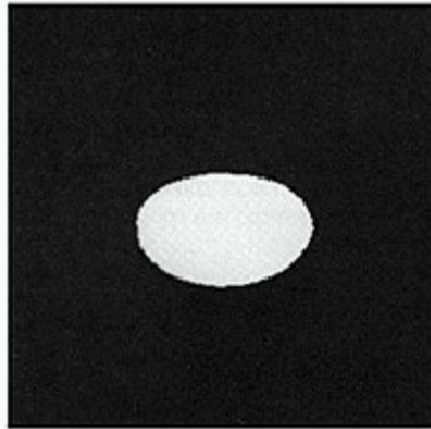
blended pyramid

Laplacian Pyramid: Blending

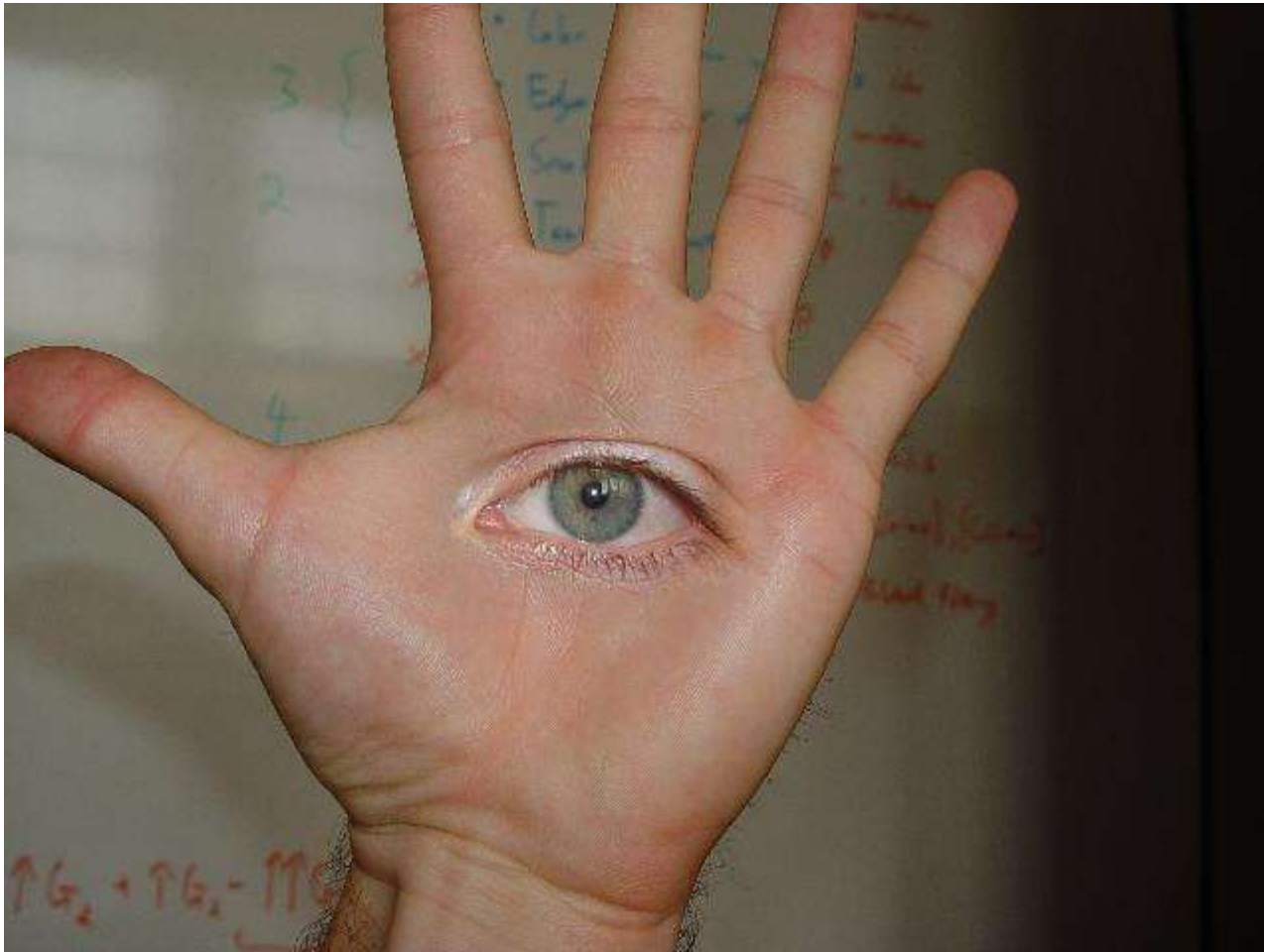
General Approach:

1. Build Laplacian pyramids LA and LB from images A and B
2. Build a Gaussian pyramid GR from selected region R
3. Form a combined pyramid LS from LA and LB using nodes of GR as weights:
 - $LS(i,j) = GR(l,j) * LA(l,j) + (1 - GR(l,j)) * LB(l,j)$
4. Collapse the LS pyramid to get the final blended image

Blending Regions



Horror Photo



© prof. dmartin