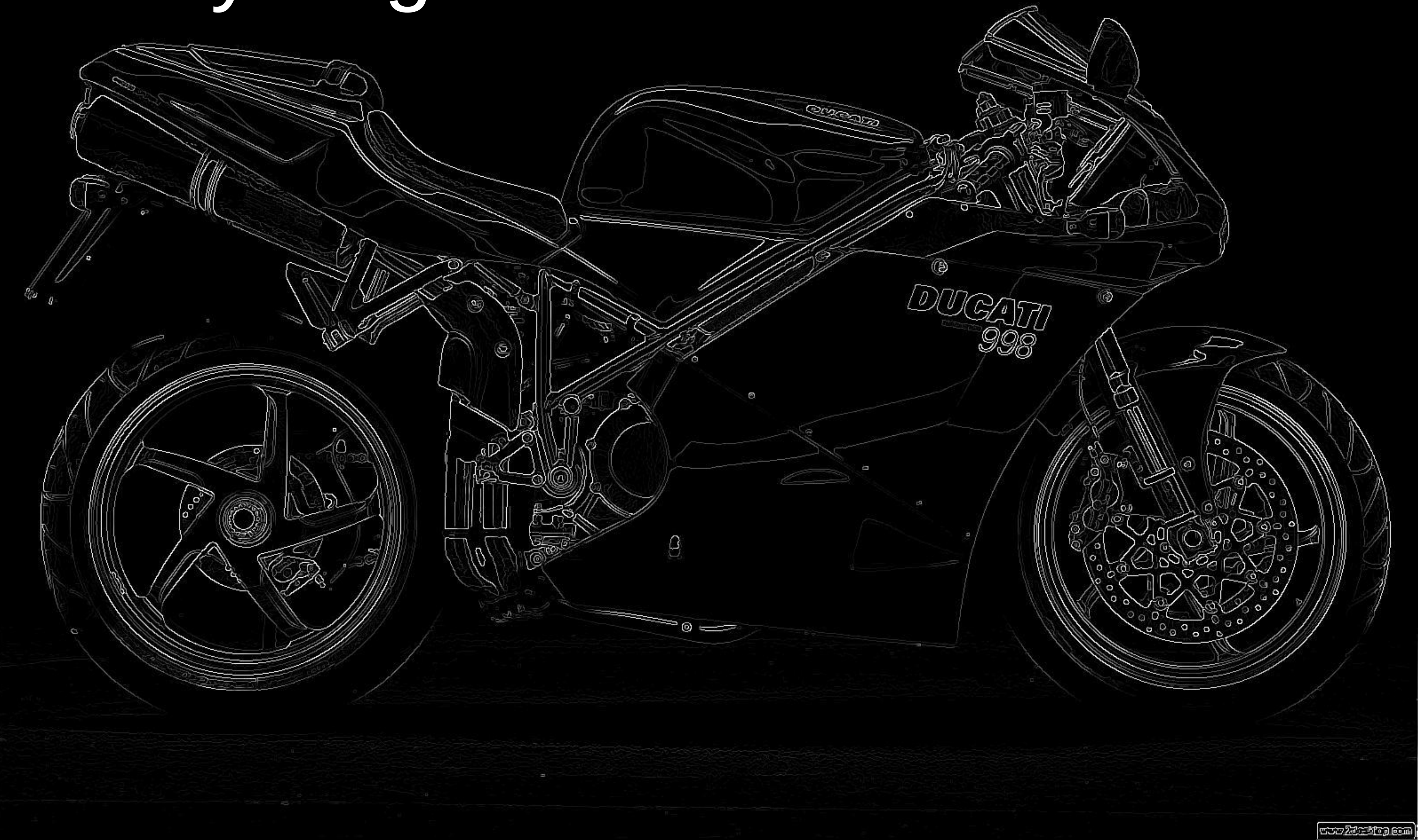
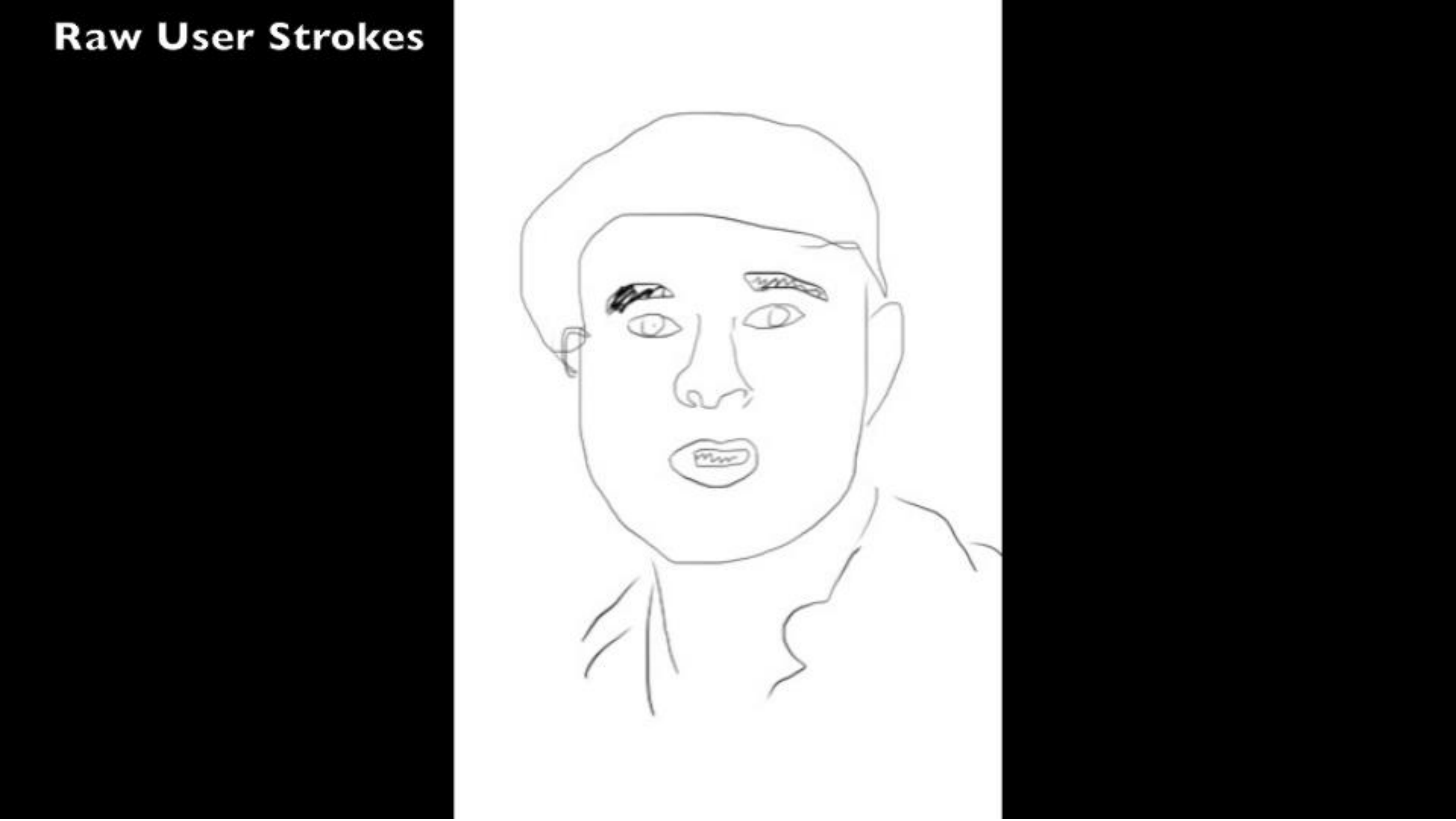
Canny Edge Detection





Real-time Drawing Assistance through Crowdsourcing

SIGGRAPH 2013

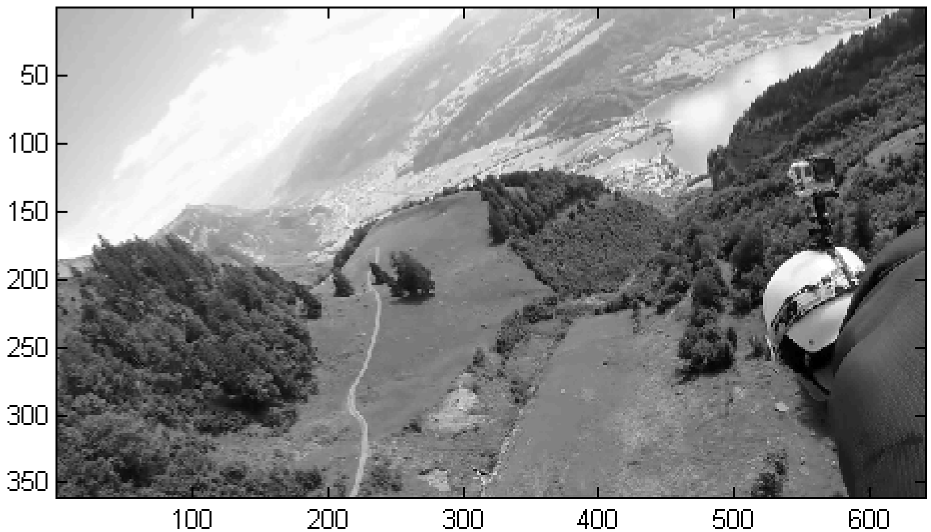
A. Limpaecher, N. Feltman, A. Treuille, and M. Cohen

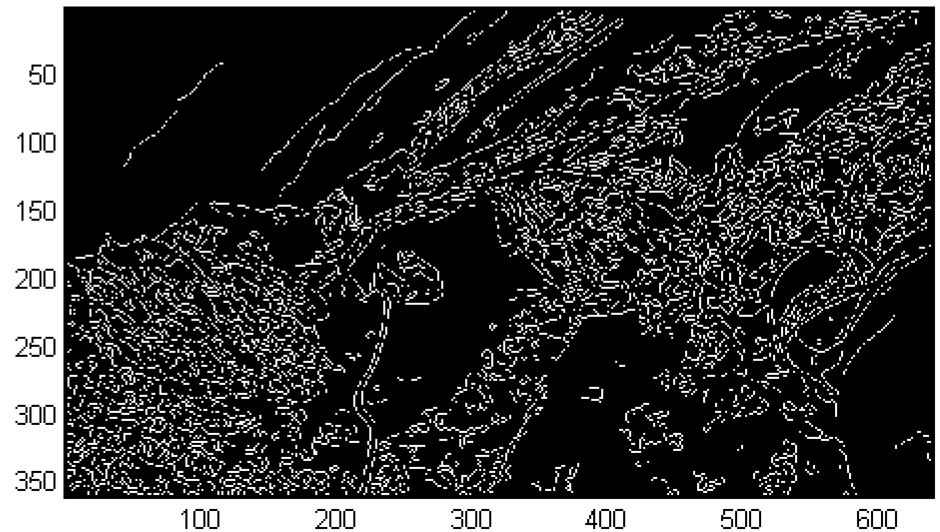
CMU and Microsoft

Edge Detection

Code

```
Jb = rgb2gray(J);

imagesc(Jb);axis image; colormap(gray);

bw = edge(Jb,'canny');
```





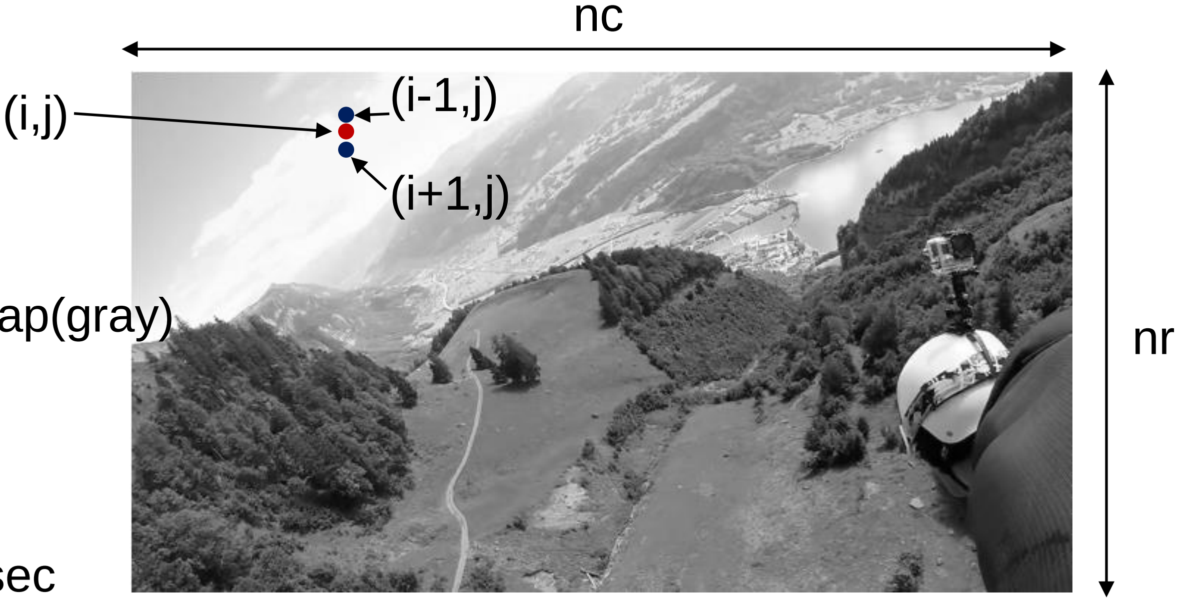
Numerical Image Filtering

Filter

-0.8
2.0
-0.8

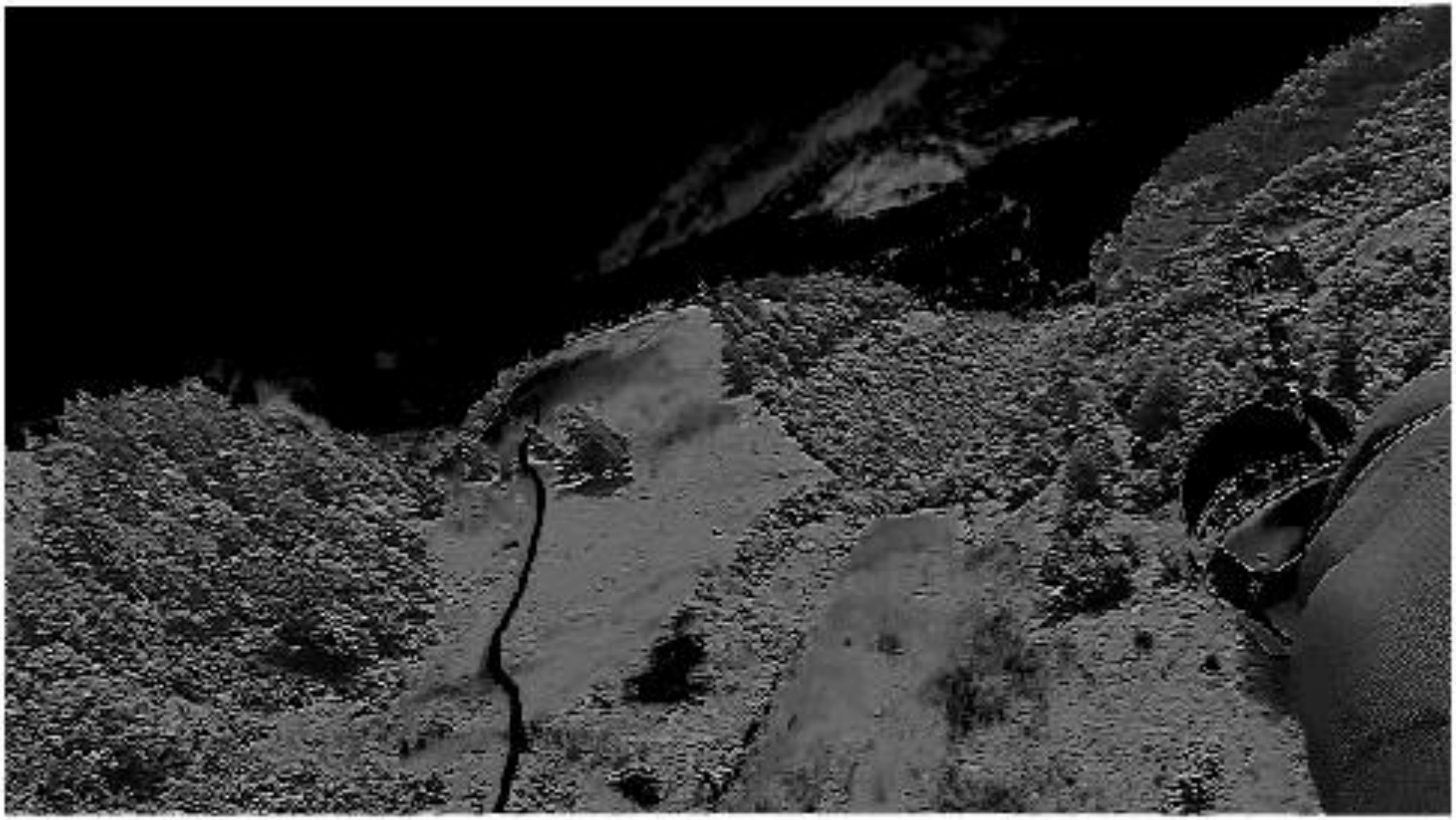
Looping through all pixels

```
[nr,nc] = size(Jb);
J_out = zeros(nr,nc);
for i=1:nr,
    for j=1:nc;
        if (i<nr) && (i>1),
            J_out(i,j) = 2*Jb(i,j) - 0.8*Jb(i+1,j) - 0.8*Jb(i-1,j);
        else
            J_out(i,j) = Jb(i,j);
        end
    end
end
figure; imagesc(J_out);colormap(gray)
```



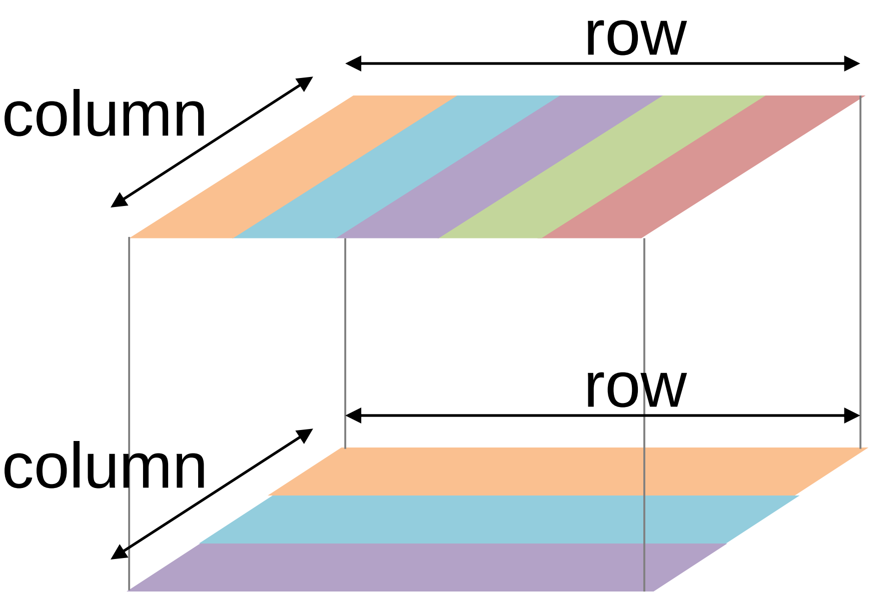
Computation time: 0.050154 sec

Numerical Image Filtering

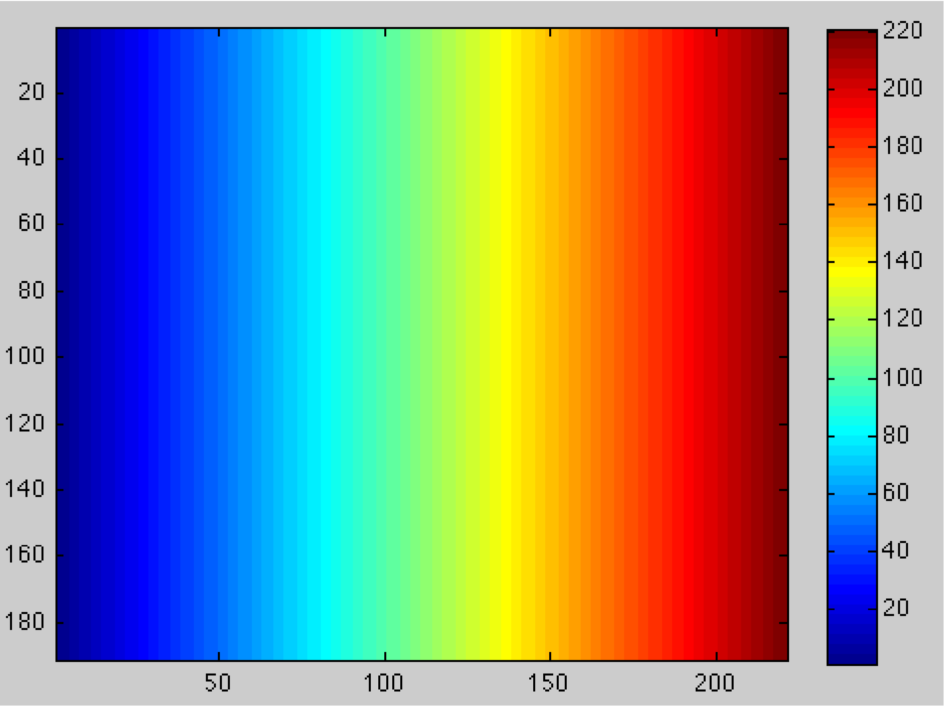


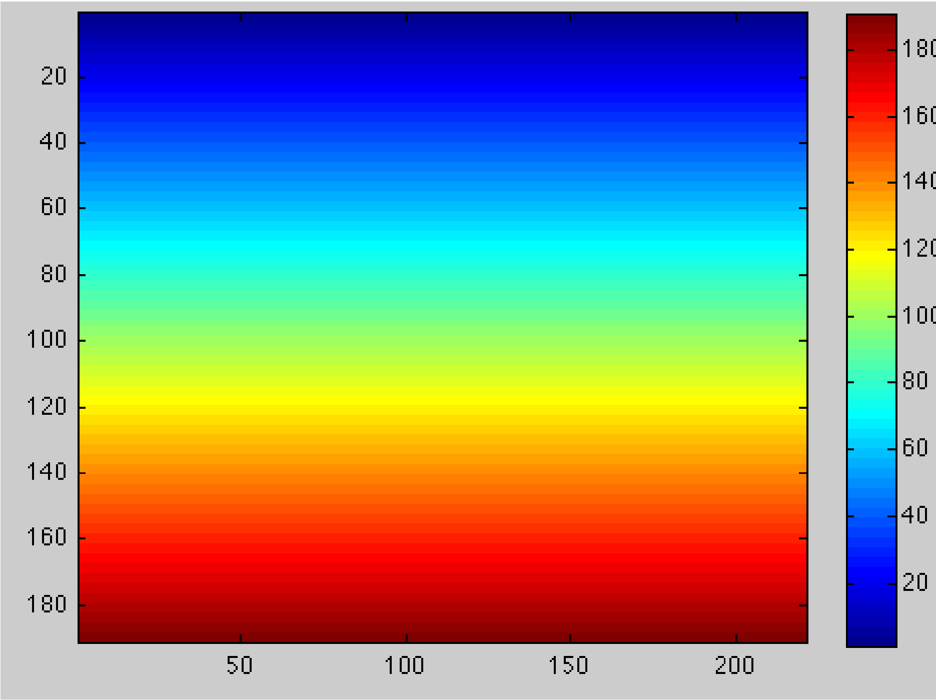
Convolution without Looping using meshgrid

```
>> [x,y] = meshgrid(1:5,1:3)

x =

     1     2     3     4     5
     1     2     3     4     5
     1     2     3     4     5


y =

     1     1     1     1     1
     2     2     2     2     2
     3     3     3     3     3
```



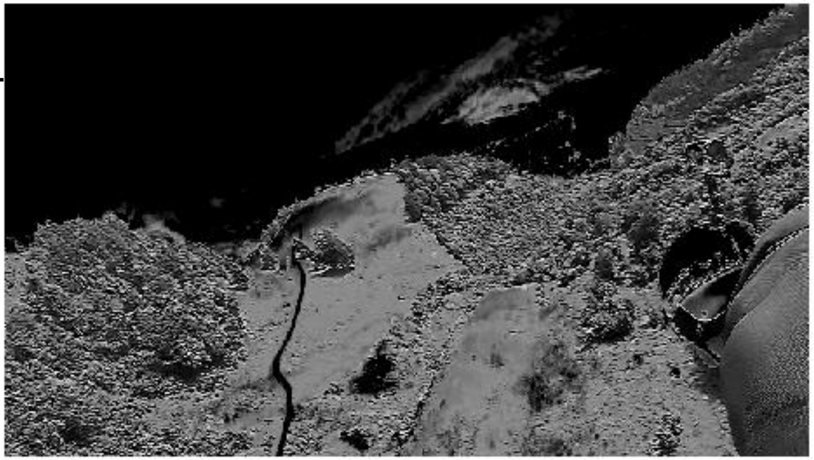
```
[x,y] = meshgrid(1:nc,1:nr);
figure(1); imagesc(x); axis image; colorbar;
colormap(jet);
figure(2); imagesc(y); axis image; colorbar;
colormap(jet);
```





Convolution without Looping using meshgrid

```
[x,y] = meshgrid(1:nc,1:nr);

y_up = y-1;
y_down = y+1;

y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));

J_out = 2*Jb(:) - 0.8*Jb(ind_up) - 0.8*Jb(ind_
J_out = reshape(J_out, nr, nc);

figure; imagesc(J_out);colormap(gray)
```



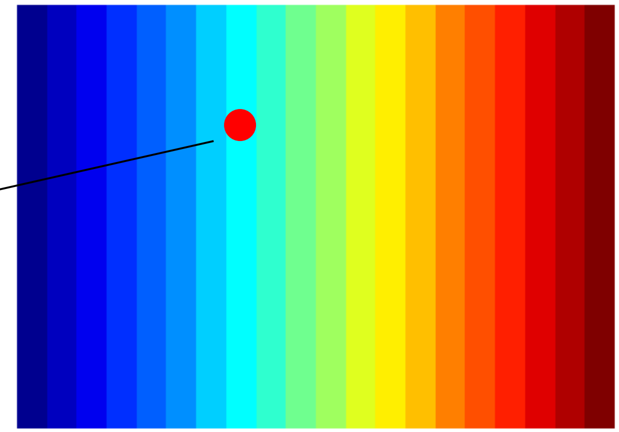
Computation time: 0.024047 sec

Convolution without Looping using meshgrid

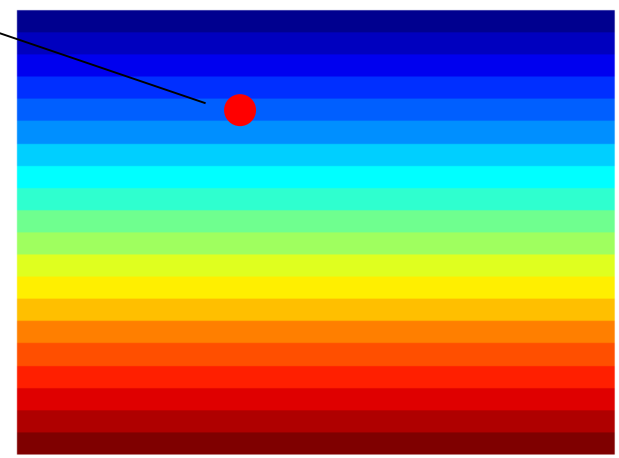
[x,y] = meshgrid(1:nc,1:nr);

x and y are subscript indice.

Jb_{xy}



x

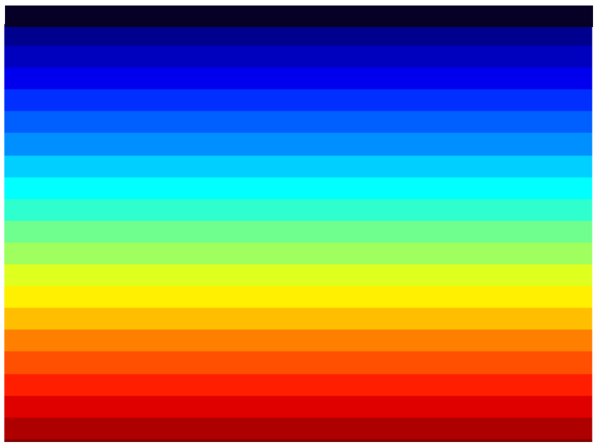


y

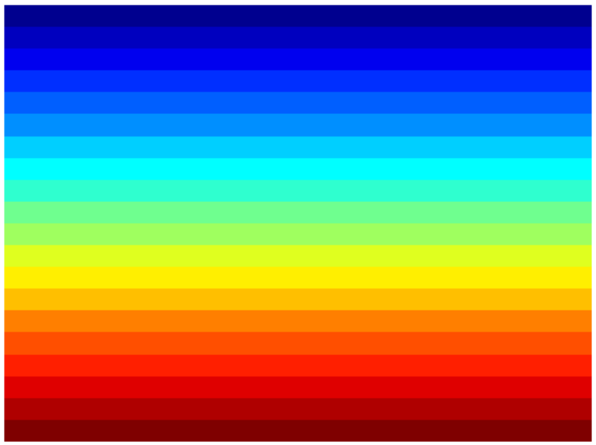
Convolution without Looping using meshgrid

```
[x,y] = meshgrid(1:nc,1:nr);

y_up = y-1;
y_down = y+1;
```

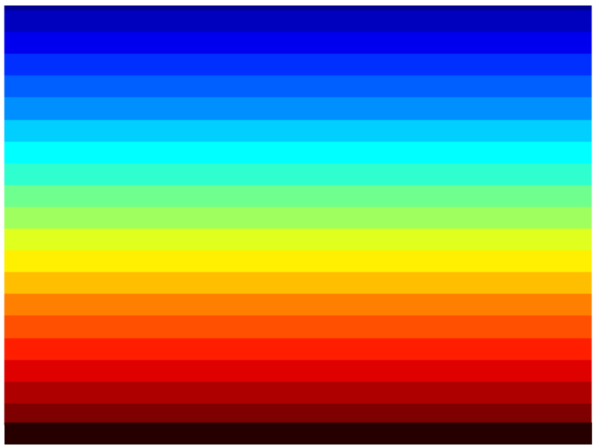
y_up



y



y_down



y_up =

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2

y =

1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

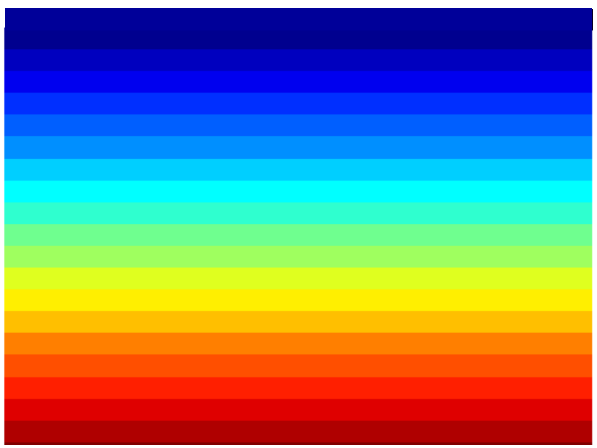
y_down =

2	2	2	2	2
3	3	3	3	3
4	4	4	4	4

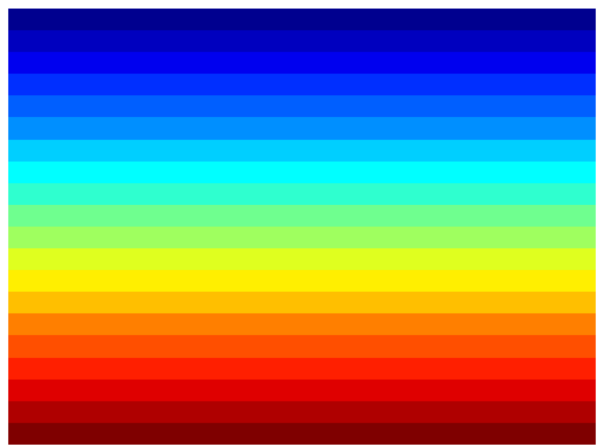
Convolution without Looping using meshgrid

```
y_up = y-1;
y_down = y+1;

y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));
```

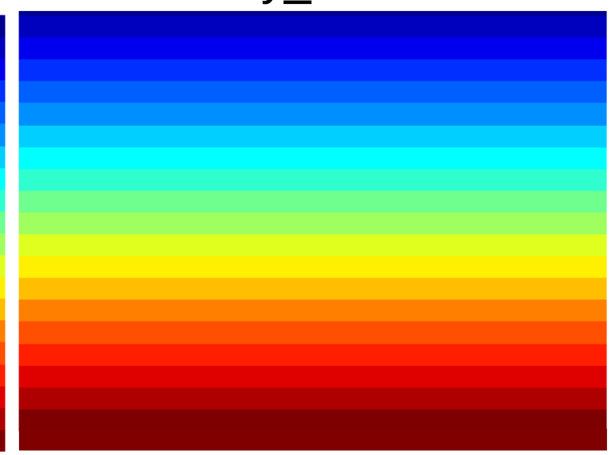
y_up



y



y_down



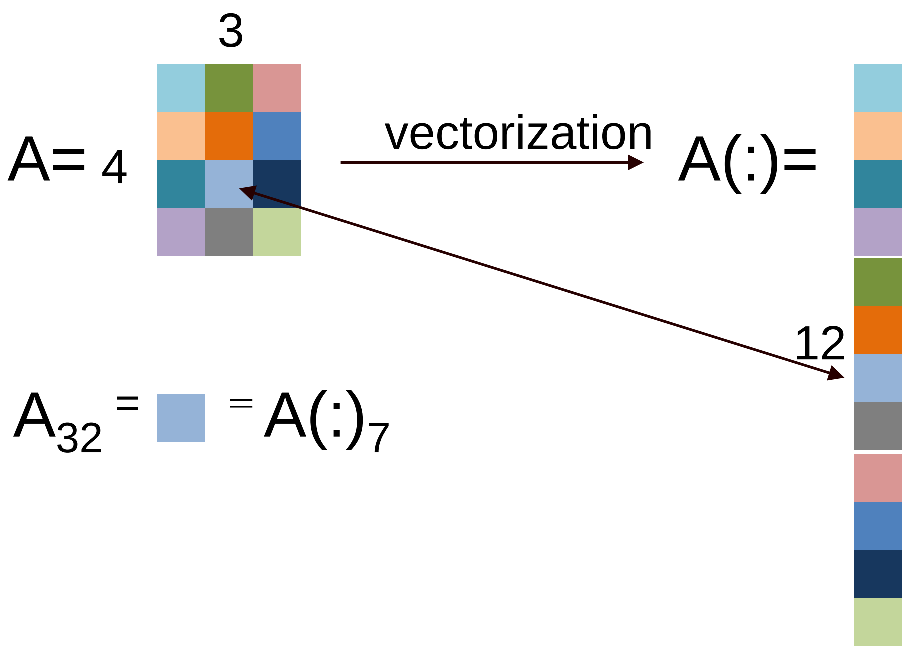
```
y_up =

     1     1     1     1     1
     1     1     1     1     1
     2     2     2     2     2
```

```
y =

     1     1     1     1     1
     2     2     2     2     2
     3     3     3     3     3
```

```
y_down =

     2     2     2     2     2
     3     3     3     3     3
     3     3     3     3     3
```

Convolution without Looping using meshgrid

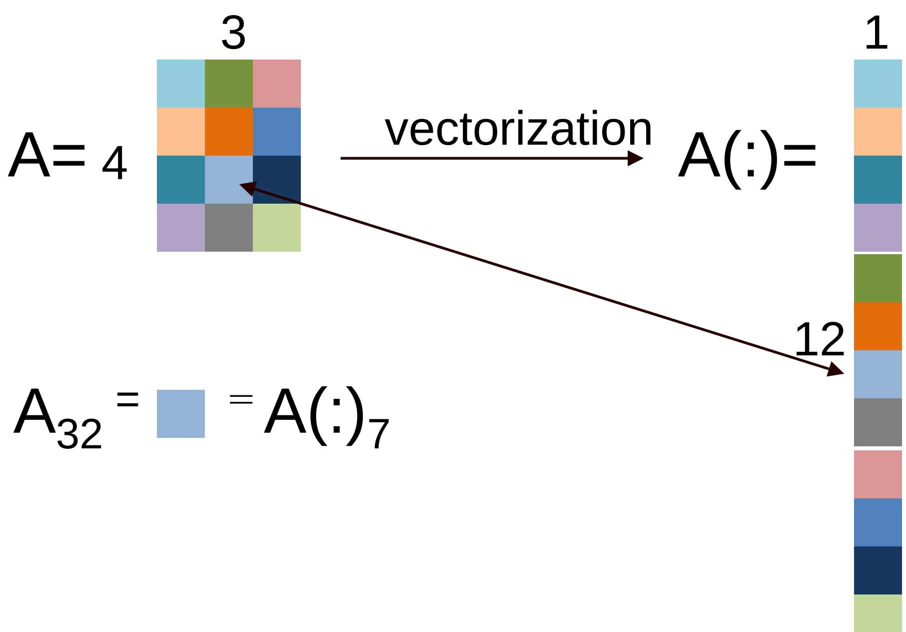
```
y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));

linear_index = sub2ind([n_row, n_col], row_subscript, col_subscript)
```



7 = sub2ind([4 3], 3,2)

Convolution without Looping using meshgrid

```
y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));

linear_index = sub2ind([n_row, n_col], row_subscript, col_subscript)
```

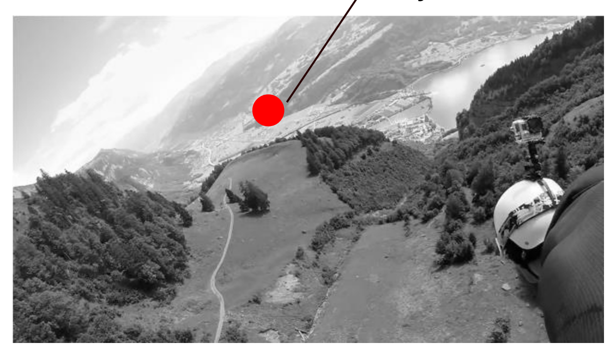


7 = sub2ind([4 3], 3,2)

ind_up = sub2ind([nr,nc],y_up(:),x(:))

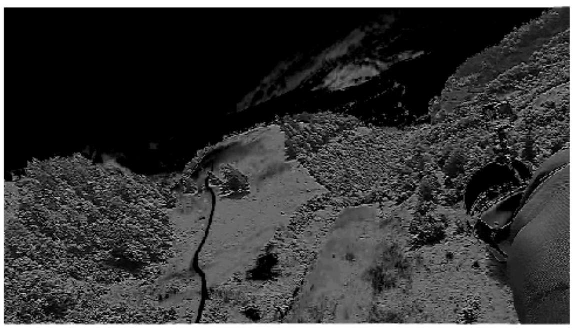
Operation on vectors

$Jb_{xy}=Jb_{idx}$



Convolution without Looping using meshgrid

```
J_out = 2*Jb(:) - 0.8*Jb(ind_up) - 0.8*Jb(ind_down);
J_out = reshape(J_out, nr, nc);

figure; imagesc(J_out);colormap(gray)
```



=

J_out

2  -0.8  -0.8 

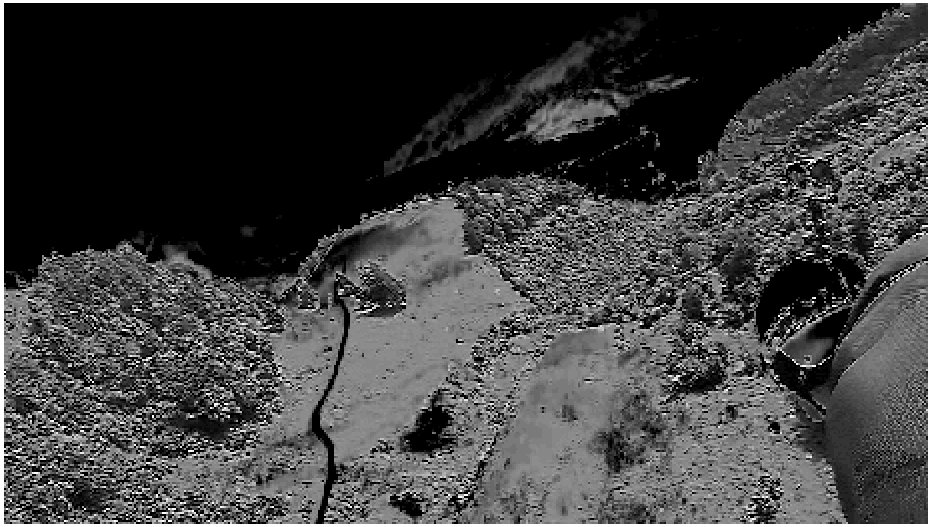
Jb

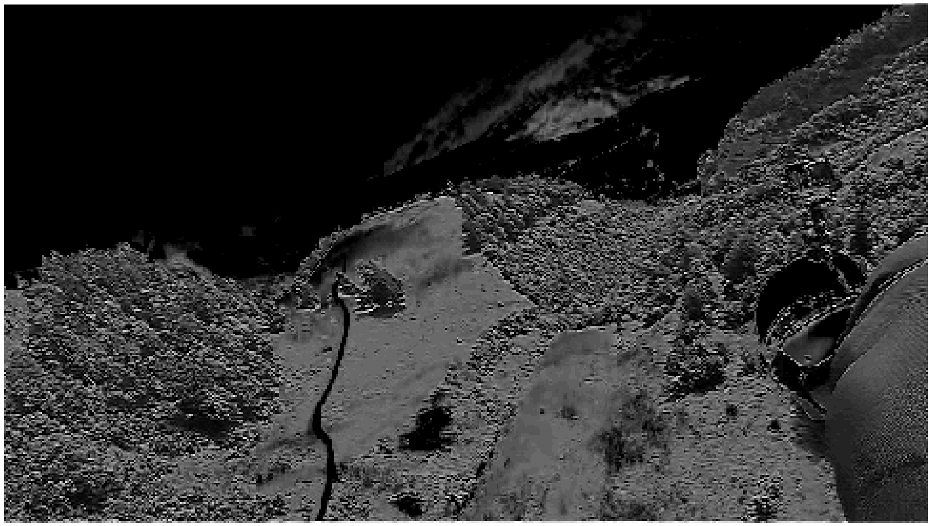
Jb(ind_up)

Jb(ind_down)

With loop

Without loop





Computation time: 0.050154 sec

Computation time: 0.024047 sec

Canny Edge Detection

$$B(i,j) = \begin{cases} 1 & \text{if } I(i,j) \text{ is edge} \\ 0 & \text{if } I(i,j) \text{ is not edge} \end{cases}$$

Objective: to localize edges given an image.

Binary image indicating edge pixels

Original image, I

Edge map image, B

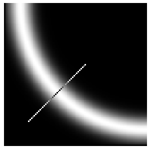
Canny Edge Detection

1. Filter image by derivatives of Gaussian

2. Compute magnitude of gradient

3. Compute edge orientation

4. Detect local maximum



5. Edge linking

CIS581, Computer Vision
Project 1, Image Edge Detection
Written part due on Sept. 15, 3:00pm
Programming part due on Sept. 24, 3:00pm

Overview

This project focuses on understanding image convolution and edge detection. Both the written part and the programming part are to be done individually. All matlab functions should follow the names and arguments stated in the problems in order for them to run properly with the grading script. A test script will be provided shortly that will call your functions to ensure they will run inside the grading script.

To submit the assignment, submit a zip file containing all your codes and pdf files (for written part only) via Canvas.

Recall the definition of convolution, $J = I \otimes g$ in 1D as

$$J(i) = \sum_k I(i-k)g(k), \tag{1}$$

and in 2D as

$$J(i,j) = \sum_{k,l} I(i-k, j-l)g(k,l). \tag{2}$$

Typically, I is an image, g is a filter, and J is the filter response of I under g.

Programming Part

5. *Edge detection*

   Write a Matlab function E = cannyEdge(I)

   Compute the Canny edges. Canny edges are defined as local maxima of the image gradient. Following the steps described in the lecture notes:

   (a) compute gradient magnitude and orientation,

   (b) seek local maximum in the gradient orientation,

   (c) continue search in the edge orientation of detected edge point.

   We provide the framework of the program. You need to follow the framework and complete the functions *findDerivatives*, *nonMaxSup* and *edgeLink*. We also

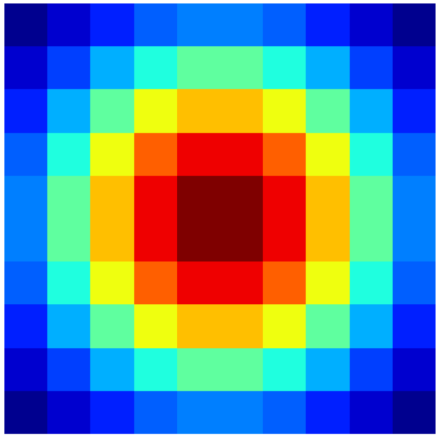
1) Compute Image Gradient

the first order derivative of Image I in x, and in y direction

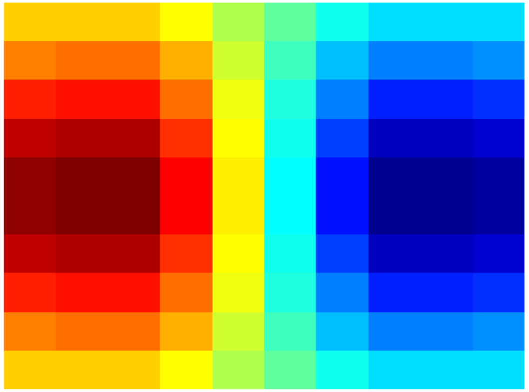
Edge Detection, Step 1,
Filter out noise and compute derivative:

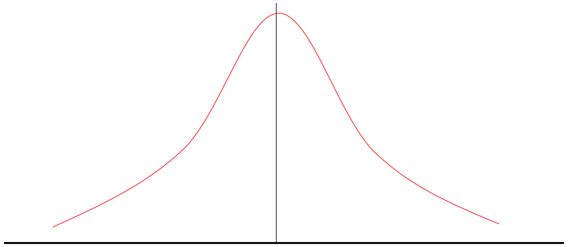
$$\left(\frac{\delta}{\delta x} \otimes G\right)$$

Gradient of Gaussian

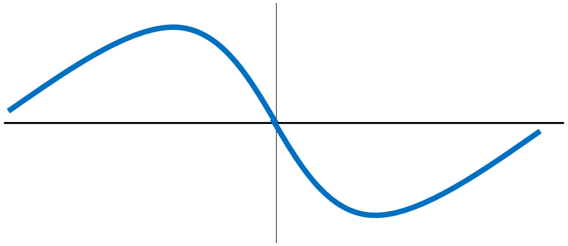


$$\frac{\delta}{\delta x} \longrightarrow$$





$$\frac{\delta}{\delta x} \longrightarrow$$

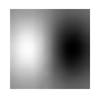


Edge Detection, Step 1,
Filter out noise and compute derivative:

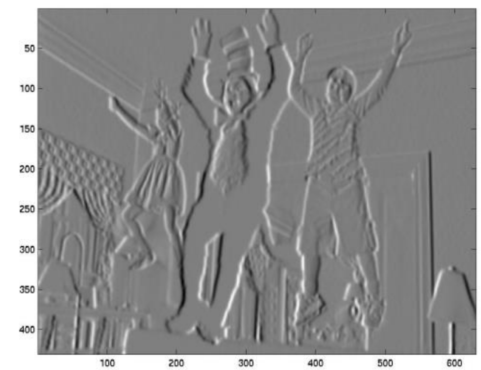
Image

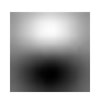
$$\otimes\left(\frac{\delta}{\delta x}\otimes G\right)$$

Smoothed Derivative

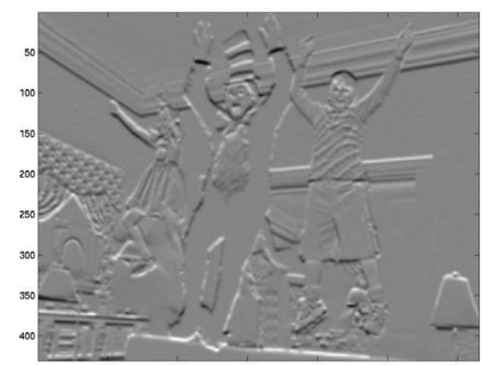


I_x





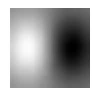
I_y



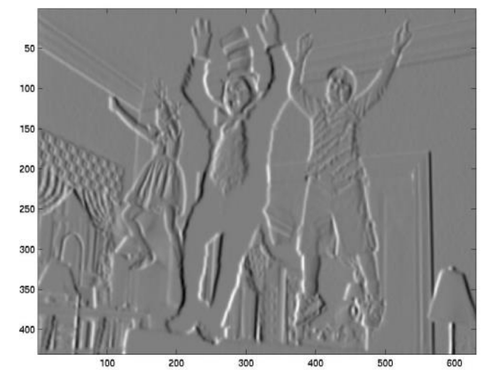
Edge Detection, Step 1,
Filter out noise and compute derivative:

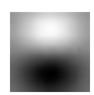
In matlab:

```
>> [dx,dy] = gradient(G); % G is a 2D gaussain
>> Ix = conv2(I,dx,'same'); Iy = conv2(I,dy,'same');
```

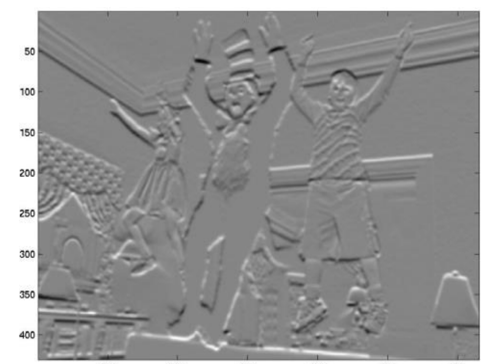


I_x





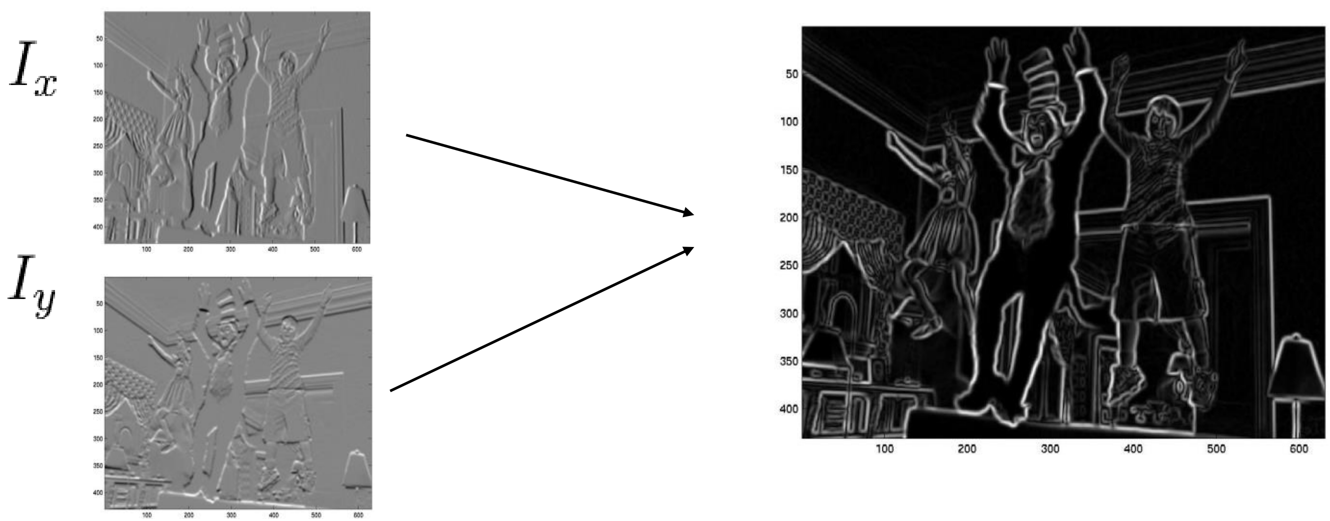
I_y



Edge Detection: Step 2
Compute the magnitude of the gradient

In Matlab:

```
>> Im = sqrt(Ix.*Ix + Iy.*Iy);
```



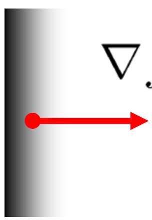
We know roughly where are the edges, but we need their precise location.

Finding the orientation of the edge

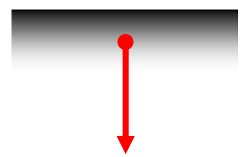
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

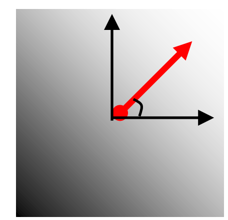
- The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

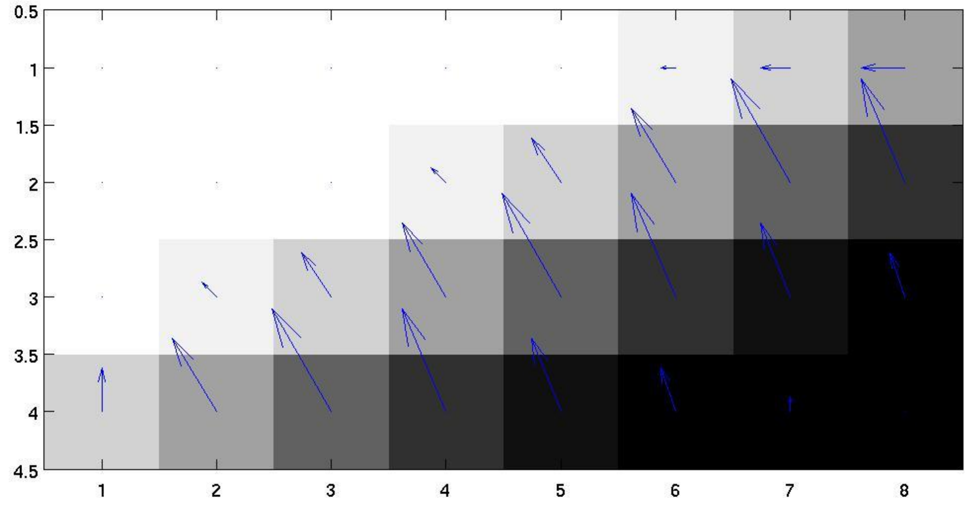
- The image gradient direction is given by:

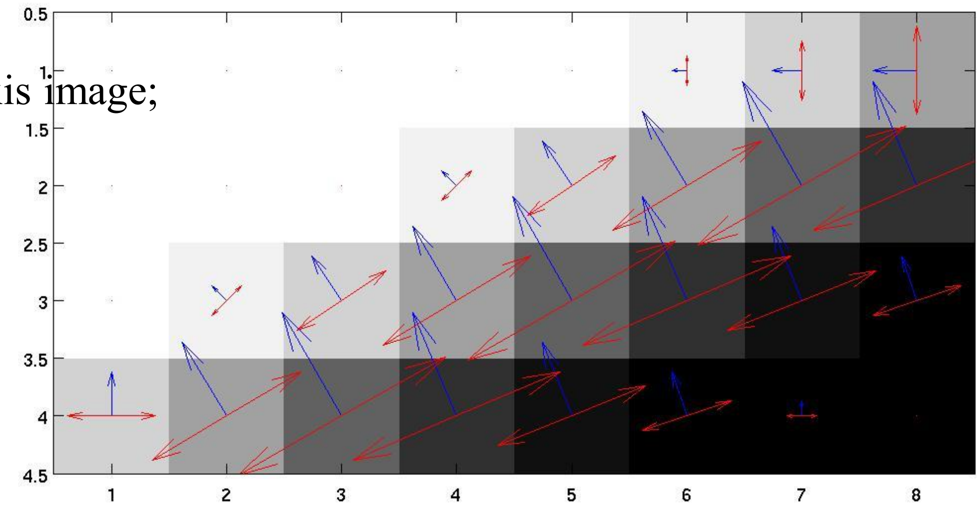
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

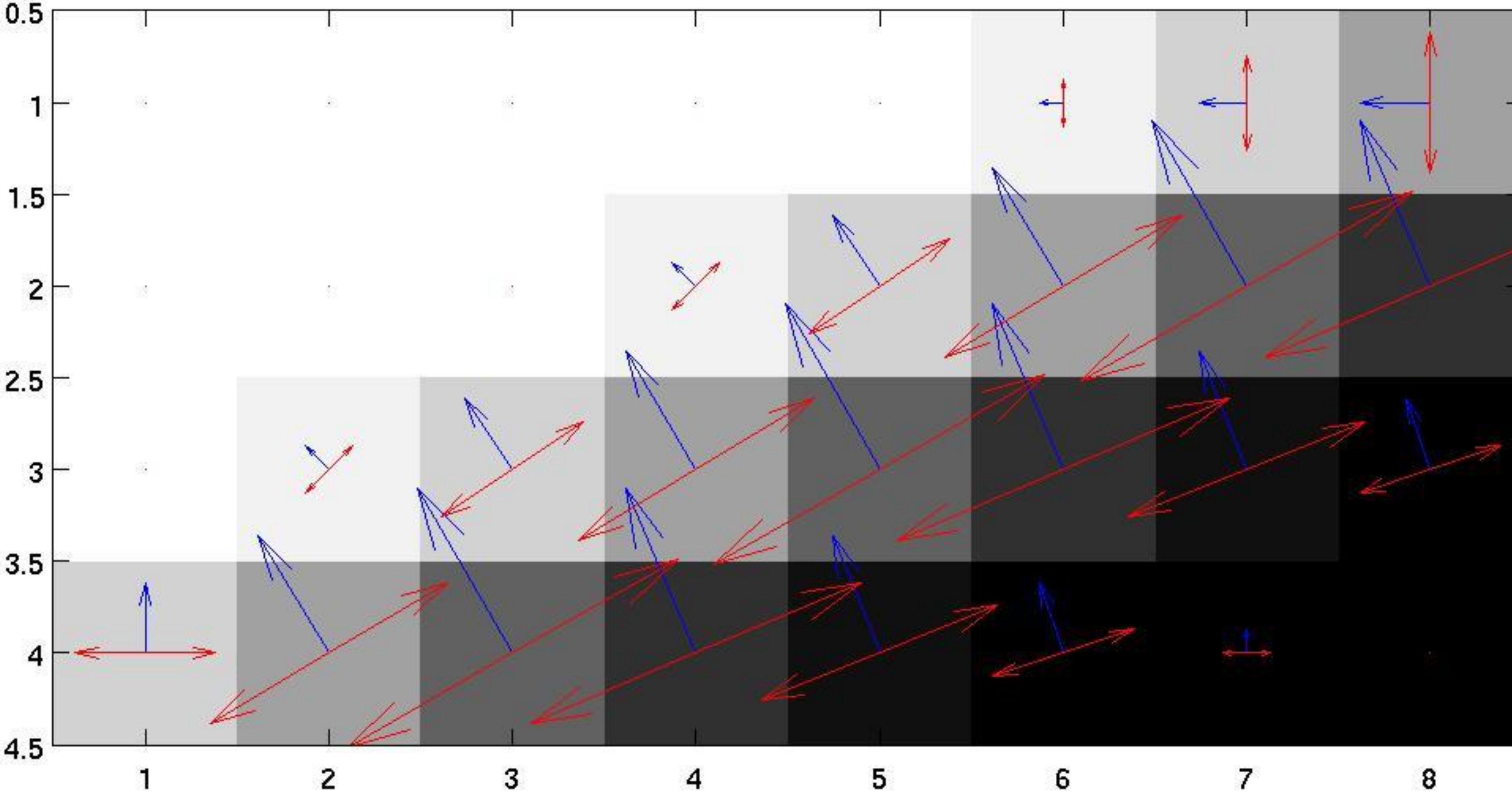
  – how does this relate to the direction of the edge?

$$\theta_{edge} = tan^{-1}\left(-\frac{\delta f}{\delta x} / \frac{\delta f}{\delta y}\right)$$

```
%% define image gradient operator
dy = [1;-1];
dx = [1,-1];

%% compute image gradient in x and y
AA_y = conv2(AA,dy,'same');
AA_x = conv2(AA,dx,'same');

Jy = AA_y(1:end-2,2:end-1);
Jy(1,:) = 0;

Jx = AA_x(2:end-1,1:end-2);
Jx(:,1) = 0;

%% display the image gradient flow
figure(3);clf;imagesc(J);colormap(gray);axis image;
hold on;
quiver(Jx,Jy);
quiver(-Jy,Jx,'r');
quiver(Jy,-Jx,'r');
```







0.5
1
1.5
2
2.5
3
3.5
4
4.5
1
2
3
4
5
6
7
8

[gx,gy] = gradient(J);
mag = sqrt(gx.*gx+gy.*gy);   imagesc(mag);colorbar

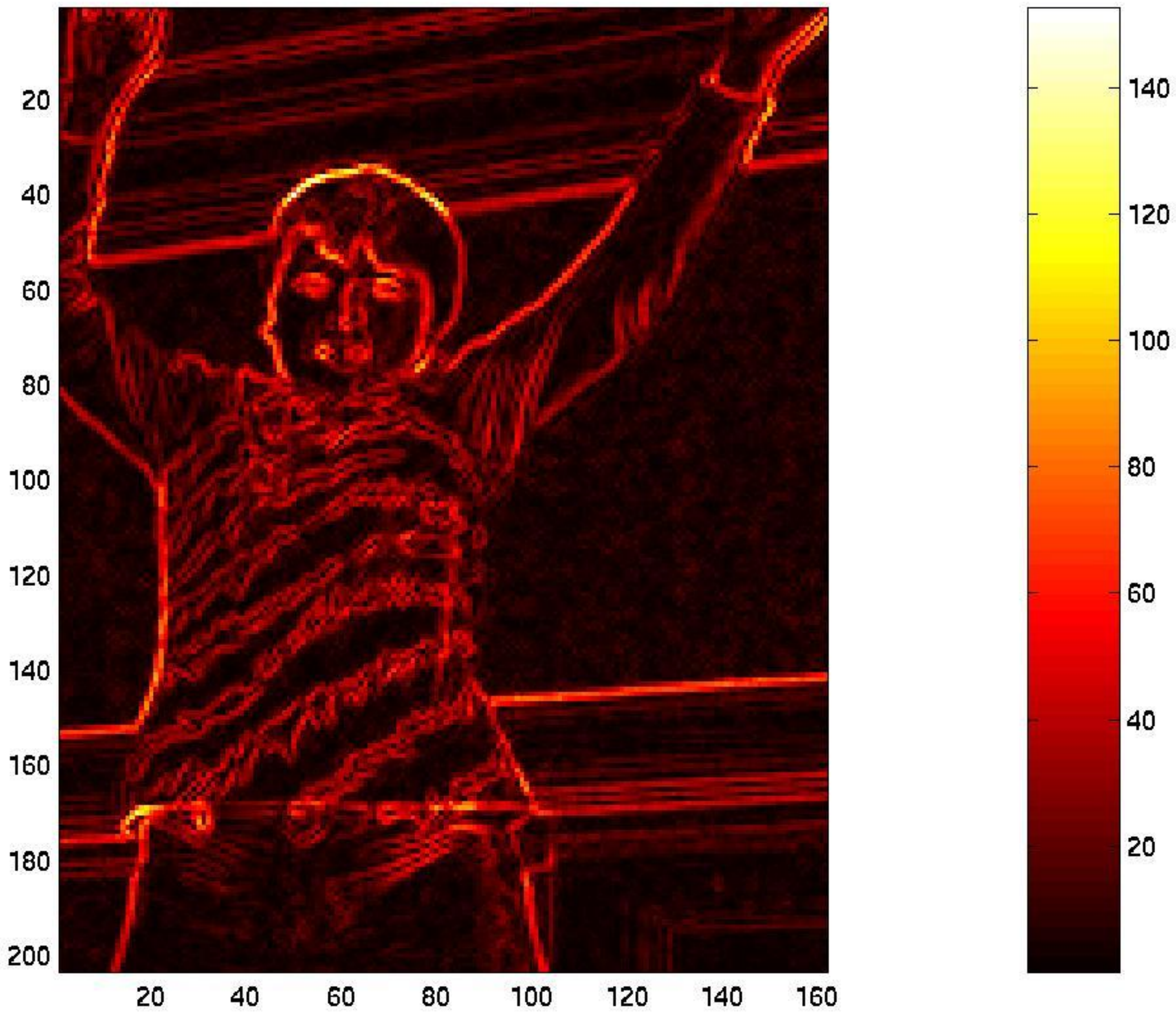
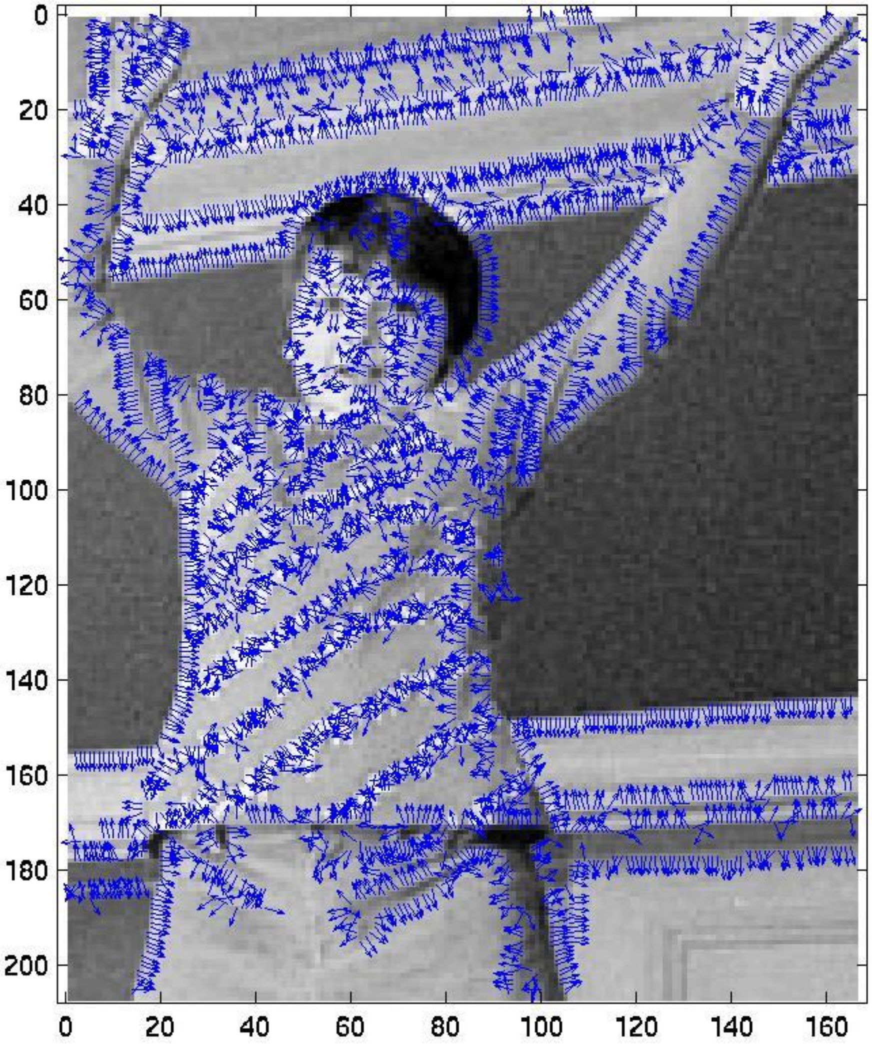
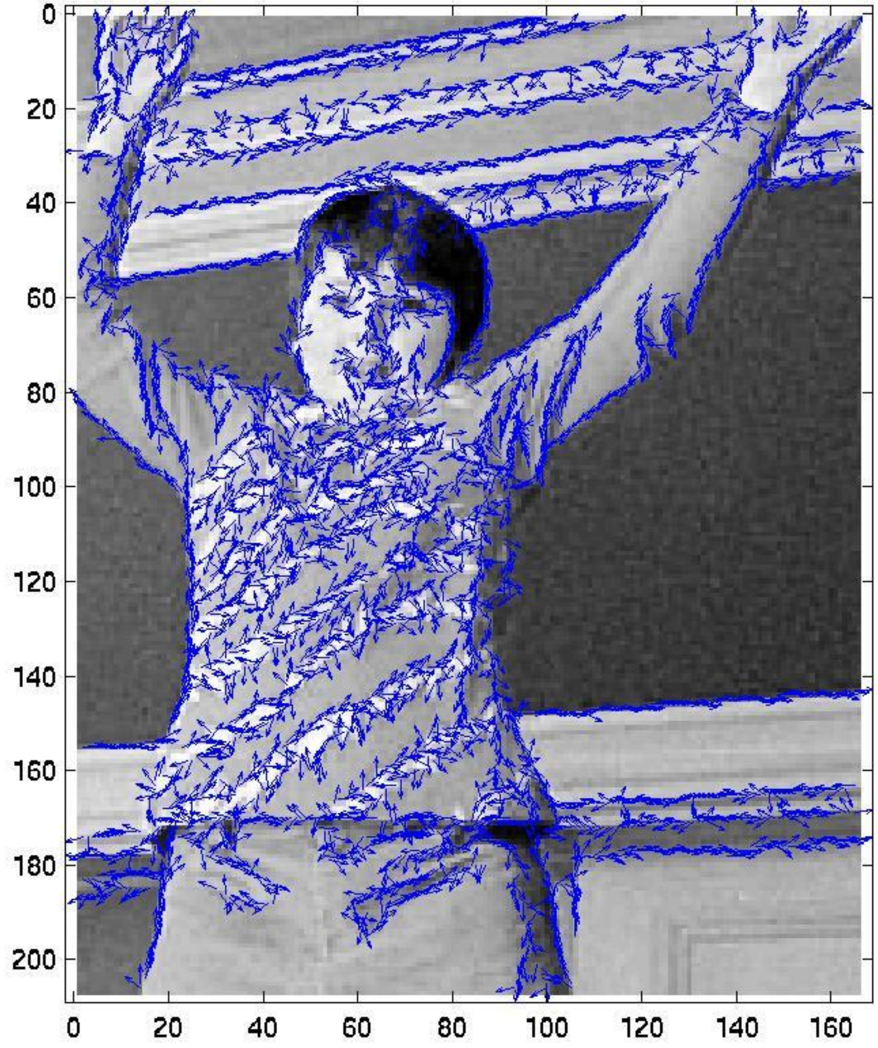


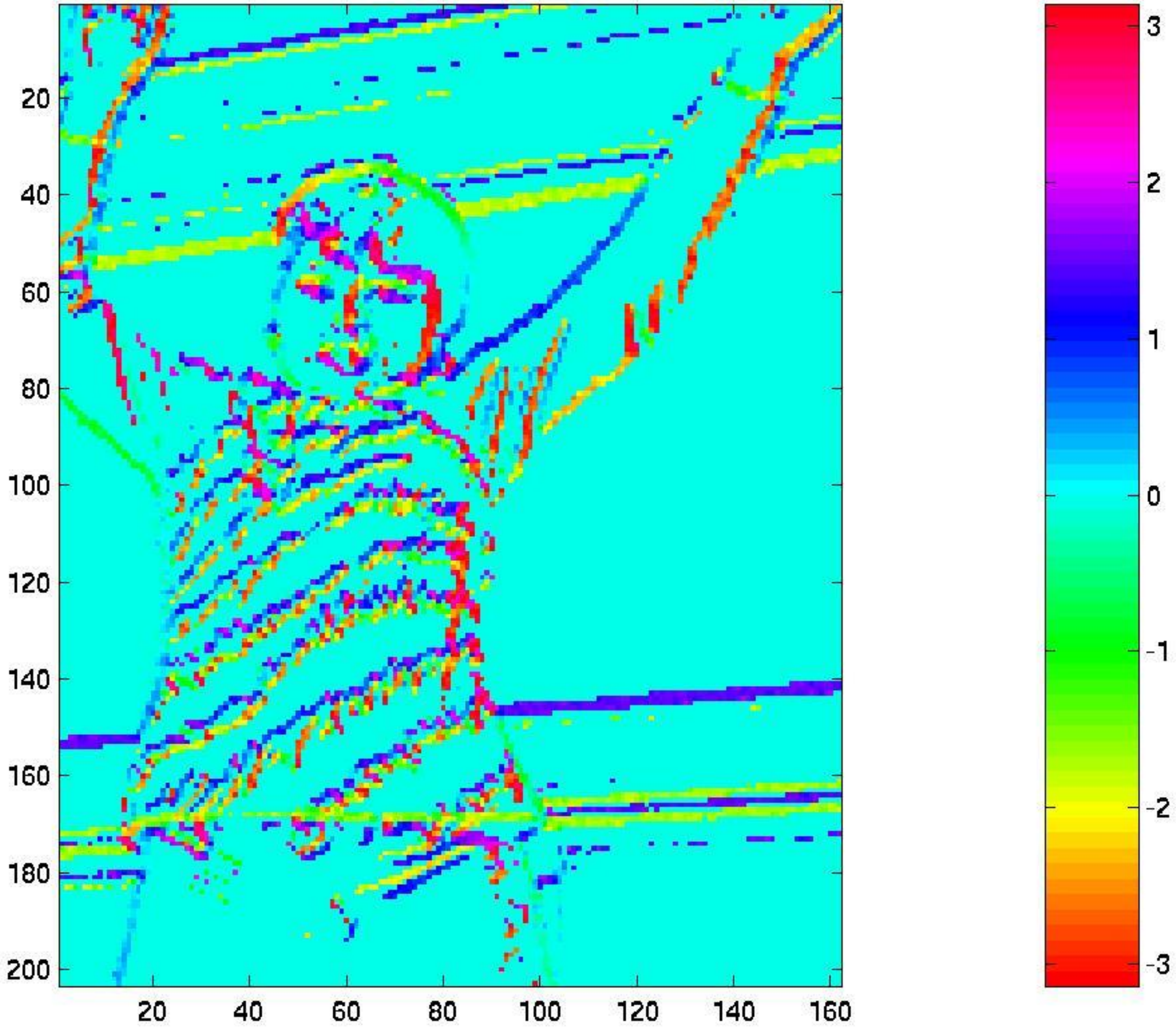
image gradient direction:



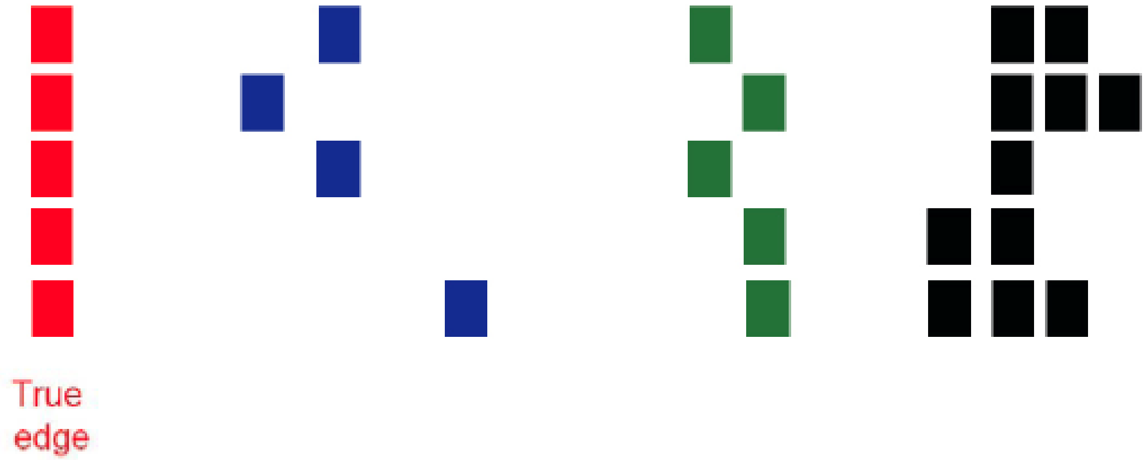
Edge orientation direction:



[gx,gy] = gradient(J);
th = atan2(gy,gx);   % or you can use:[th,mag] = cart2pol(gx,gy);
imagesc(th.*(mag>20));colormap(hsv); colorbar

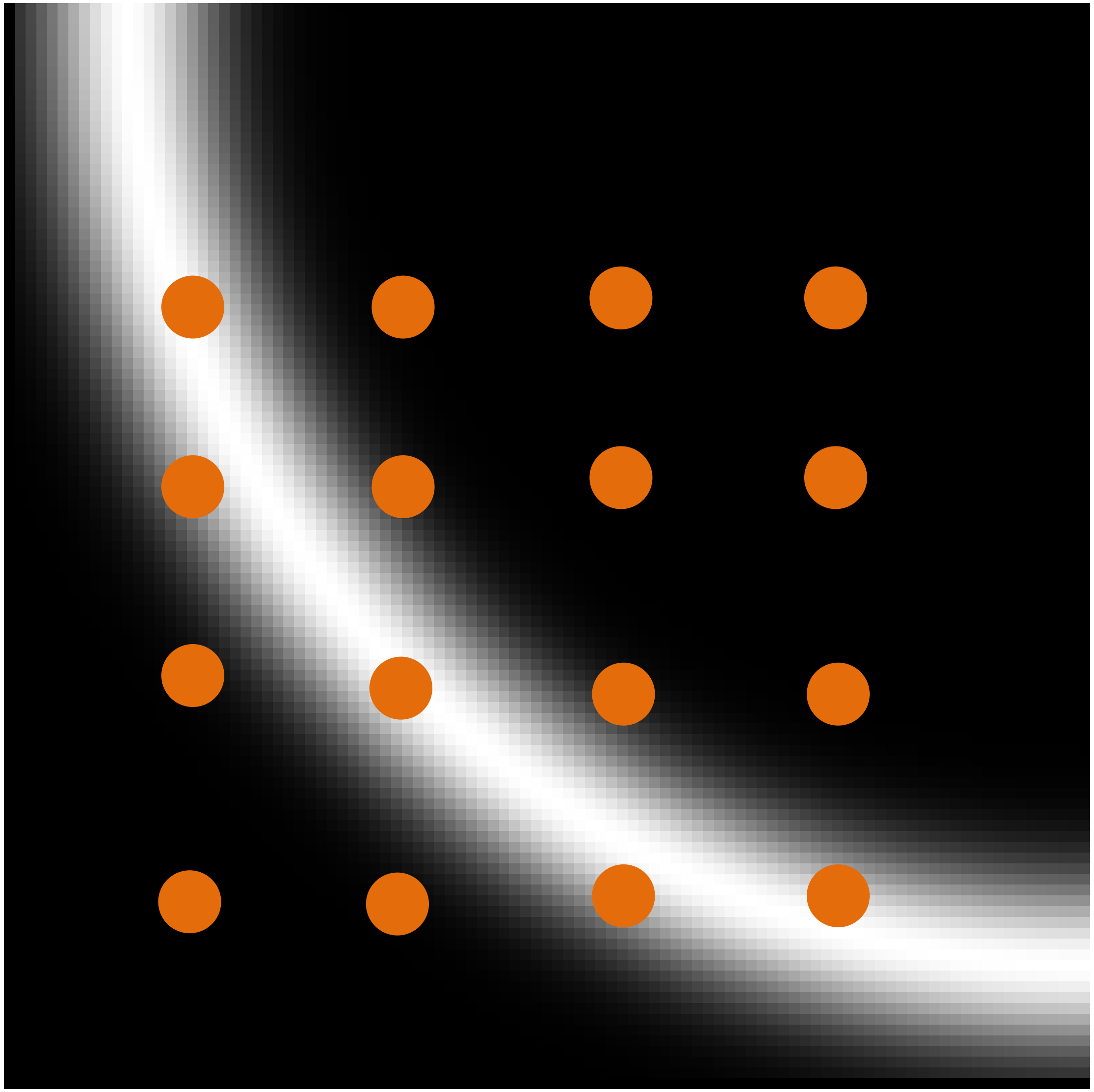


- Criteria for an “optimal” edge detector:
  - **Good detection:** the optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges)
  - **Good localization:** the edges detected must be as close as possible to the true edges
  - **Single response:** the detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge



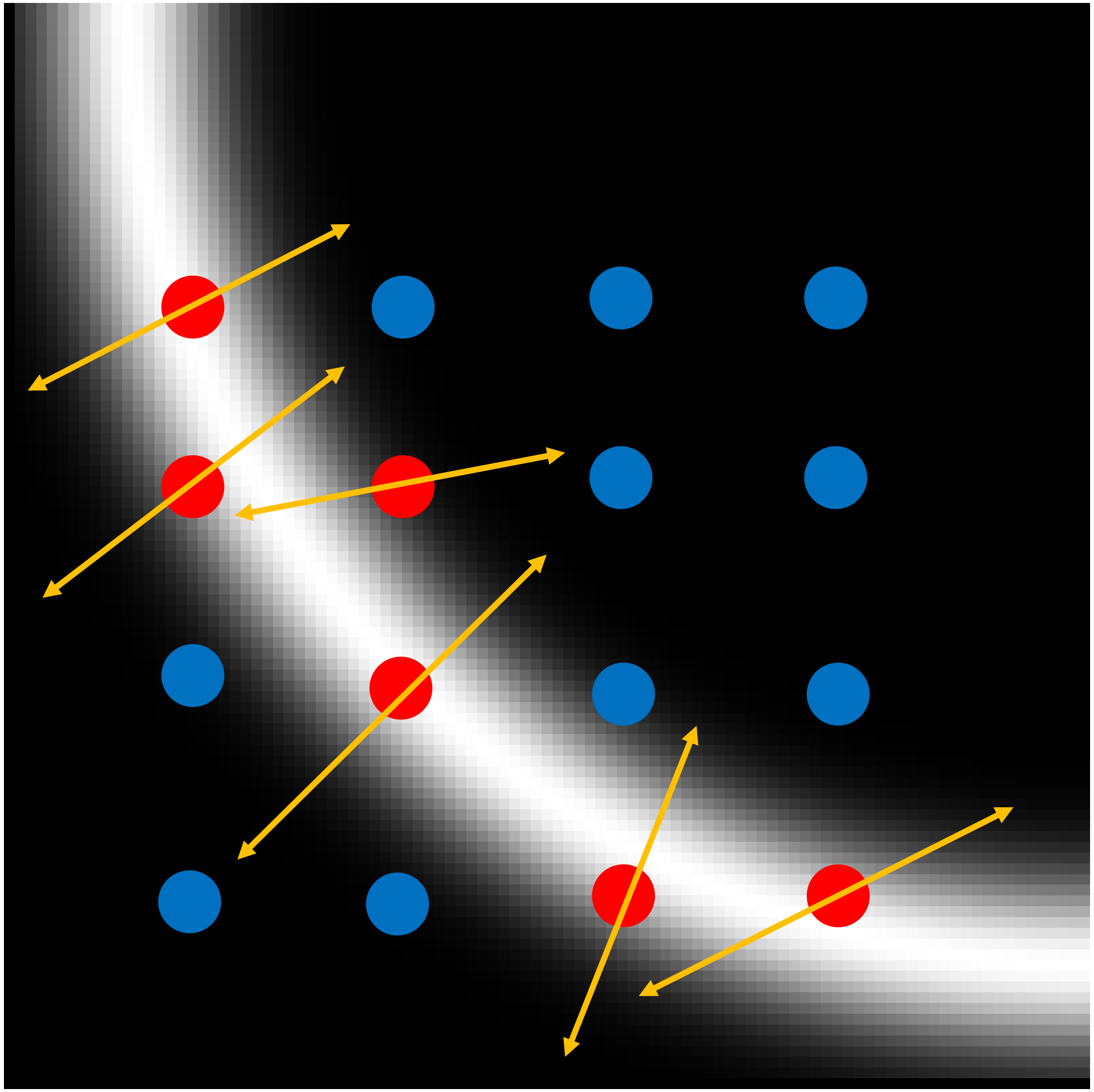
Source: L. Fei-Fei

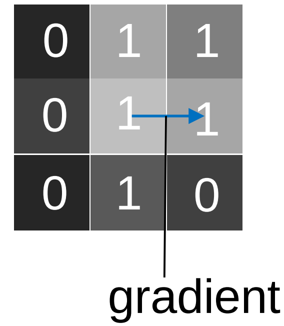
Discretized pixel locations



(Forsyth & Ponce)

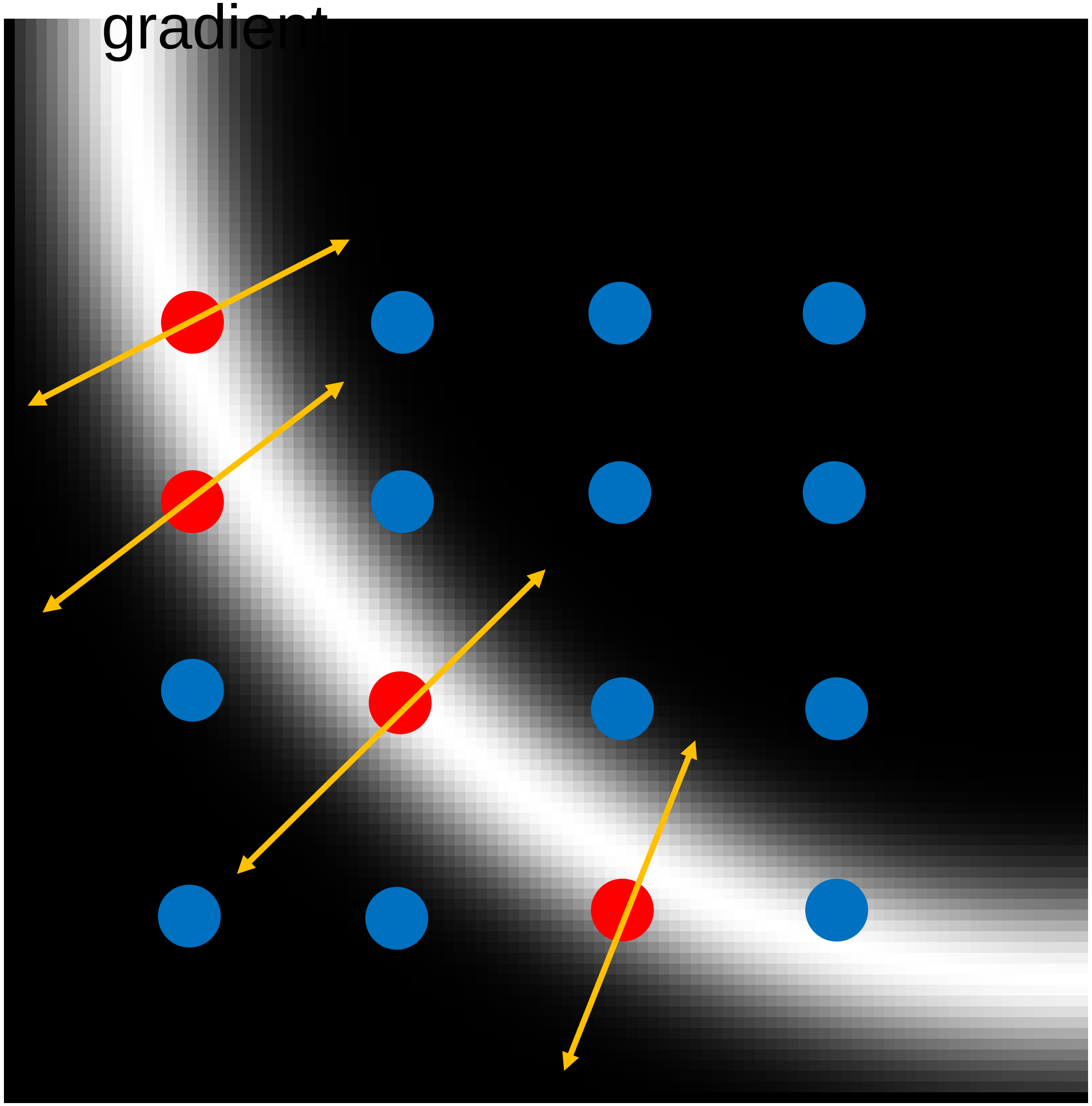
Thesholding

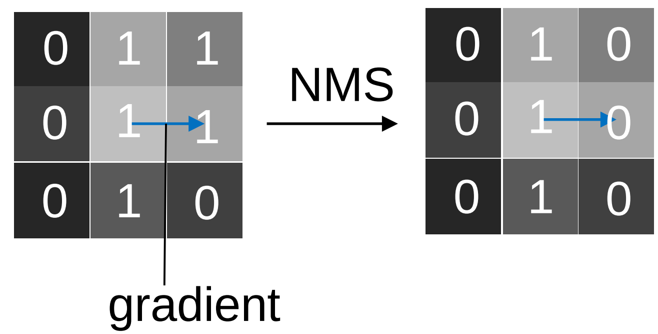




(Forsyth & Ponce)

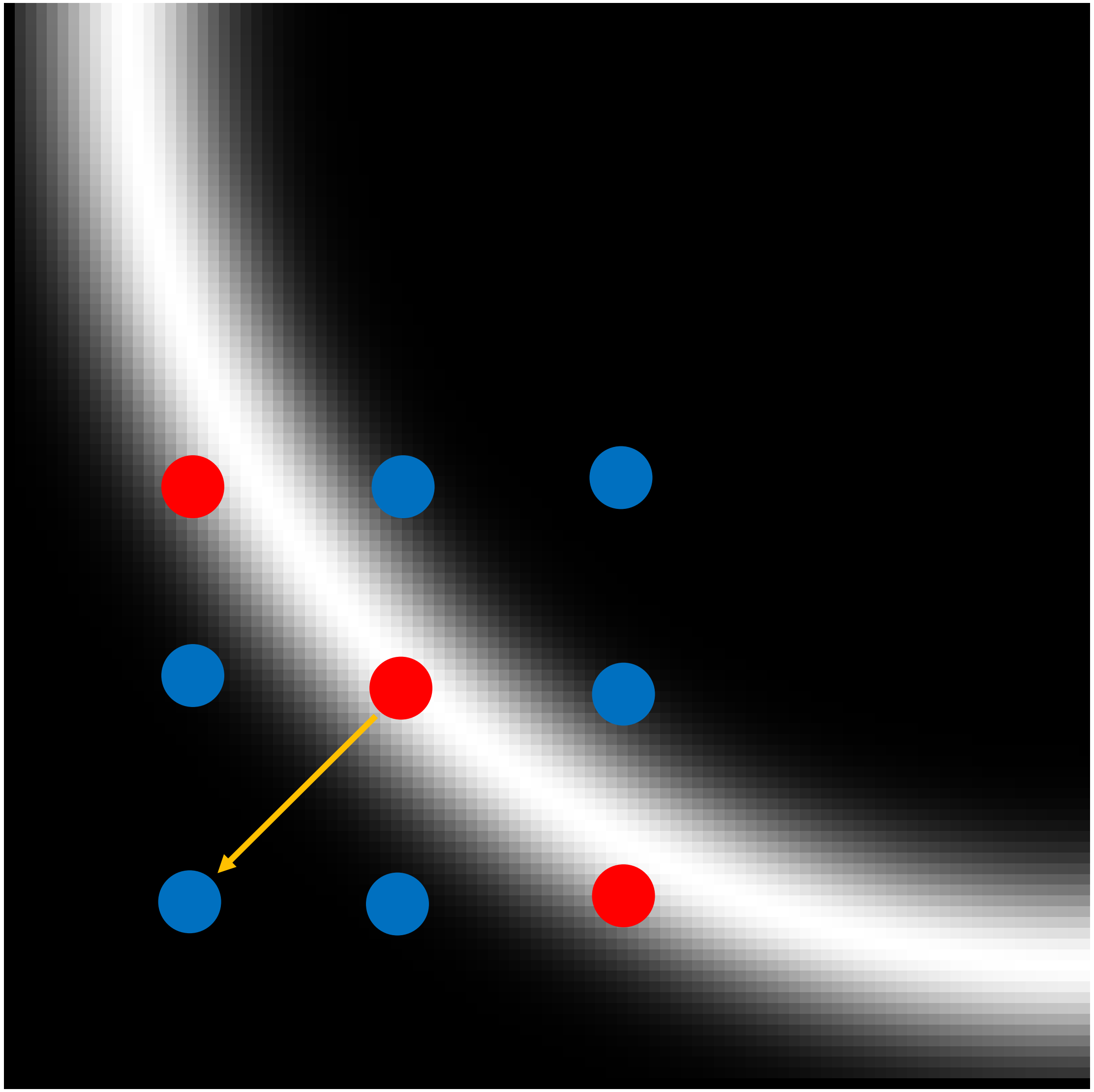
Non-maximum suppression along the line of the gradient

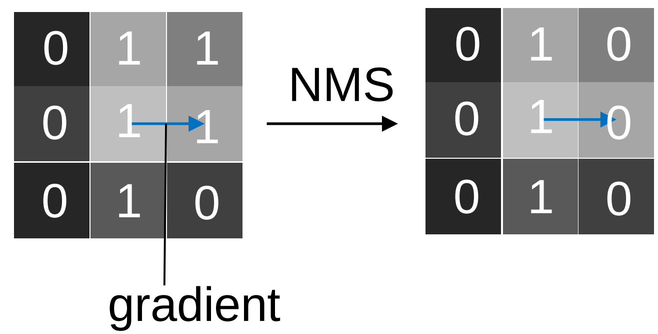




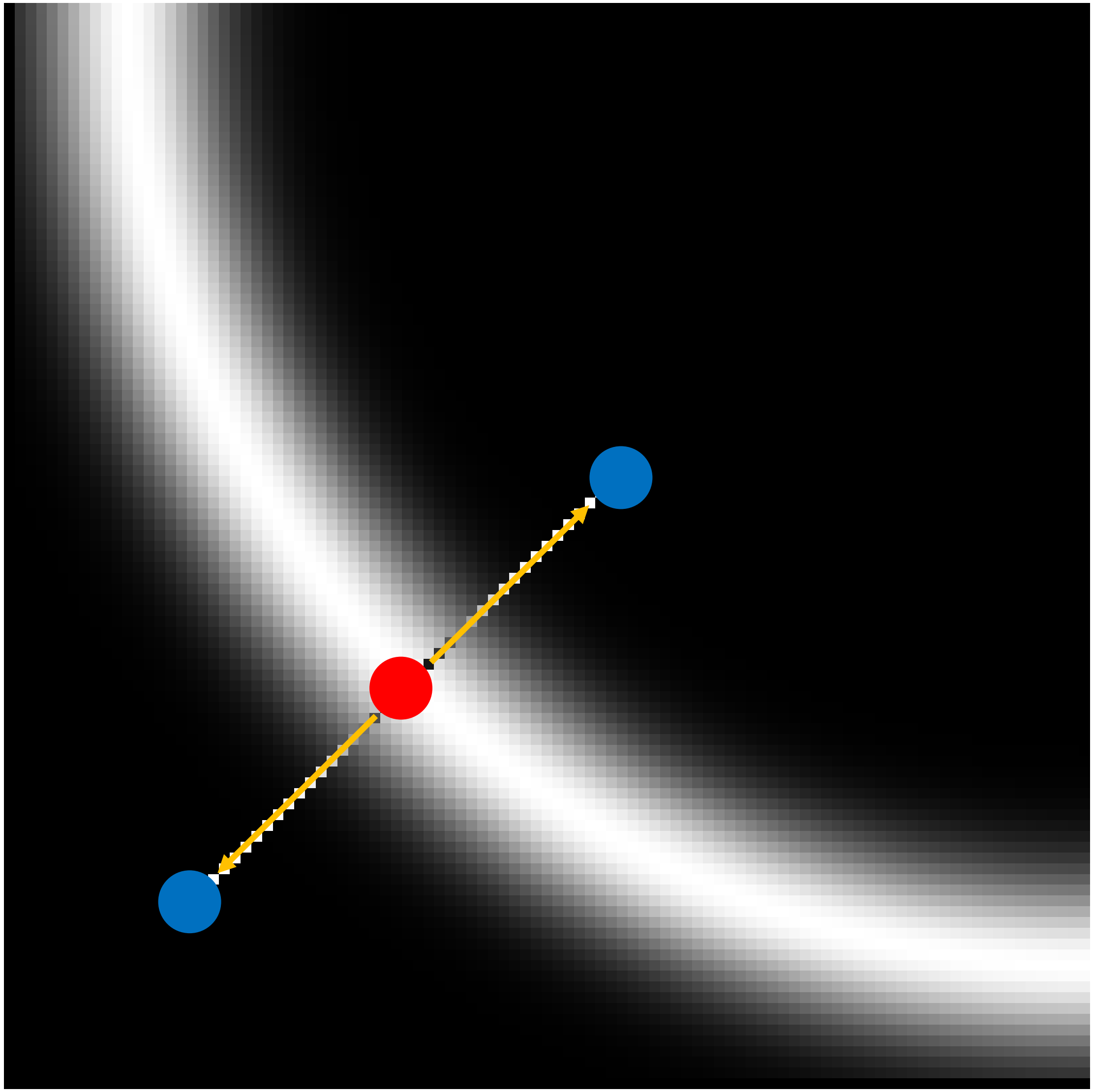
(Forsyth & Ponce)

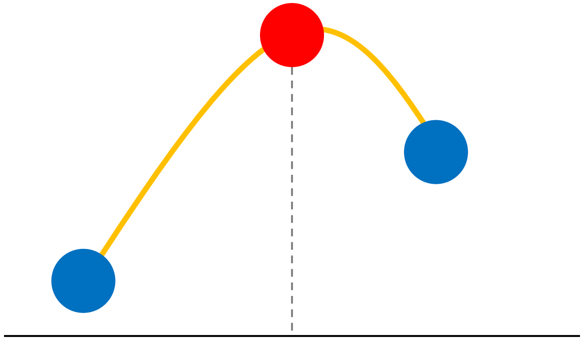
Gradient direction





(Forsyth & Ponce)

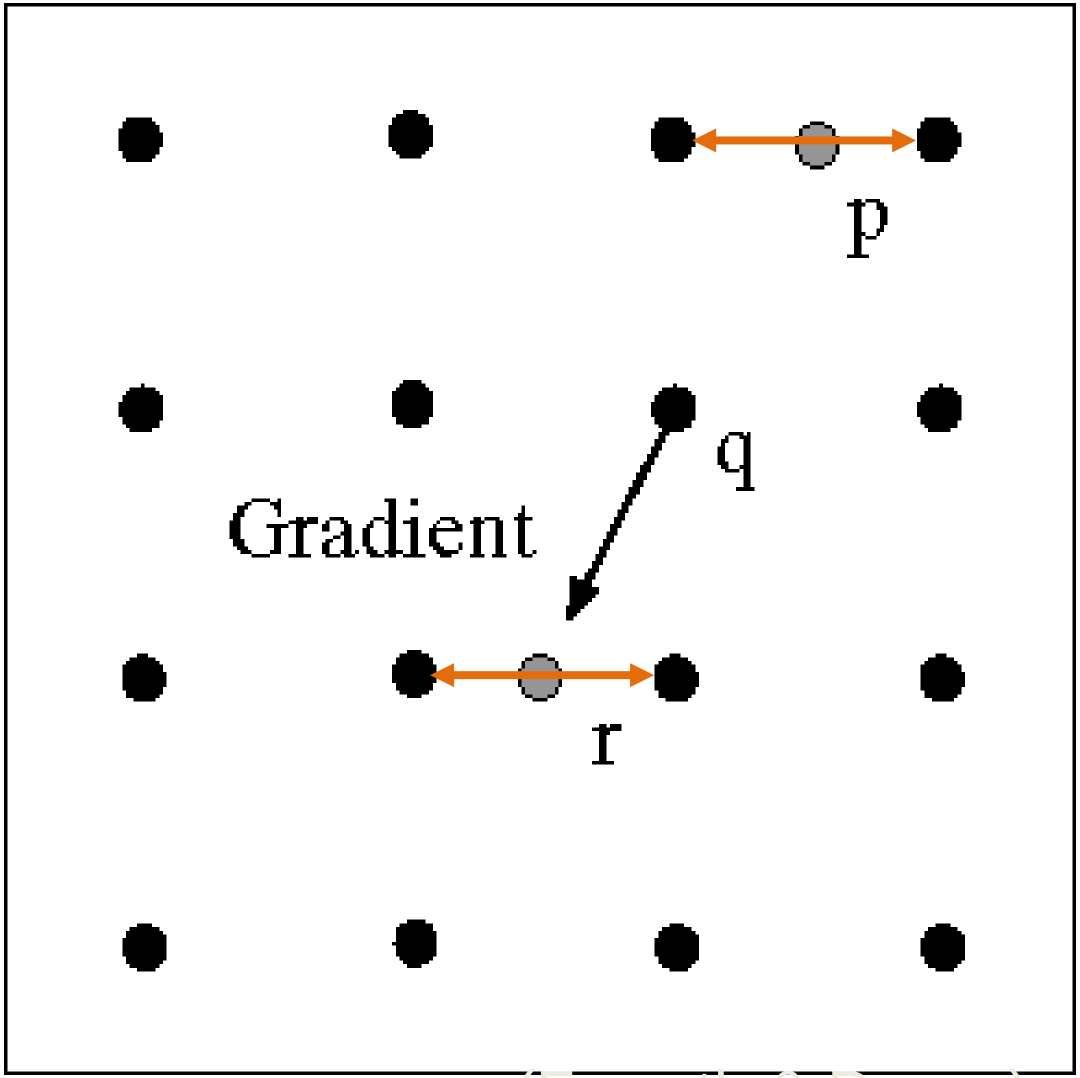




Local maximum

(Forsyth & Ponce)

No intensity values at r and p:
Interpolate these intensities using neighbor pixels.

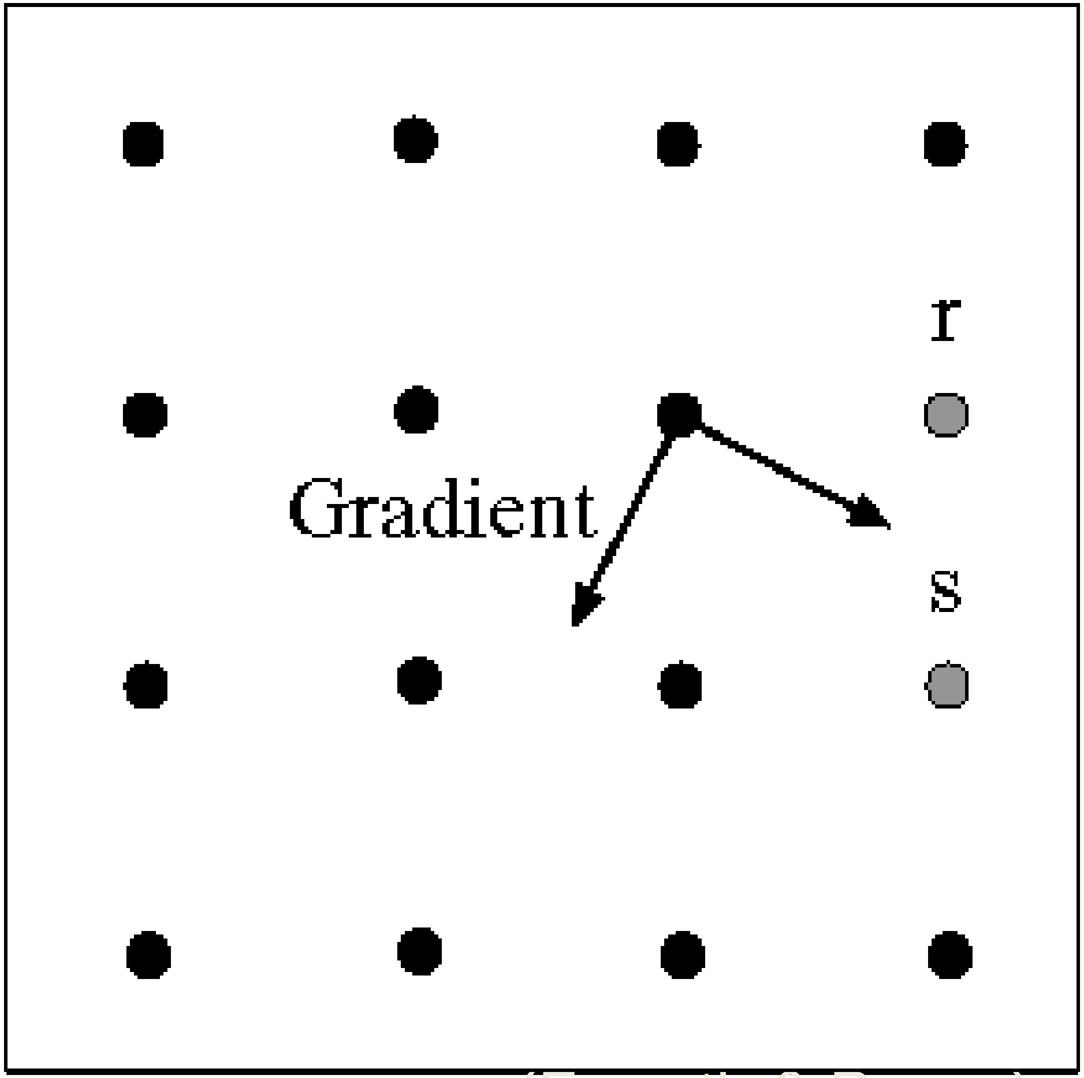


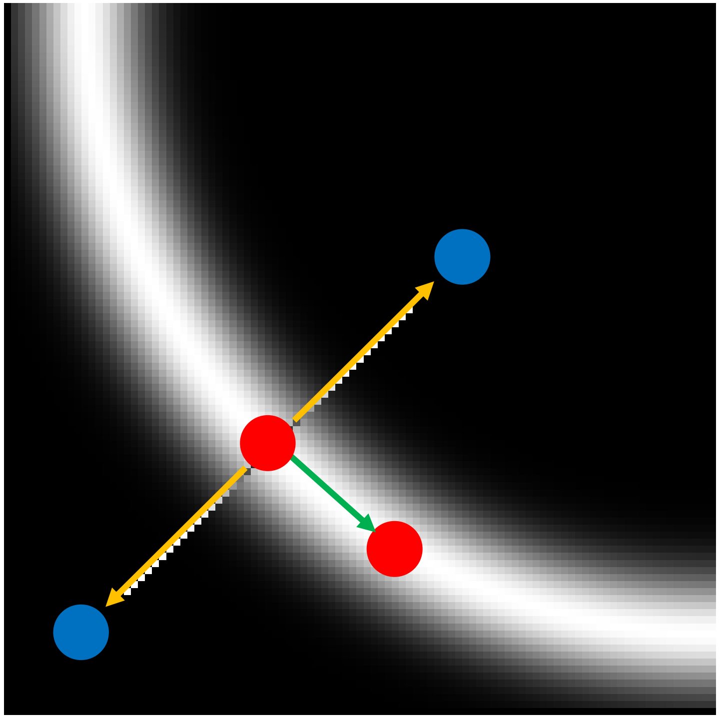
Where is next edge poin

(Forsyth & Ponce)

Where is next edge point?

we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points





(Forsyth & Ponce)

Where is next edge point?

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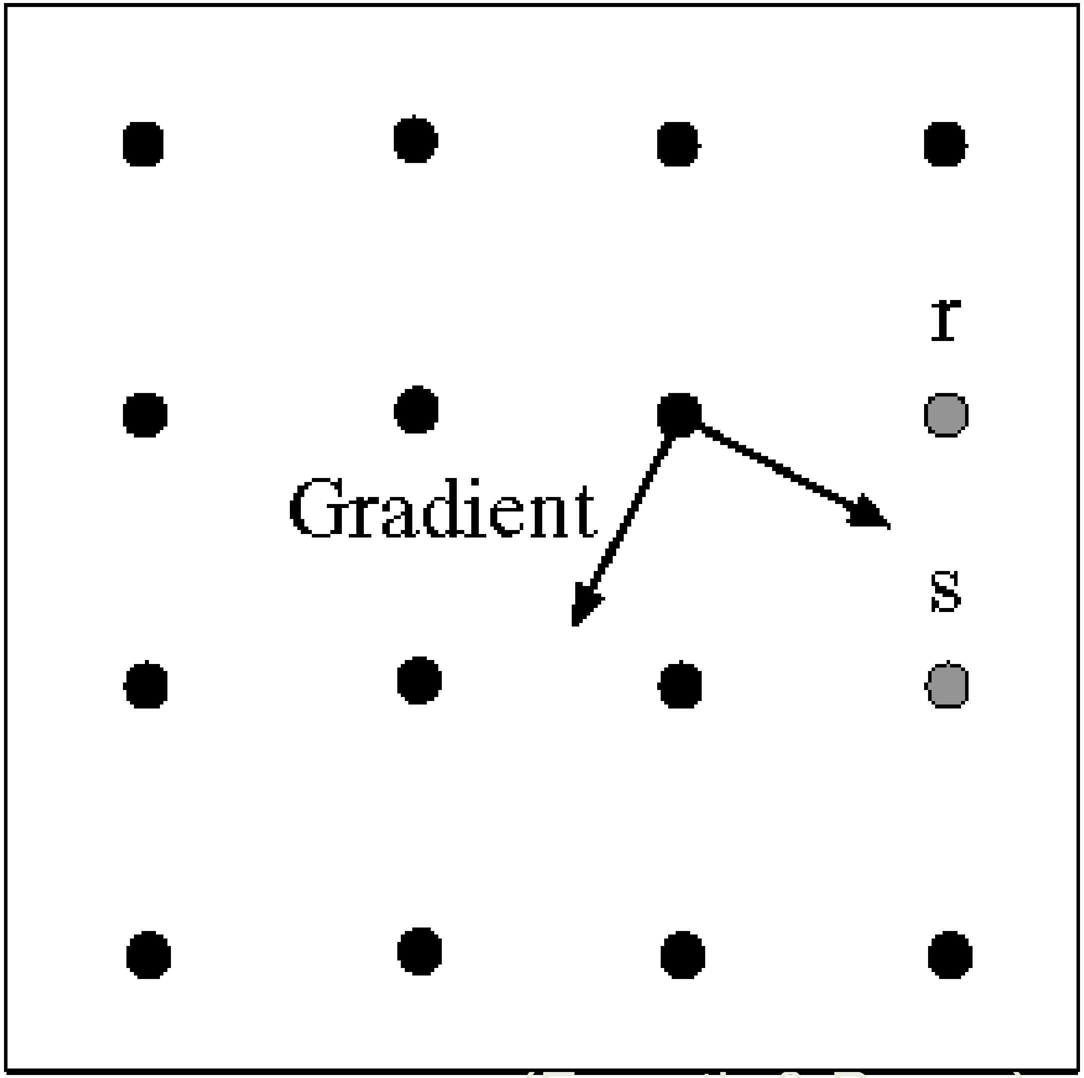
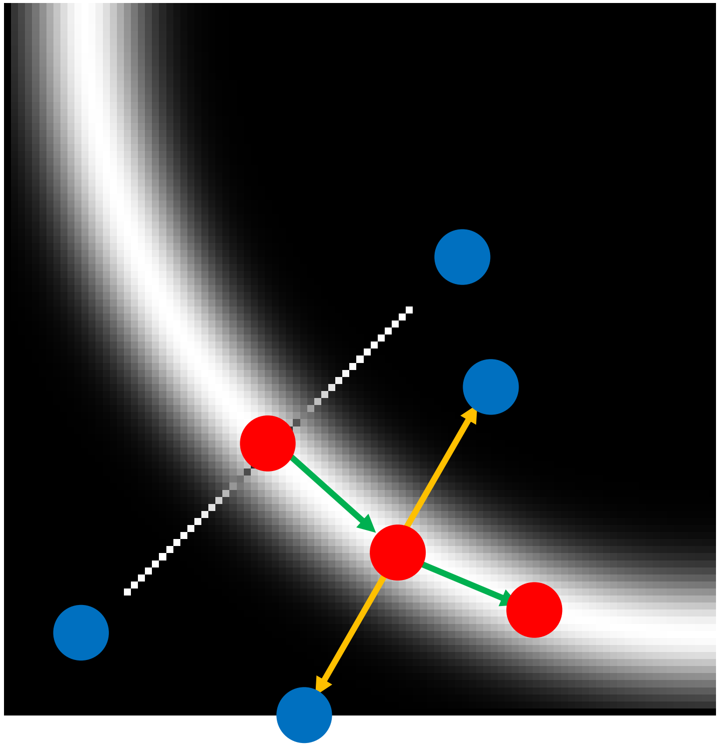
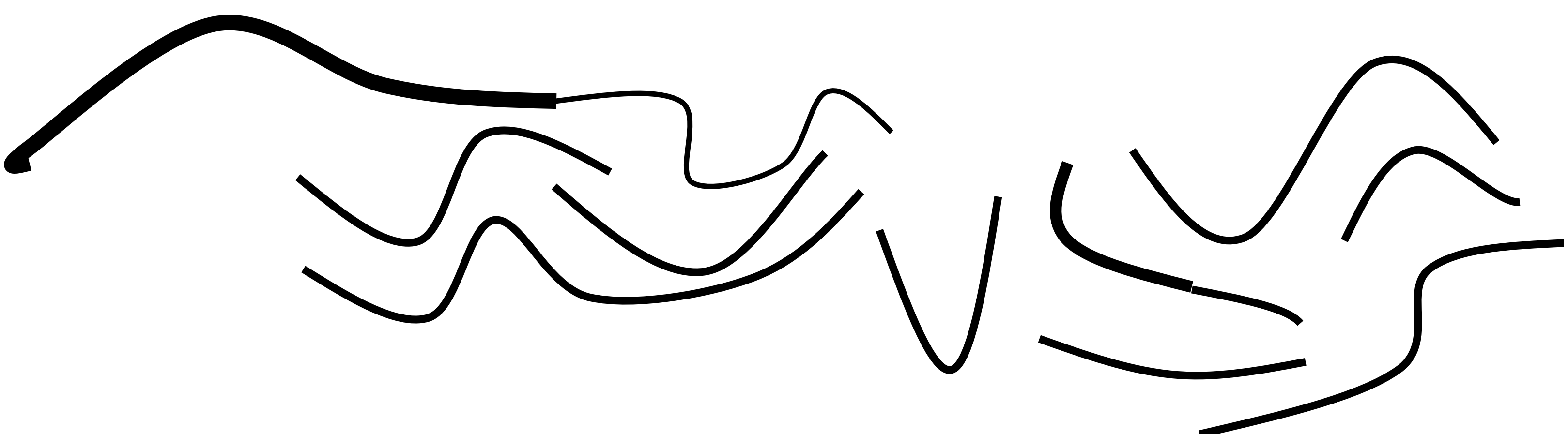




(Forsyth & Ponce)

Edge Linking: Hysteresis

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.



Edge Linking: Hysteresis

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.

threshold_high

0	1	0	0
0	1	0	1
0	0	0	0
0	1	0	0

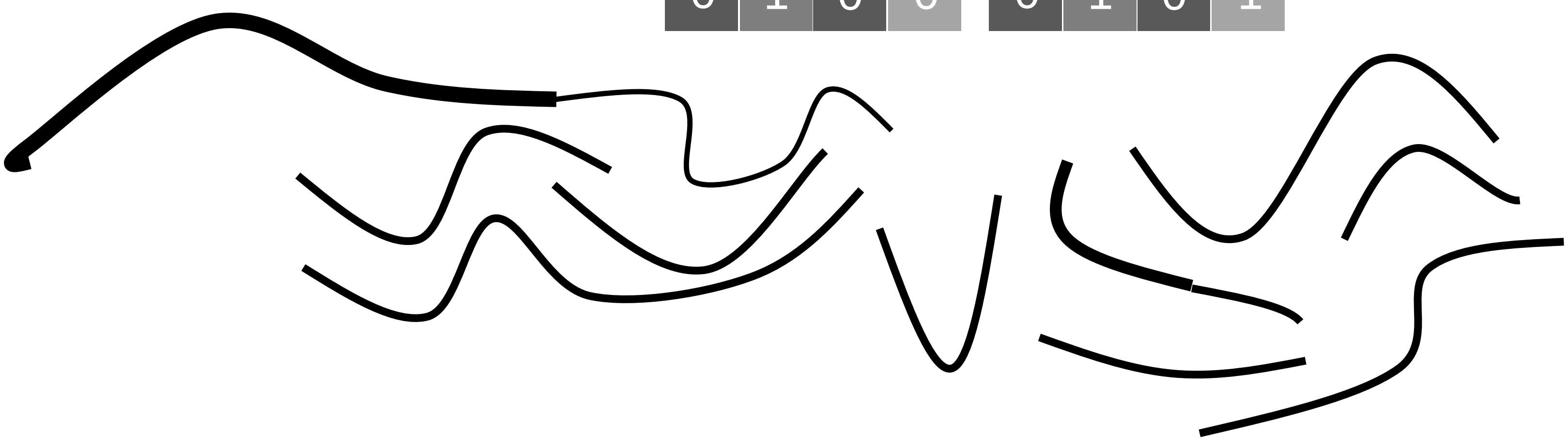
Edge Linking: Hysteresis

threshold_high

0	1	0	0
0	1	0	1
0	0	0	0
0	1	0	0

threshold_low

0	1	0	0
0	1	0	1
0	1	0	0
0	1	0	1



Edge Linking: Hysteresis

threshold_high

0	1	0	0
0	1	0	1
0	0	0	0
0	1	0	0

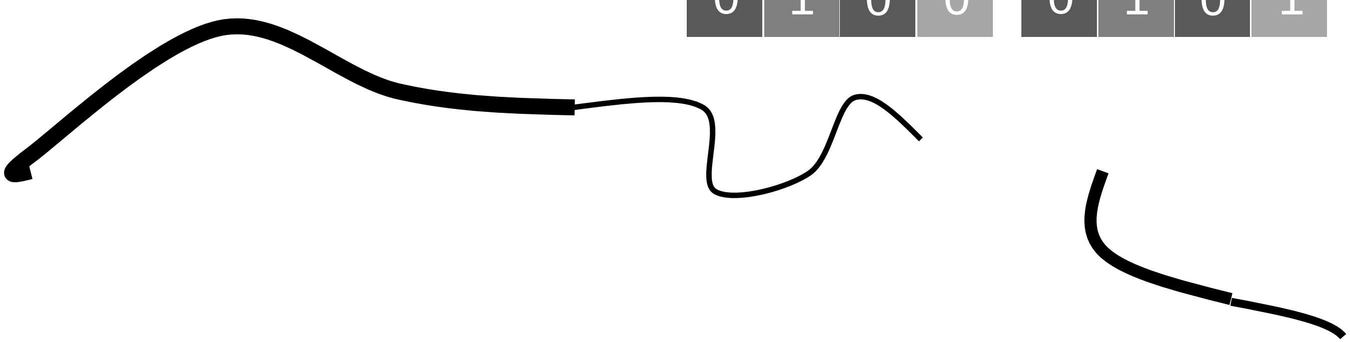
threshold_low

0	1	0	0
0	1	0	1
0	1	0	0
0	1	0	1

→

hysteresis

0	1	0	0
0	1	0	1
0	1	0	0
0	1	0	0



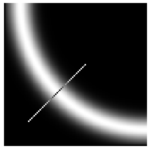
Canny Edge Detection

1. Filter image by derivatives of Gaussian

2. Compute magnitude of gradient

3. Compute edge orientation

4. Detect local maximum



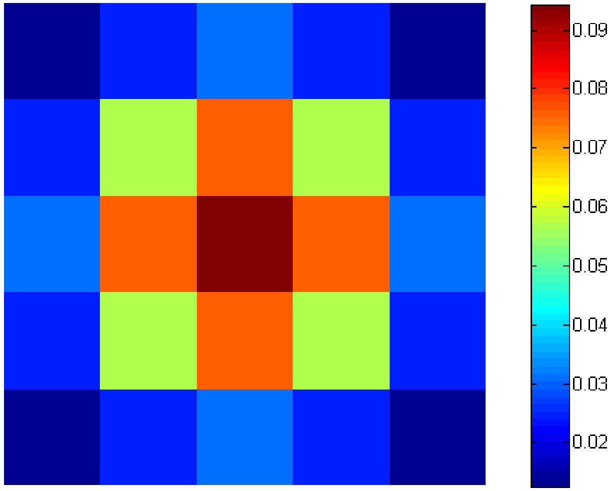
5. Edge linking

Canny Edge Implementation

```
img = imread ('Lenna.png');
img = rgb2gray(img);
img = double (img);

% Value for high and low thresholding
threshold_low = 0.035;
threshold_high = 0.175;

%% Gaussian filter definition (https://en.wikipedia.org/wiki/Canny_edge_detector)
G = [2, 4, 5, 4, 2; 4, 9, 12, 9, 4;5, 12, 15, 12, 5;4, 9, 12, 9, 4;2, 4, 5, 4, 2];
G = 1/159.* G;
```

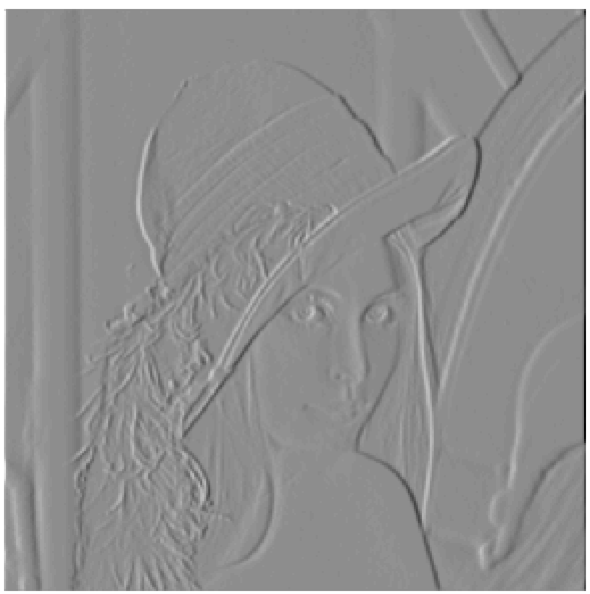



```
%Filter for horizontal and vertical direction
dx = [1 0 -1];
dy = [1; 0; -1];
```

Canny Edge Implementation

```
% % Convolution of image with Gaussian
Gx = conv2(G, dx, 'same');
Gy = conv2(G, dy, 'same');

% Convolution of image with Gx and Gy
Ix = conv2(img, Gx, 'same');
Iy = conv2(img, Gy, 'same');
```



Ix

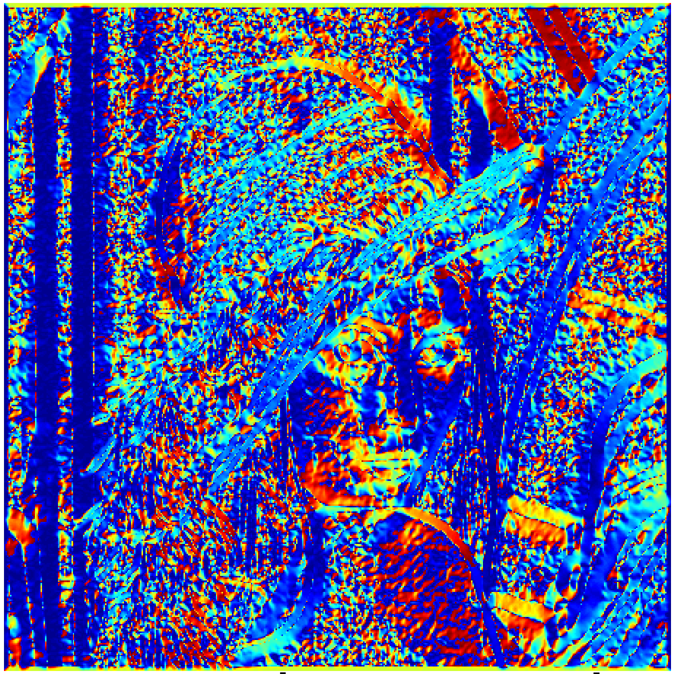
Iy

Canny Edge Implementation

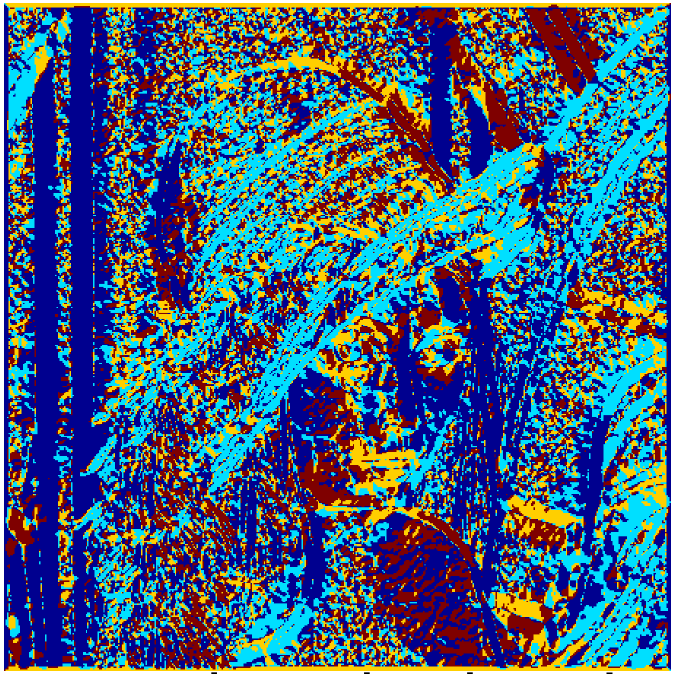
```
angle = atan2(Iy, Ix);

%% Edge angle conditioning
angle(angle<0) = pi+angle(angle<0);
angle(angle>7*pi/8) = pi-angle(angle>7*pi/8);
```

```
% Edge angle discretization into 0, pi/4, pi/2, 3*pi/4
angle(angle>=0&angle<pi/8) = 0;
angle(angle>=pi/8&angle<3*pi/8) = pi/4;
angle(angle>=3*pi/8&angle<5*pi/8) = pi/2;
angle(angle>=5*pi/8&angle<=7*pi/8) = 3*pi/4;
```



Continuous angle



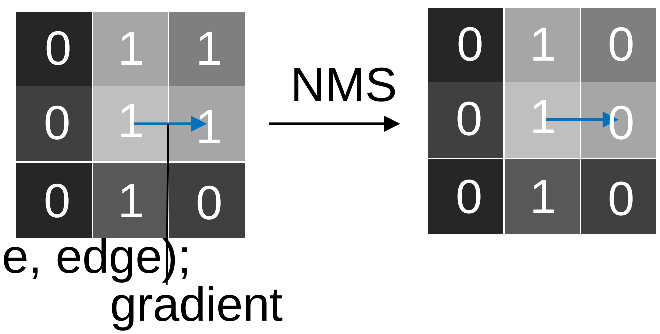
Discretized angle

Canny Edge Implementation

```
%Calculate magnitude
magnitude = sqrt(Ix.*Ix+Iy.*Iy);
edge = zeros(nr, nc);

%% Non-Maximum Supression
edge = non_maximum_suppression(magnitude, angle, edge);

edge = edge.*magnitude;
```

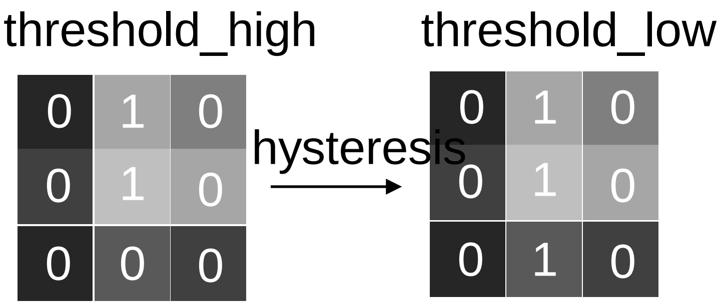


Gradient magnitude

Localized edge

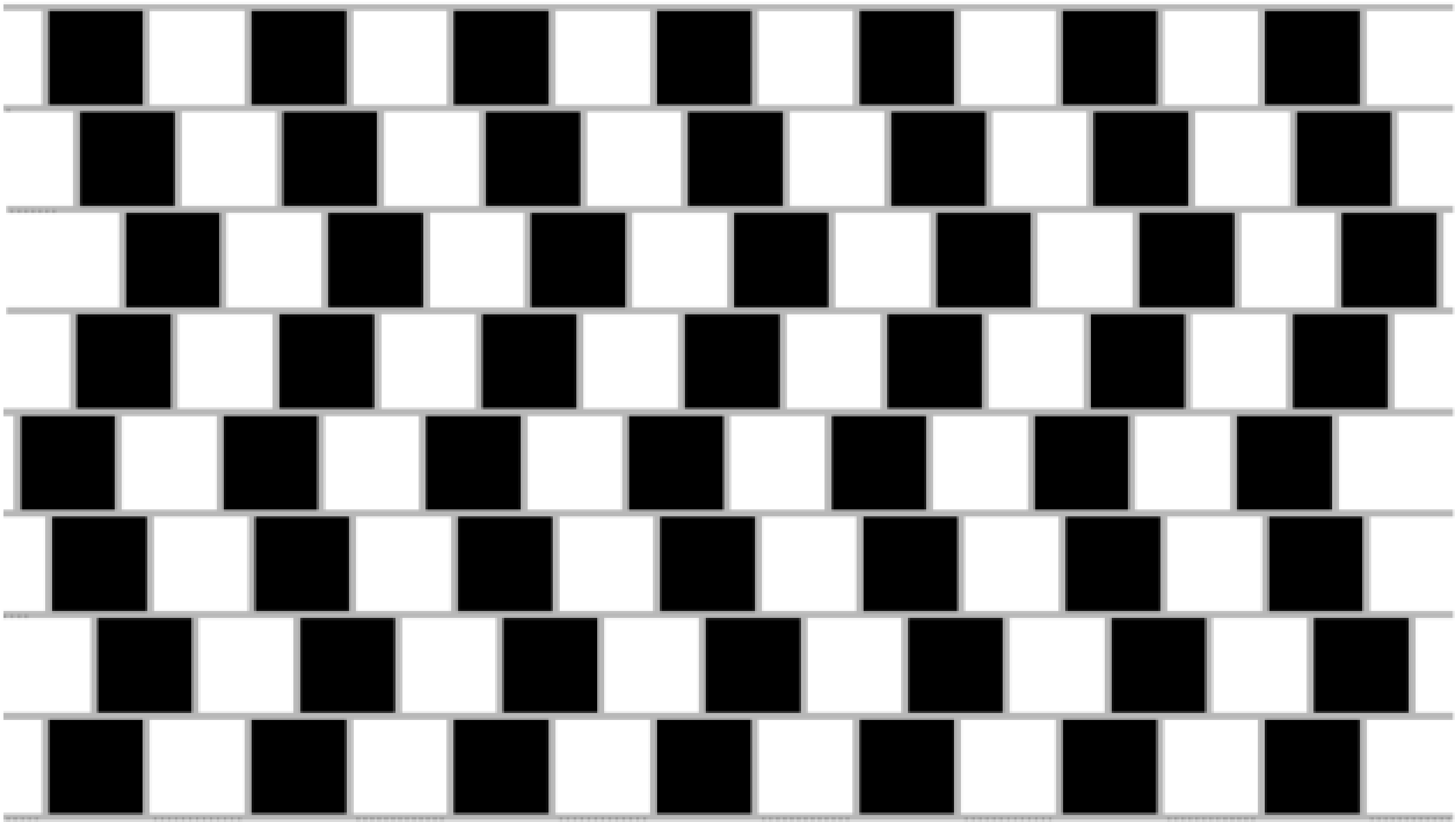
Canny Edge Implementation

```
%% Hysteresis thresholding
% for weak edge
threshold_low = threshold_low * max(edge(:));
% for strong edge
threshold_high = threshold_high * max(edge(:));
linked_edge = zeros(nr, nc);
linked_edge = hysteresis_thresholding(threshold_low, threshold_high, linked_edge, ed
```



http://www.cfar.umd.edu/~fer/optical/index.html

The rows of black and white squares are all parallel.
The vertical zigzag patterns disrupt our horizontal perception.



Cornelia Fermüller

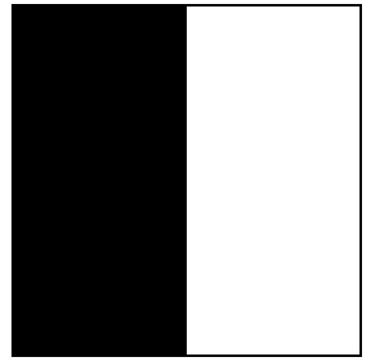


Figure 1: Type 1 Edge

This edge can be represented by a function. The picture below shows the gray value changes from black to white in the horizontal direction.

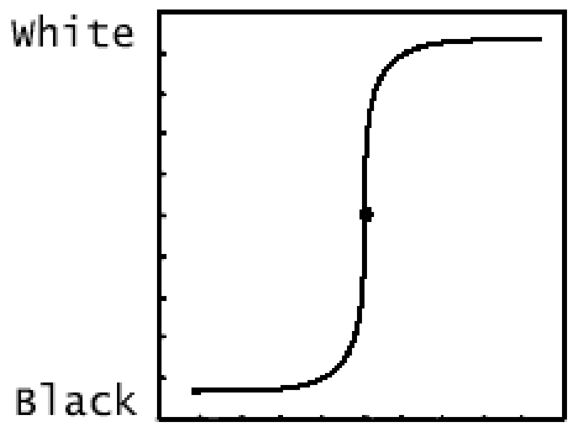


Figure 2

The edge is at the location of the inflection point on the curve, indicated by the black dot. If we were to smooth this image, it would look like this.

Cornelia Fermüller

The edge is at the location of the inflection point on the curve, indicated by the black dot. If we were to smooth this image, it would look like this.

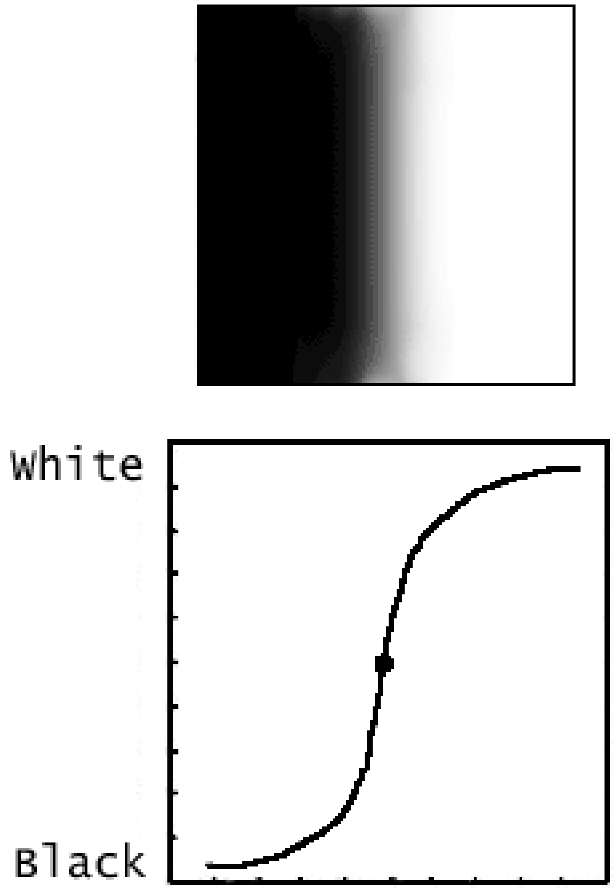


Figure 3

As you can see, smoothing the image does not change the location of the edge.

The second case is a line on a background of different intensity.

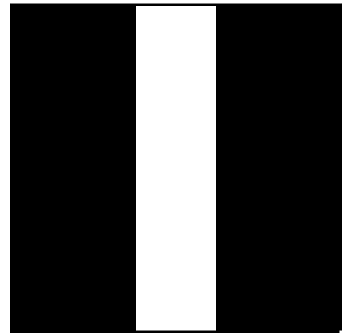


Figure 4: Type 2 Edge

This again can be represented as a function with two inflection points representing the edges at the boundaries of the line and the background regions.

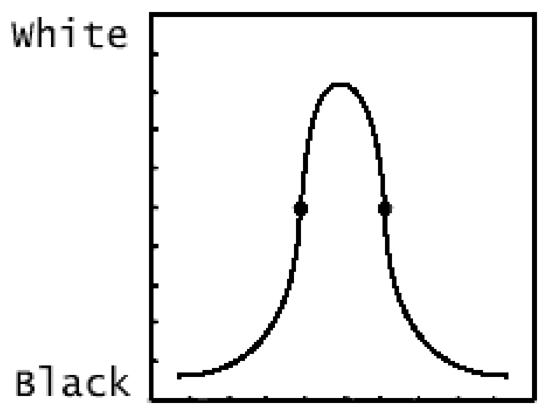


Figure 5

When the image is smoothed the edges drift apart as shown.

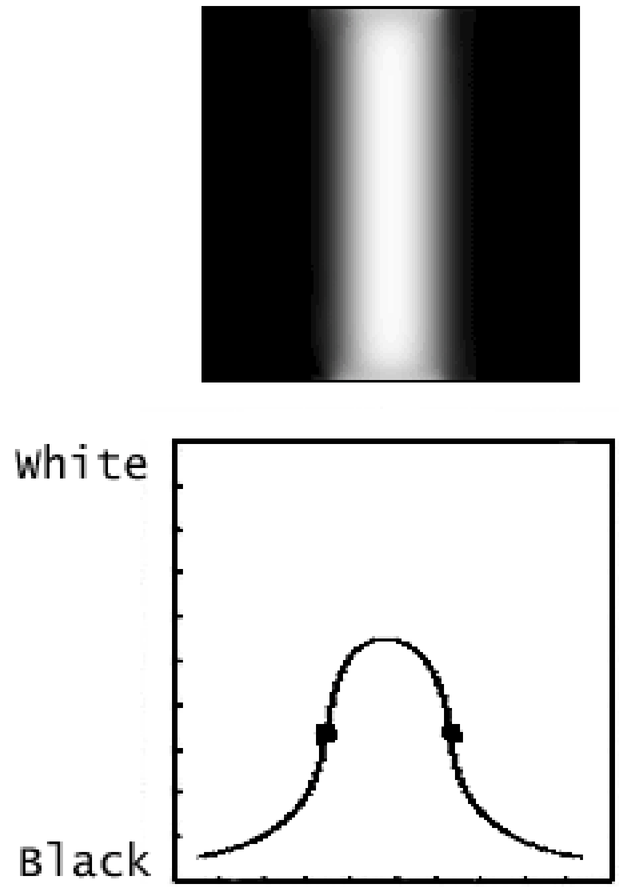


Figure 6

The third case is a gray line between a bright and a dark region.

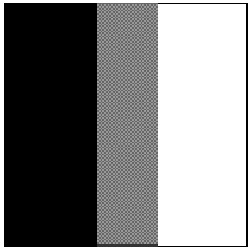


Figure 7: Type 3 Edge

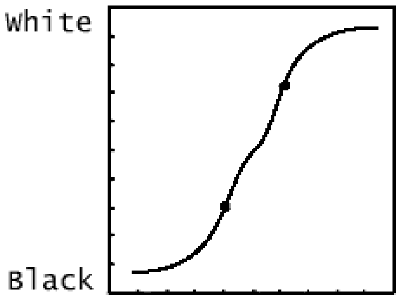
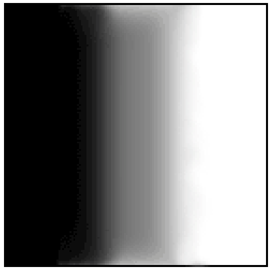
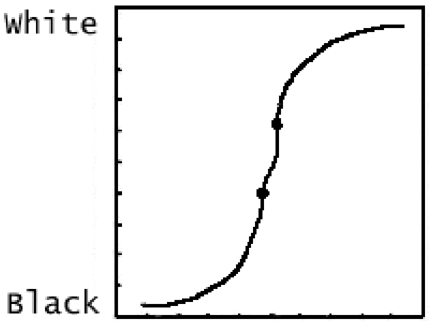
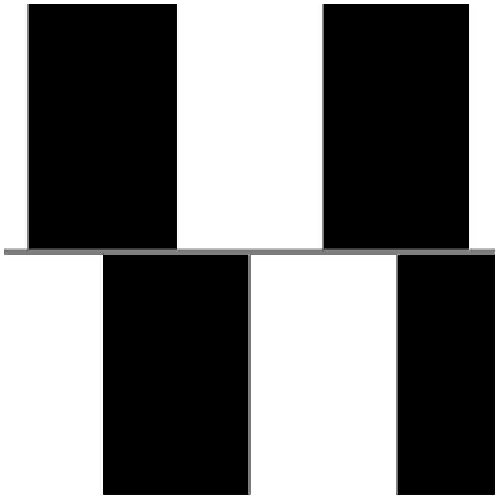


Figure 8

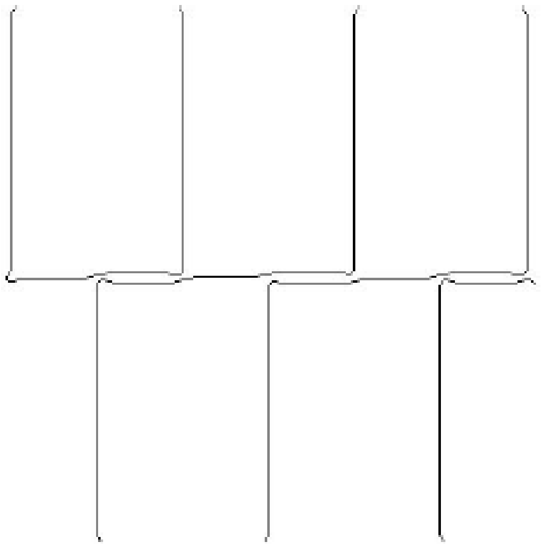
When this image is smoothed the edges at the boundary of the line move toward each other.







If we smooth the image and then apply edge detection, we obtain edges as shown below.



Cornelia Fermüller

http://www.cfar.umd.edu/~fer/optical/index.html

The rows of black and white squares are all parallel.
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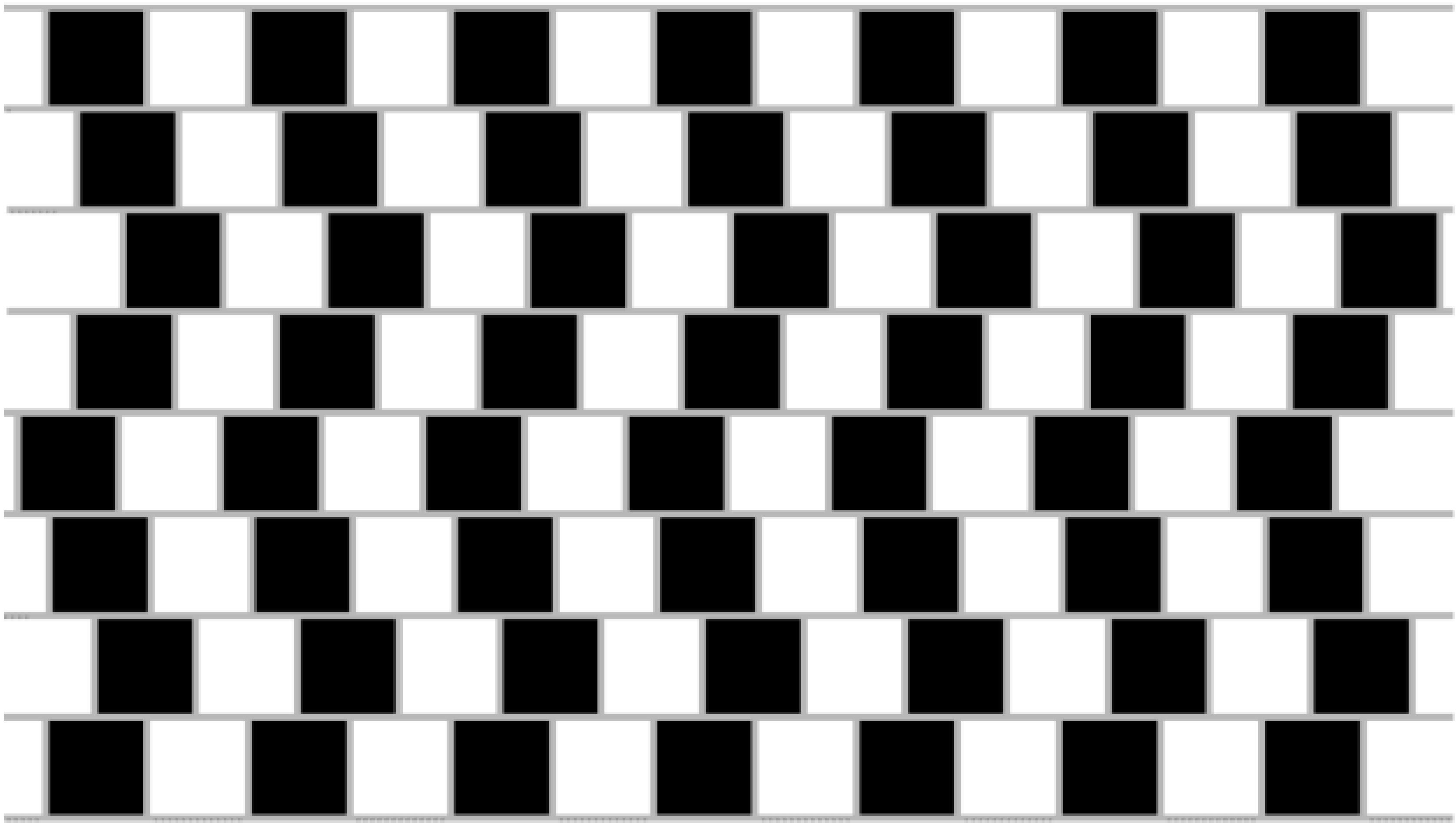


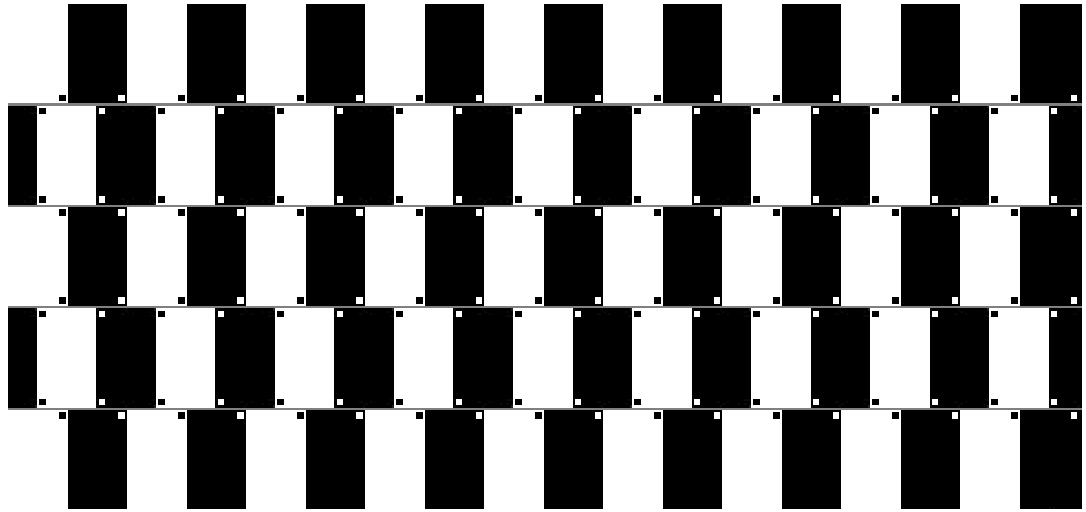
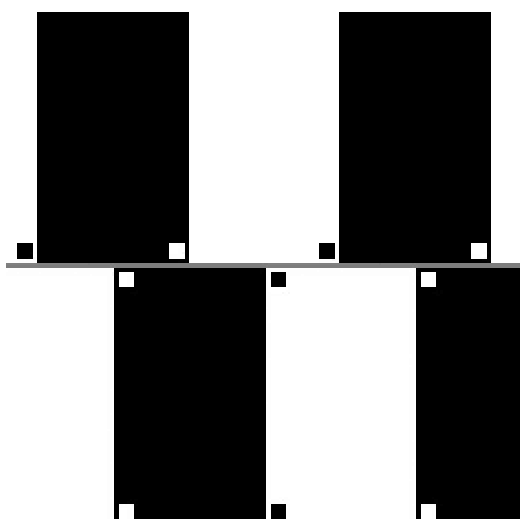
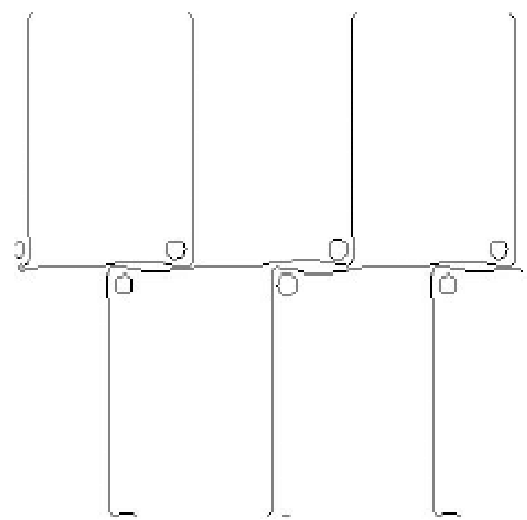


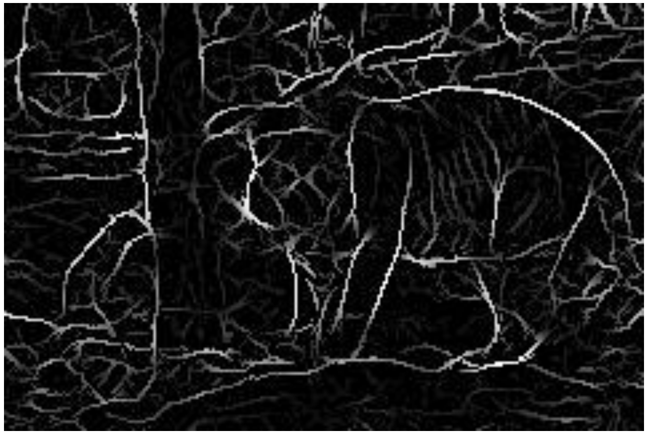
Figure 4

If we zoom in and look at the edges we notice that the added squares compensate for the drifting of the lines. There is still "waviness" to the edges, but it is too weak to be perceived.

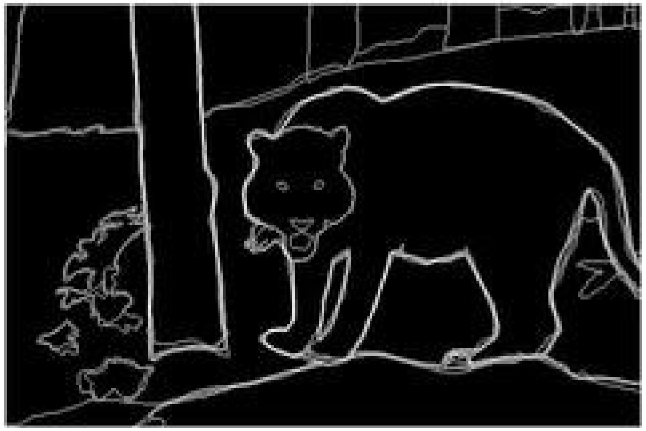




Is Edge Detection Solved?



⇒ Edge detector



⇒ Human segmentation

Edge Formation Factors

Surface normal discontinuity

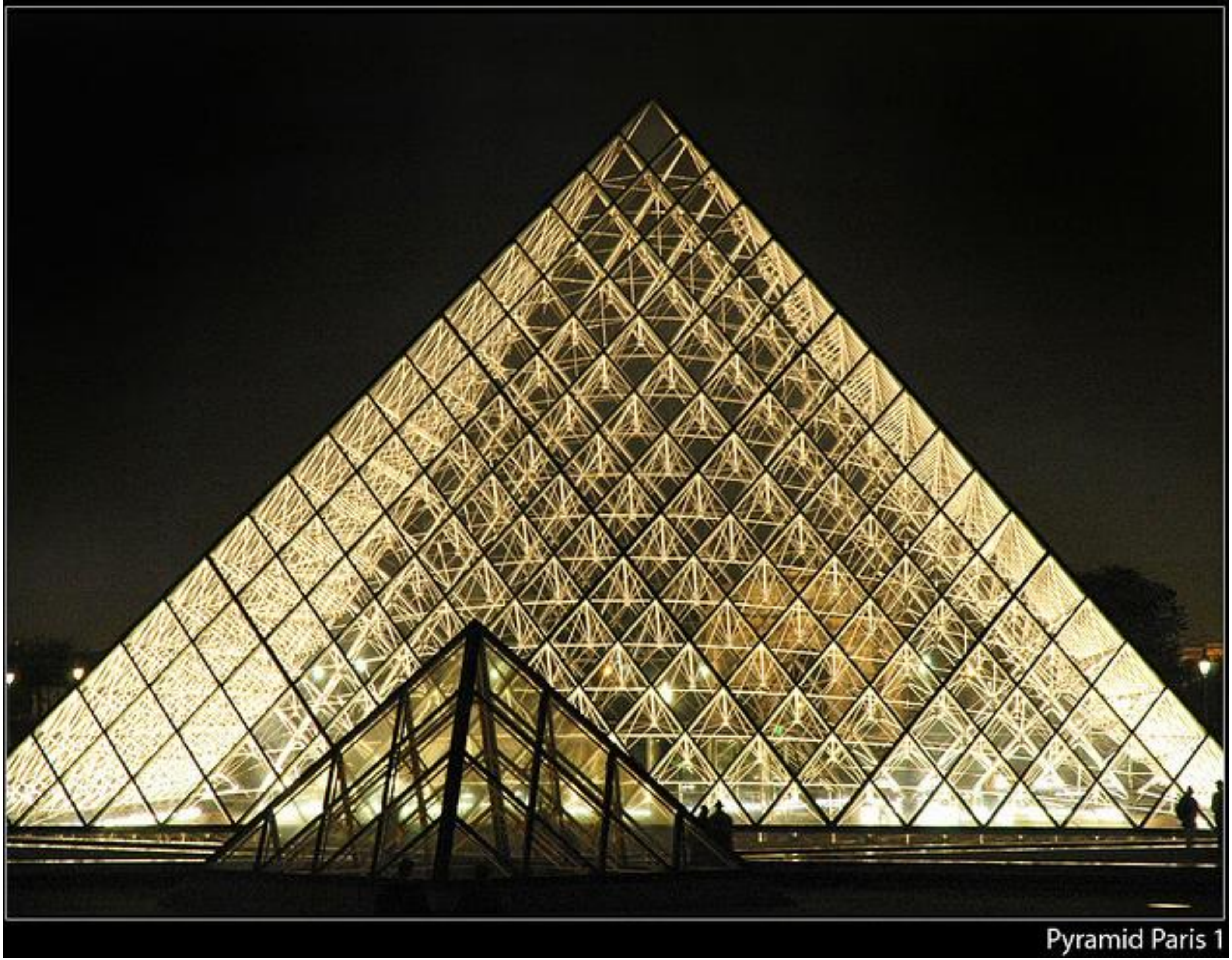
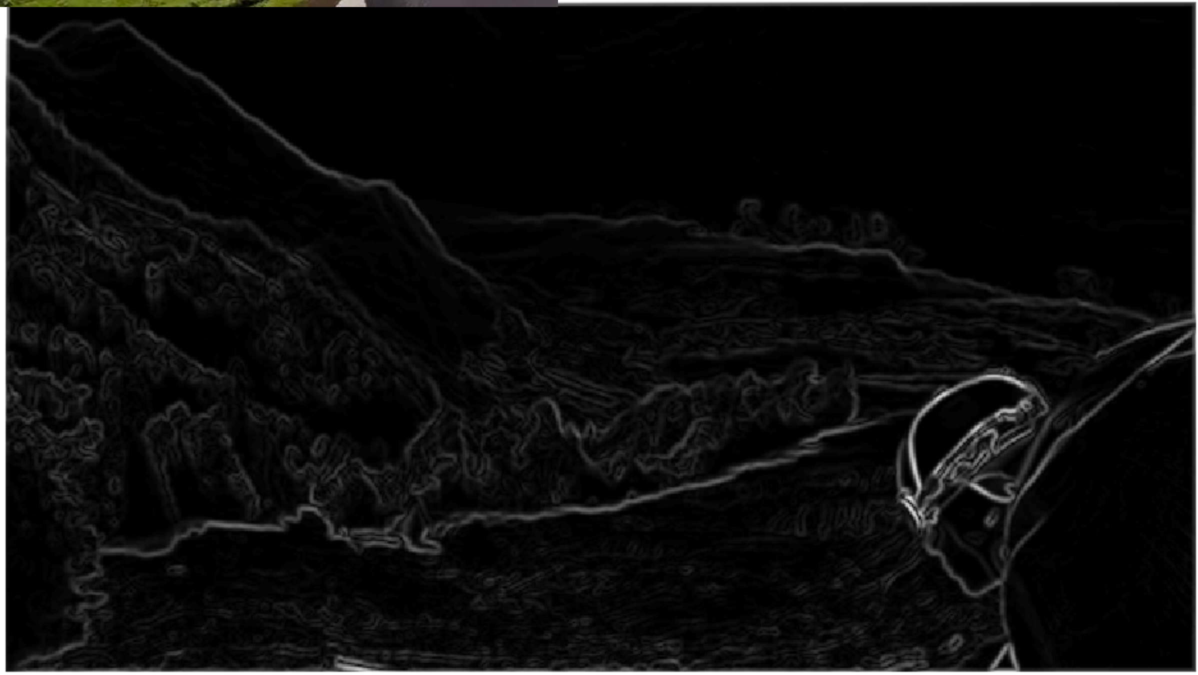
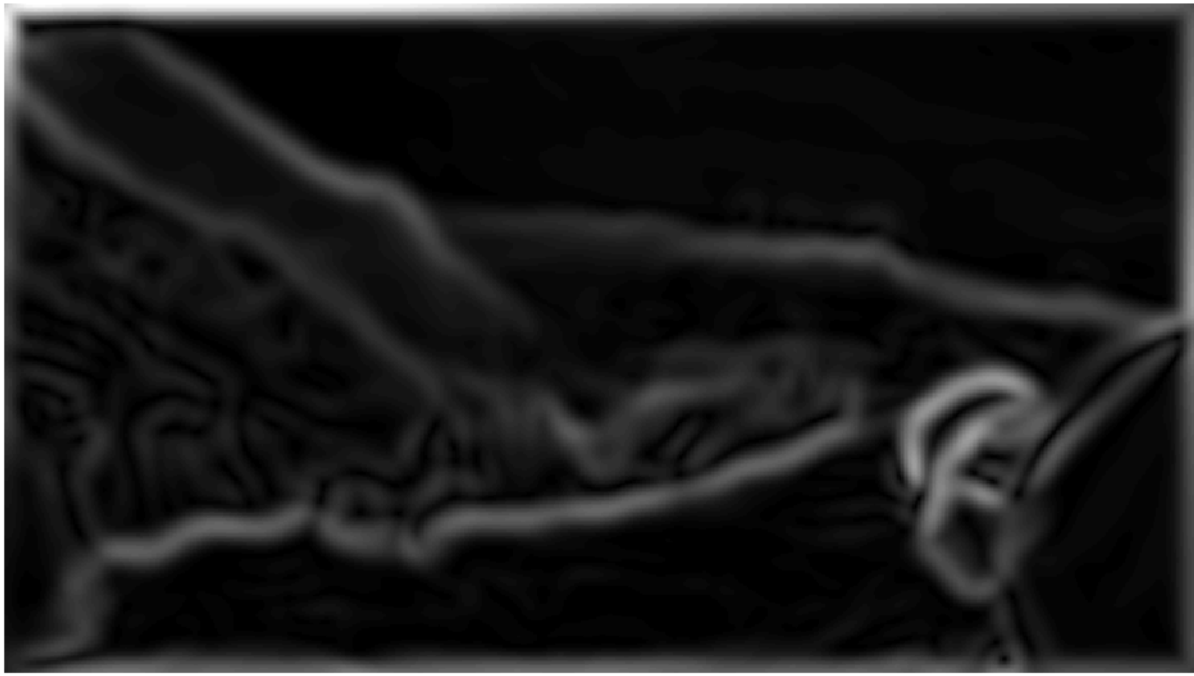
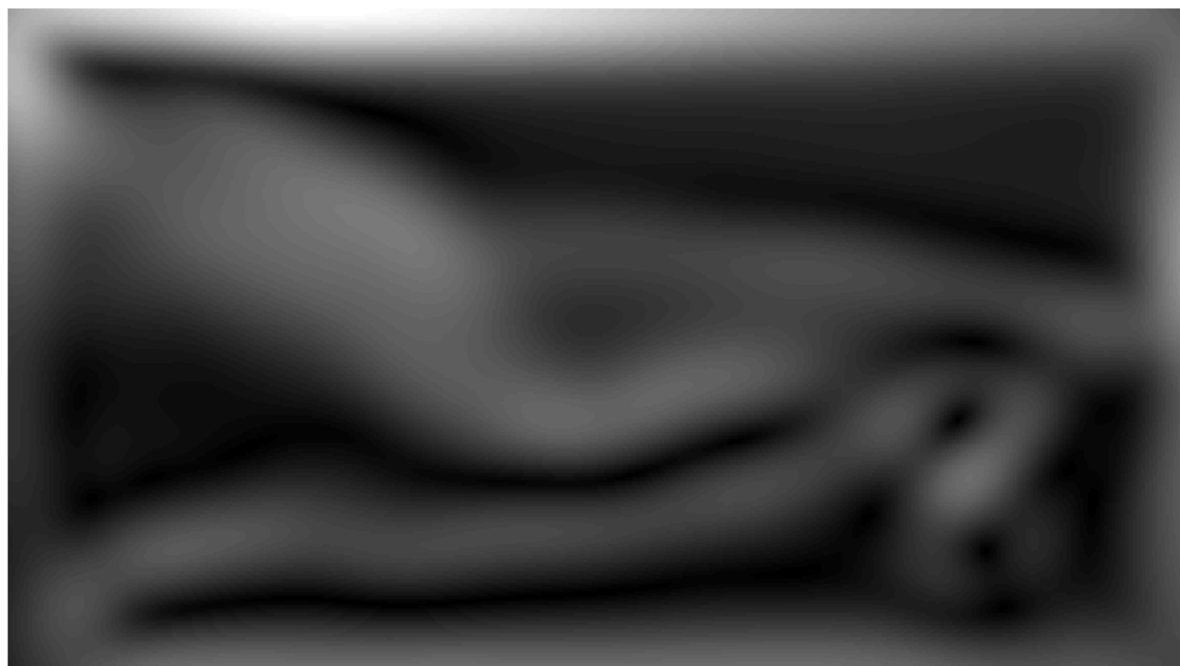


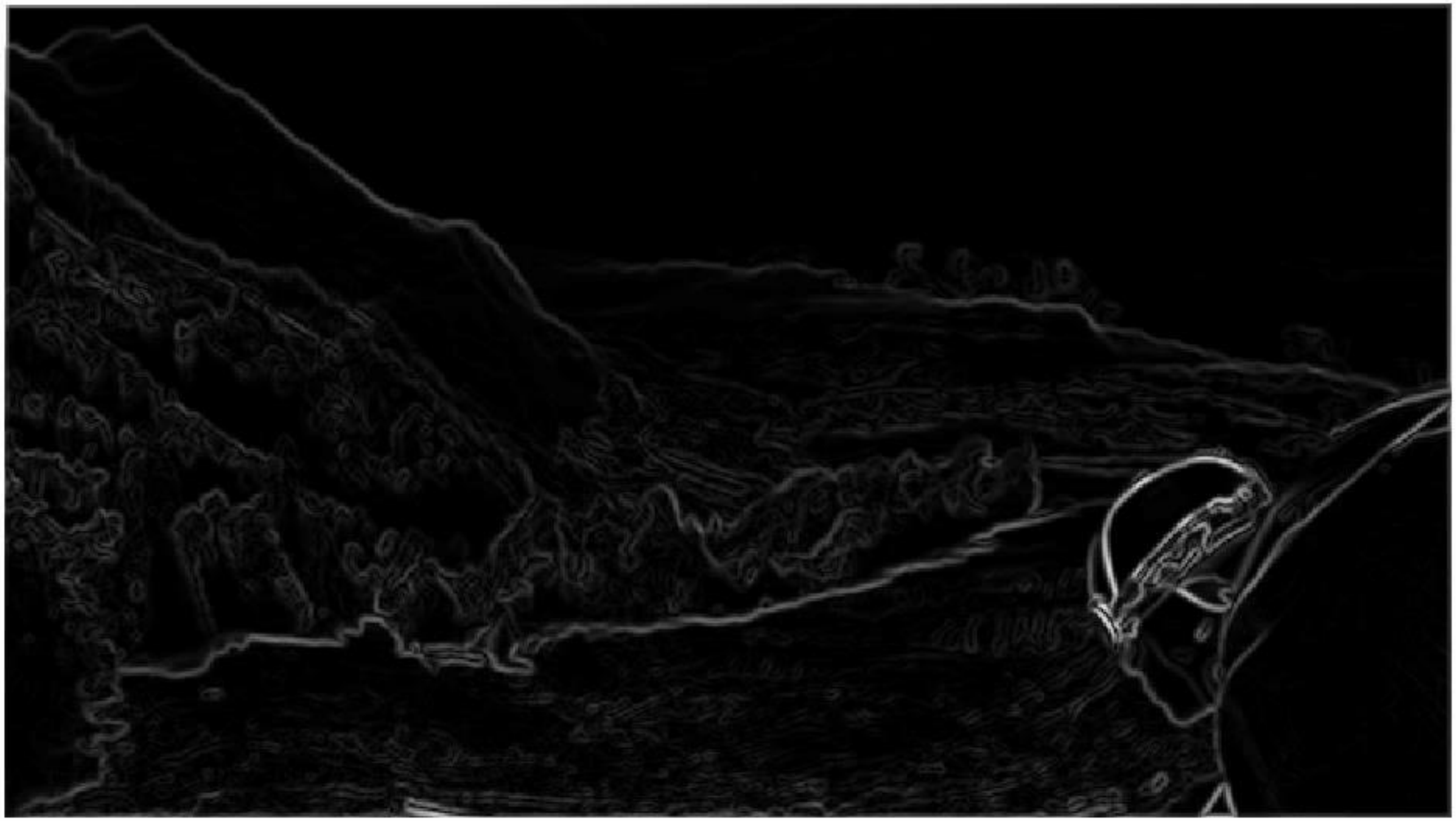
Image Pyramid, CIS581

Image Scale



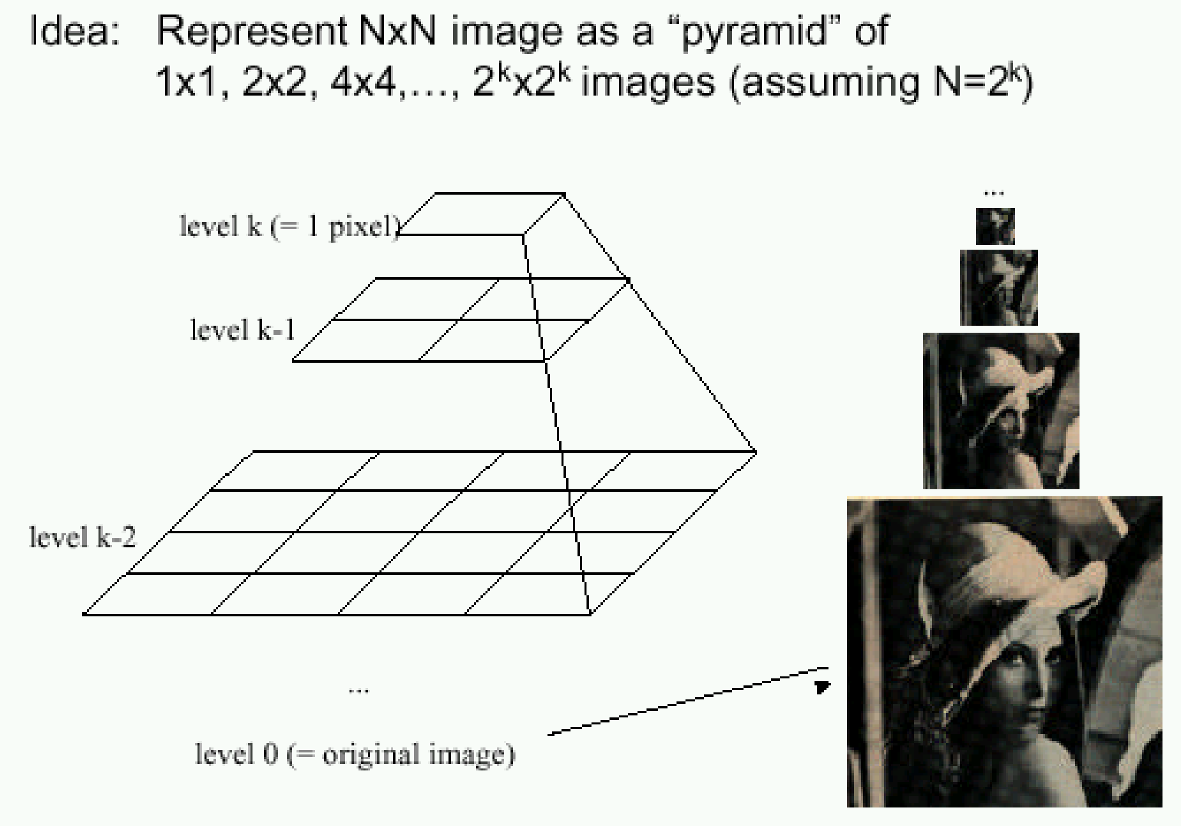






Different scale of image encodes different edge response.

Image Pyramids



Known as a Gaussian Pyramid [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

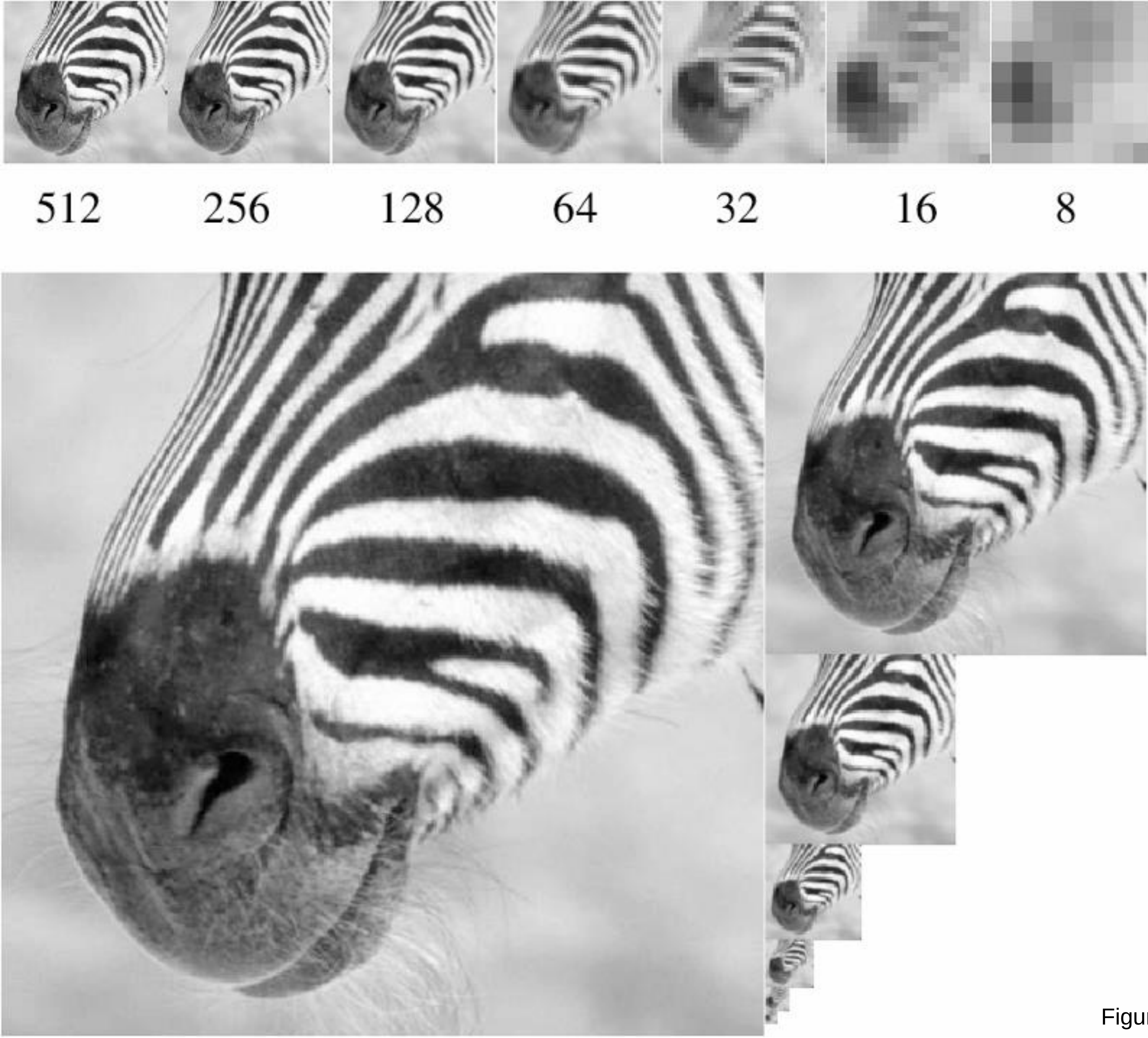
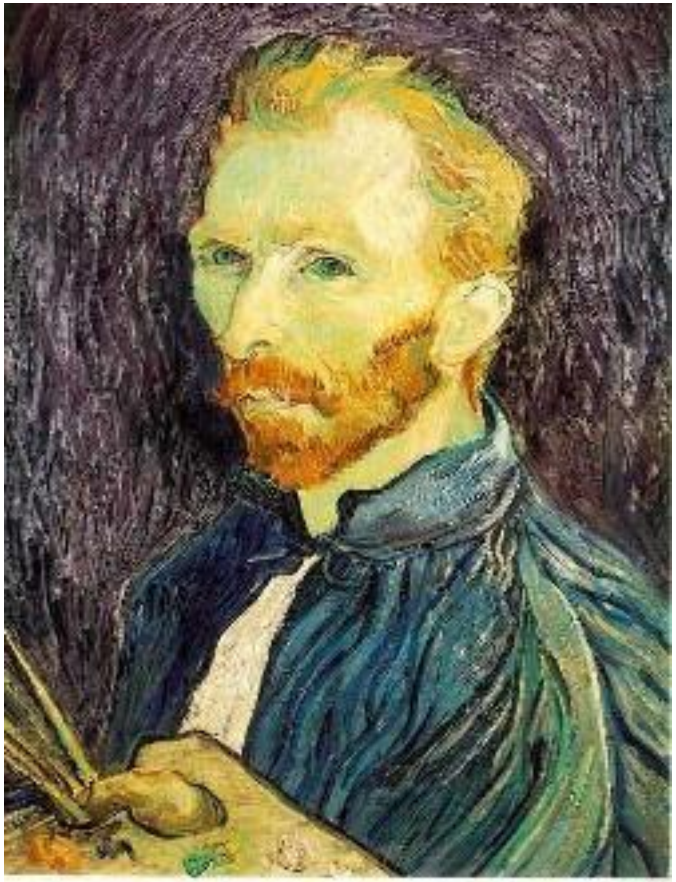


Figure from David Forsyth

Image sub-sampling

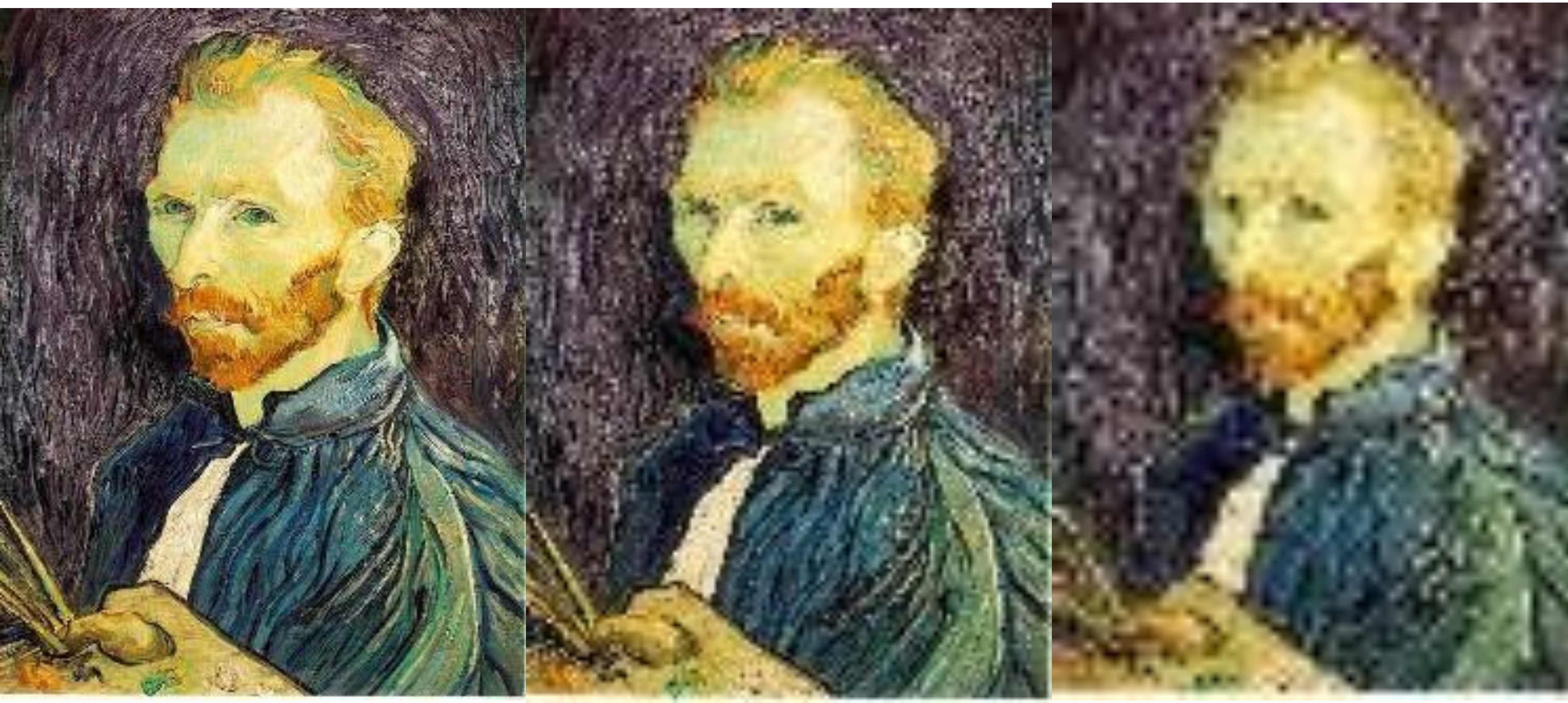


Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

1/4

1/8

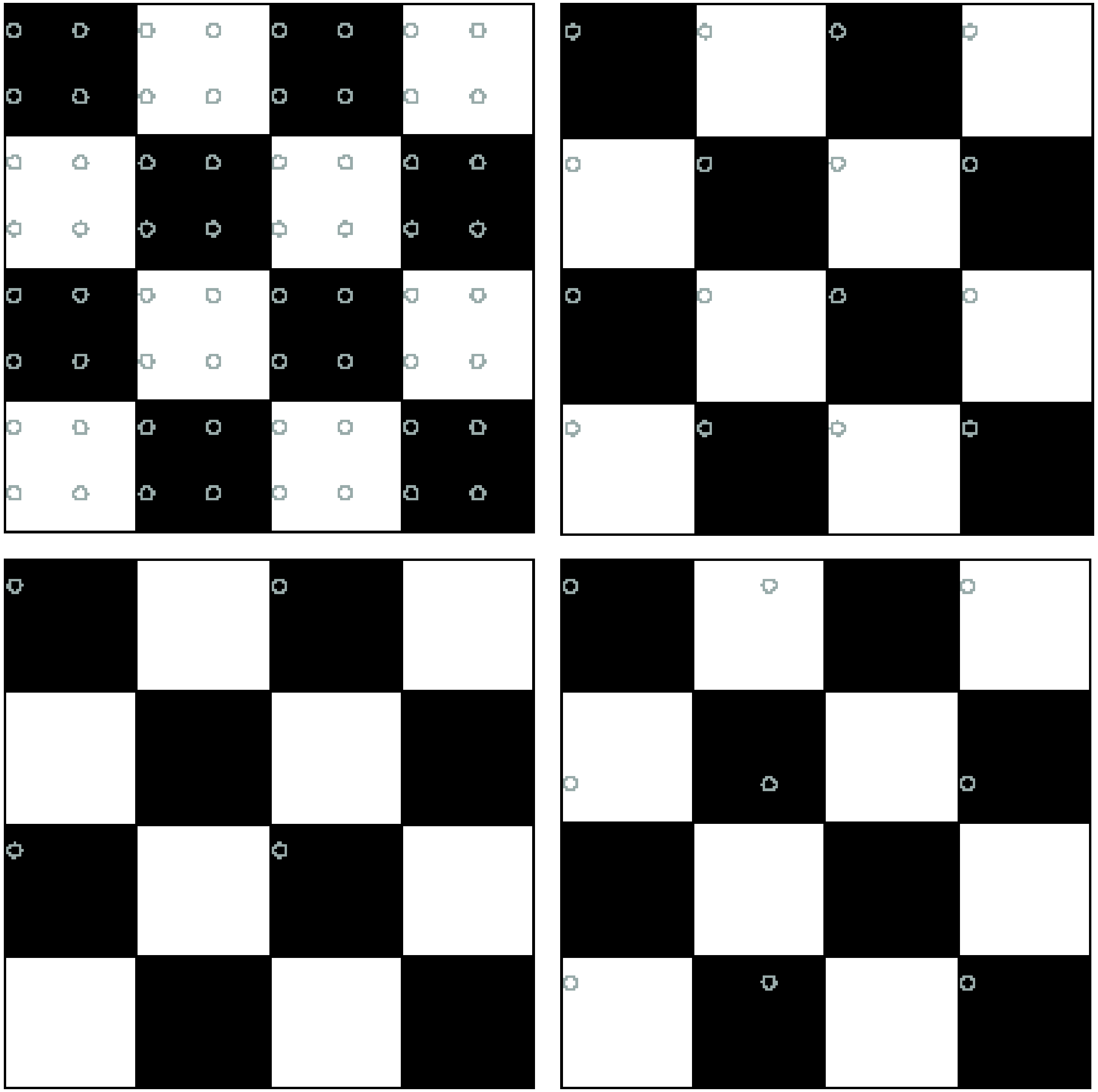
Image sub-sampling



1/2 1/4 (2x zoom) 1/8 (4x zoom)

Why does this look so bad?

Sampling



Good sampling:

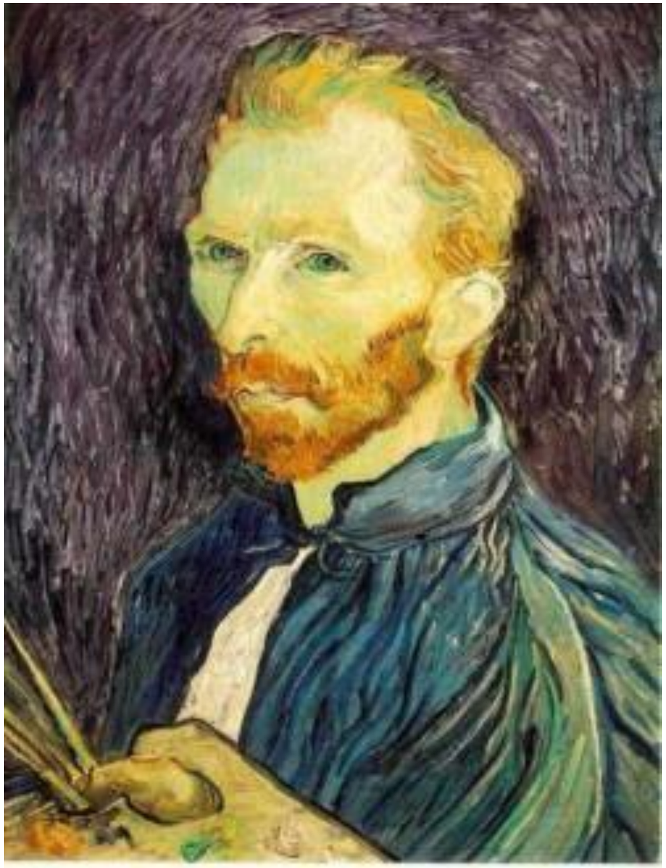
•Sample often or,

•Sample wisely

Bad sampling:

•see aliasing in action!

Gaussian pre-filtering



Gaussian 1/2

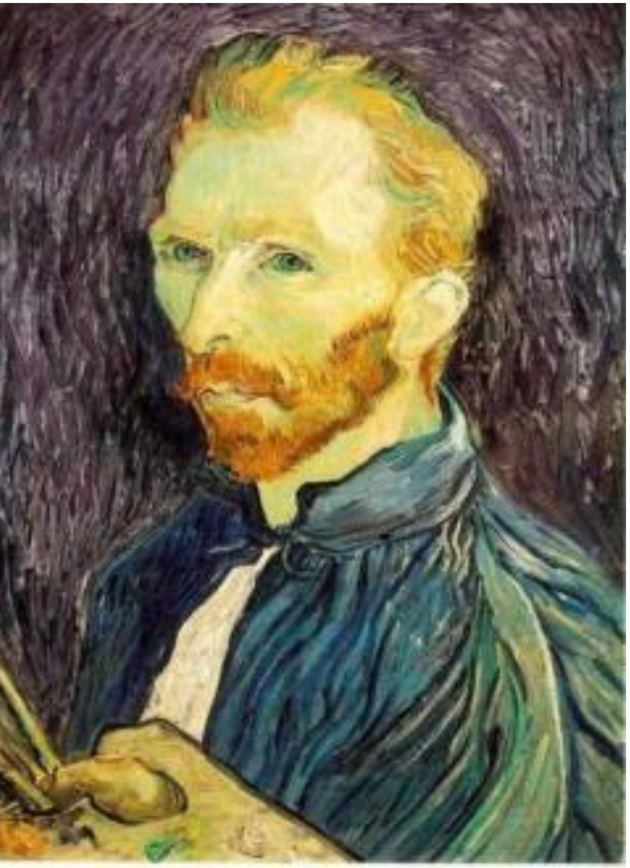
G 1/4

G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?

Subsampling with Gaussian pre-filtering



Gaussian 1/2

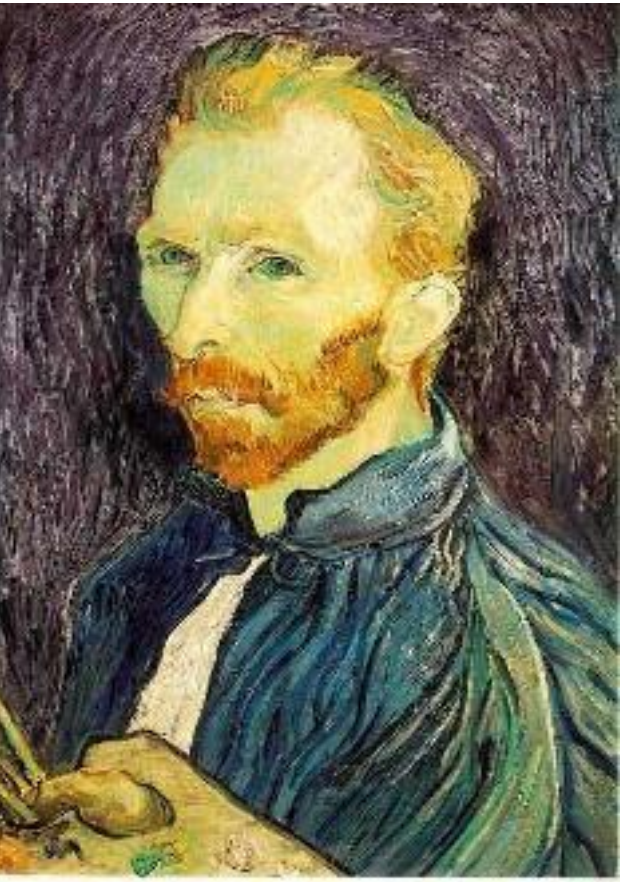
G 1/4

G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?
- How can we speed this up?

Comparison



1/2

1/4 (2x zoom)

1/8 (4x zoom)