Image filtering

$$g[m,n] = \sum_{k,l} I(m+k, n+l) * f(k,l)$$

Image I 8x8

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Kernel f 3x3

1	2	3
4	5	6
7	8	9

Output g

28	39	39	39	39	39	39	24
33	45	45	45	45	45	45	27
33	45	45	45	45	45	45	27
16	21	21	21	21	21	21	12
5	6	6	6	6	6	6	3
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Same position

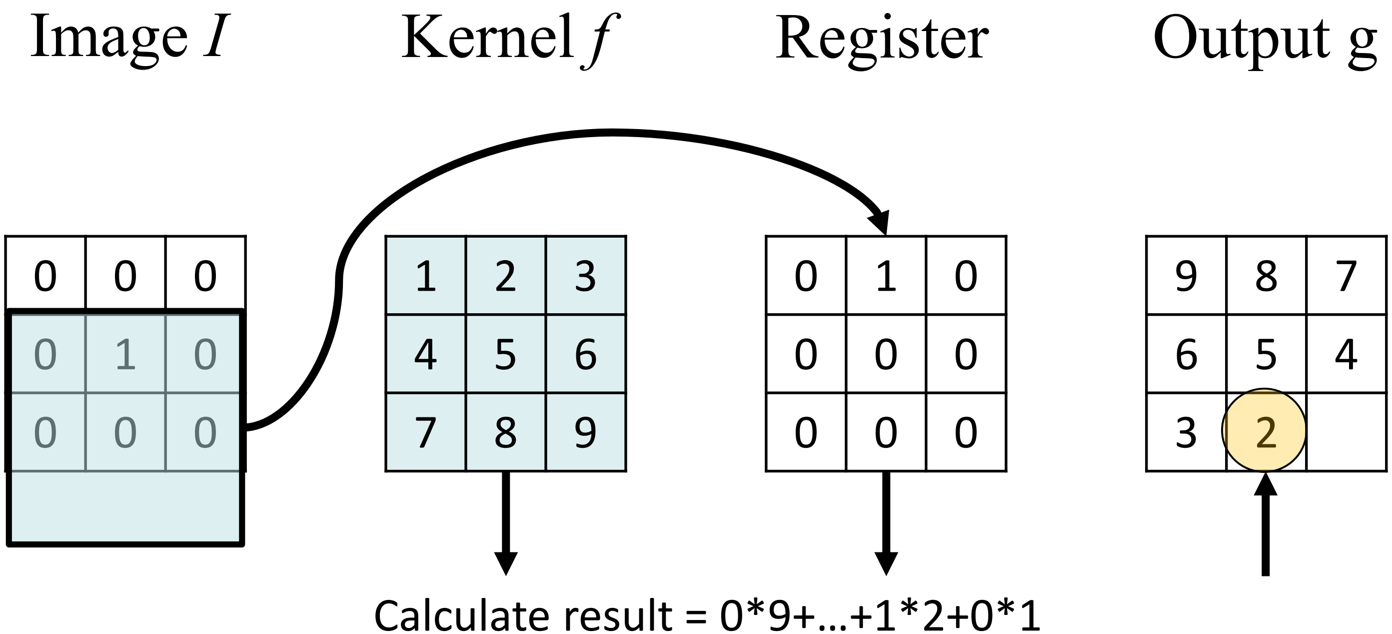
Register

a	b	c
d	e	f
g	h	i

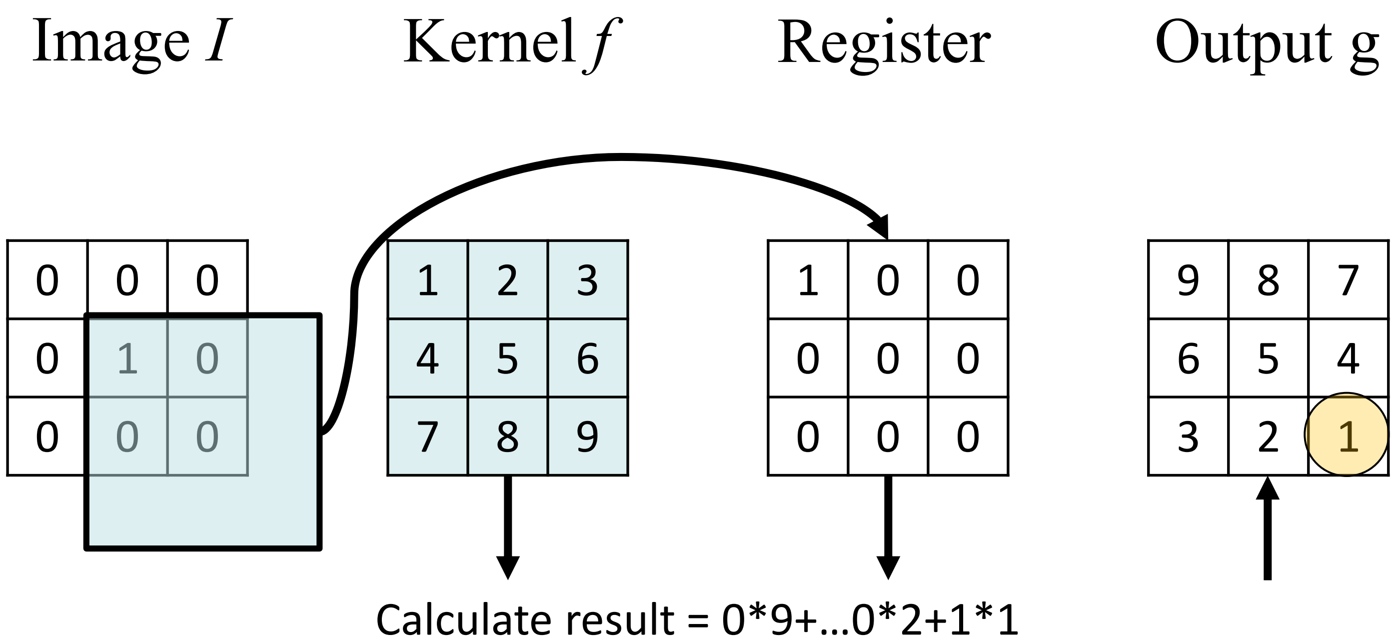
Loop over every pixel

Calculate result = a*1+b*2+...+i*9

Special case: impulse function



Special case: impulse function



<Note> The output is the kernel flipped left-right, up-down!

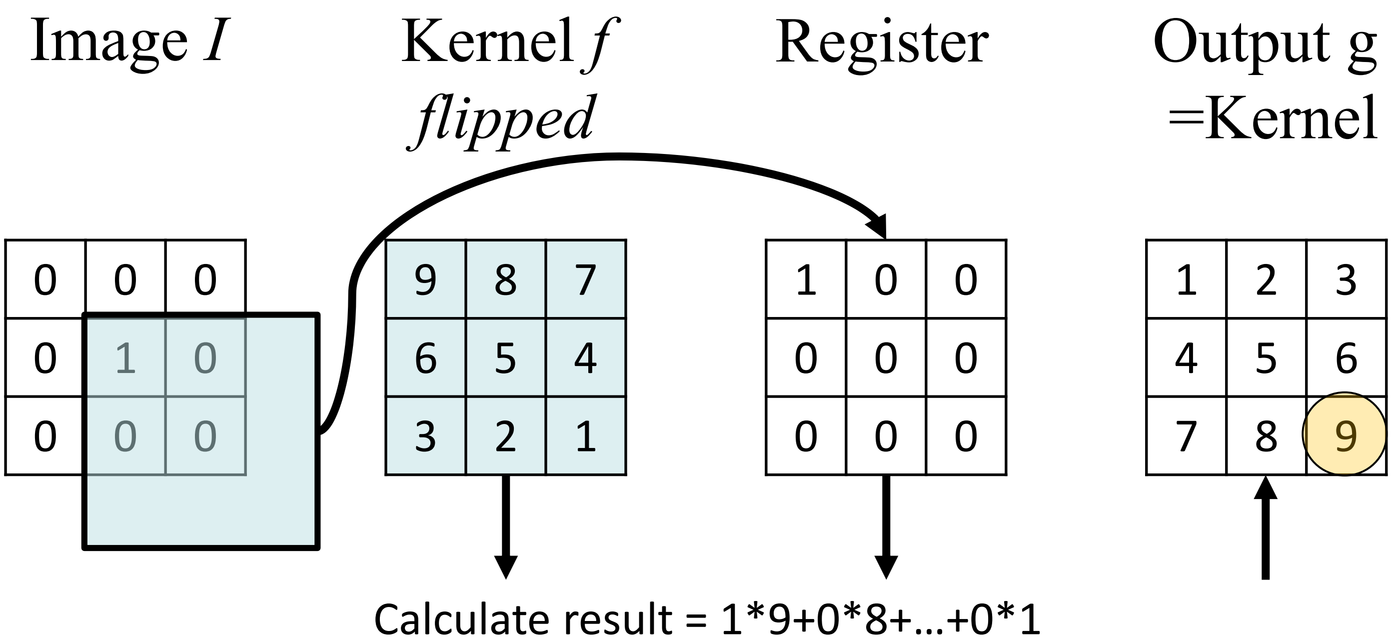
Convolution

- Let I be an Signal(image), Convolution kernel g,

$$f[m,n] = I \otimes g = \sum_{k,l} I[m-k, n-l]g[k,l]$$

Convolution

- $g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$
- Convolution is filtering with kernel flipped



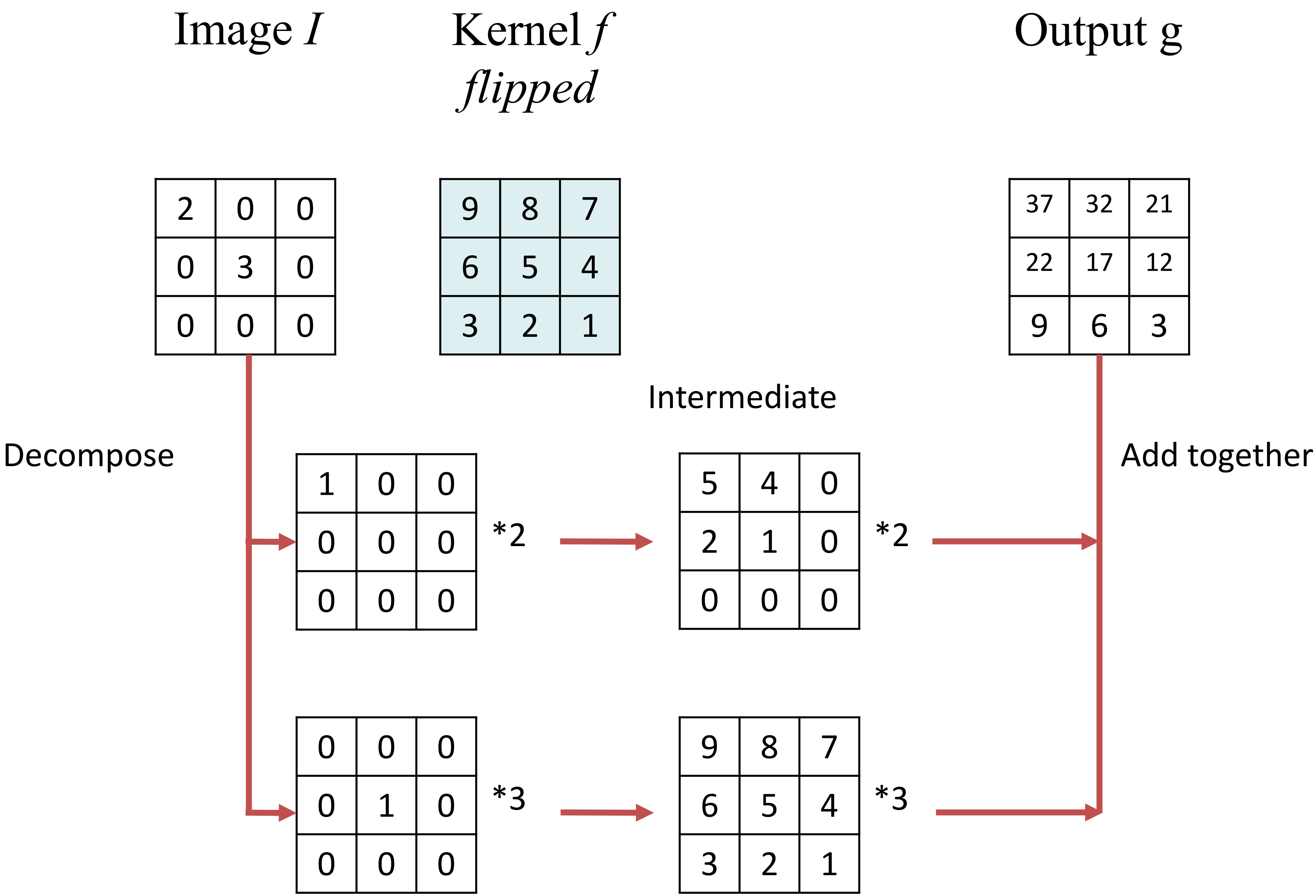


Image I
Kernel f flipped
Output g
2 0 0
0 3 0
0 0 0
9 8 7
6 5 4
3 2 1
37 32 21
22 17 12
9 6 3
Intermediate
Decompose
Add together
1 0 0
0 0 0
0 0 0
*2
5 4 0
2 1 0
0 0 0
*2
0 0 0
0 1 0
0 0 0
*3
9 8 7
6 5 4
3 2 1
*3

- Convolution has commutative property $I \otimes f$

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
1	0	0
1	1	0

- Convolution has commutative property $I \otimes f$

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
1	0	0
1	1	0

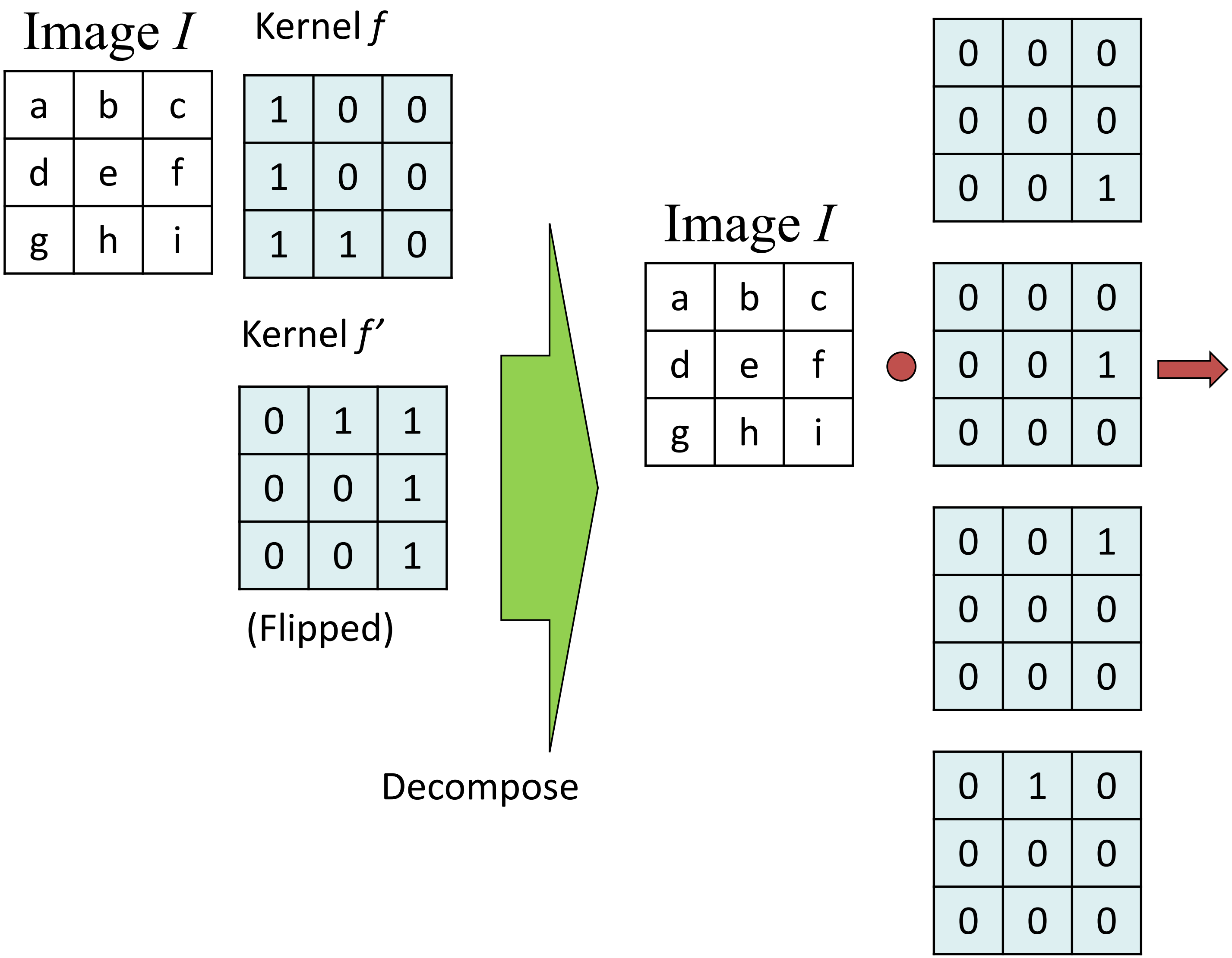
Kernel f'

0	1	1
0	0	1
0	0	1

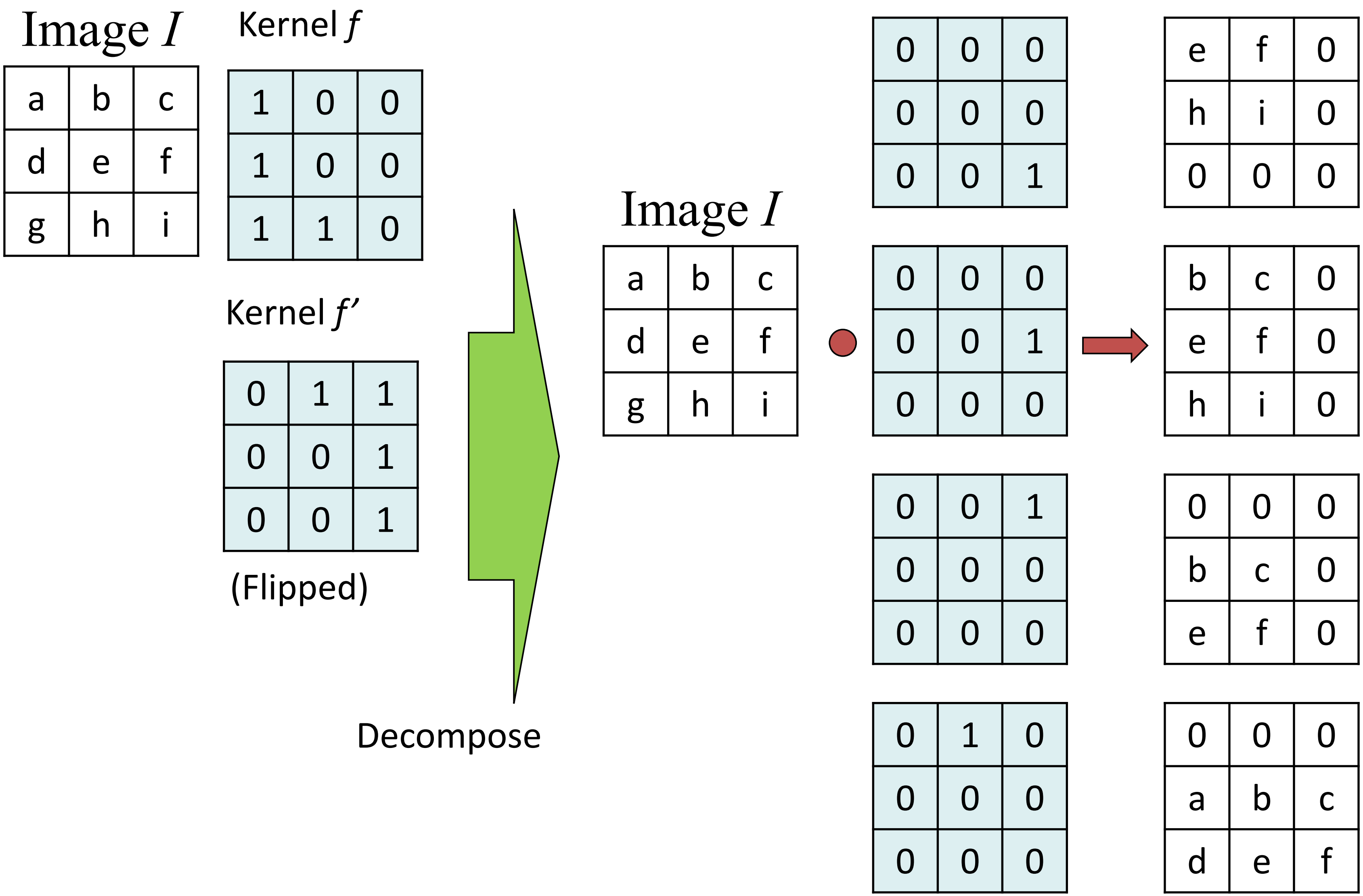
(Flipped)

Decompose

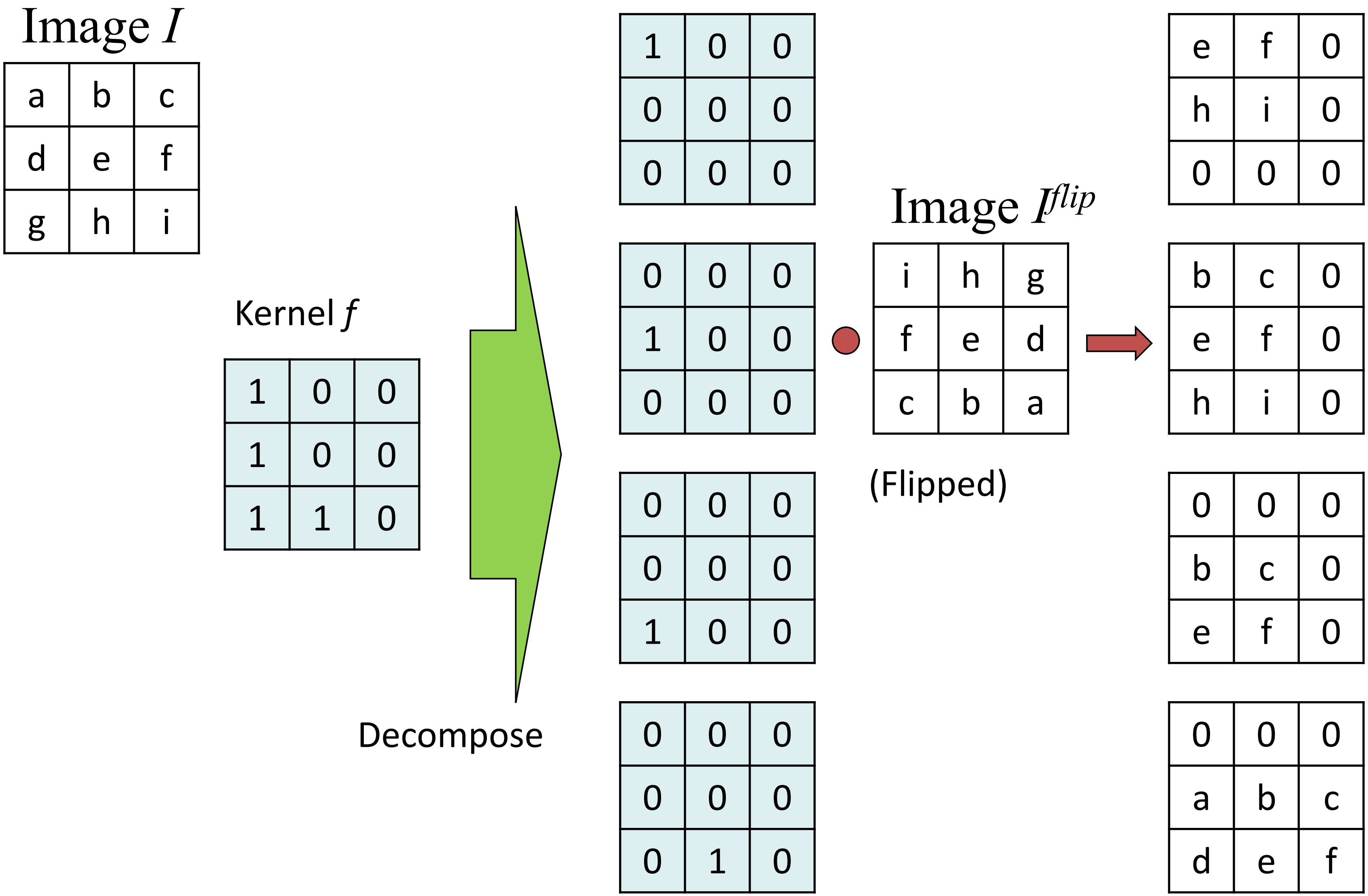
- Convolution has commutative property $I \otimes f$



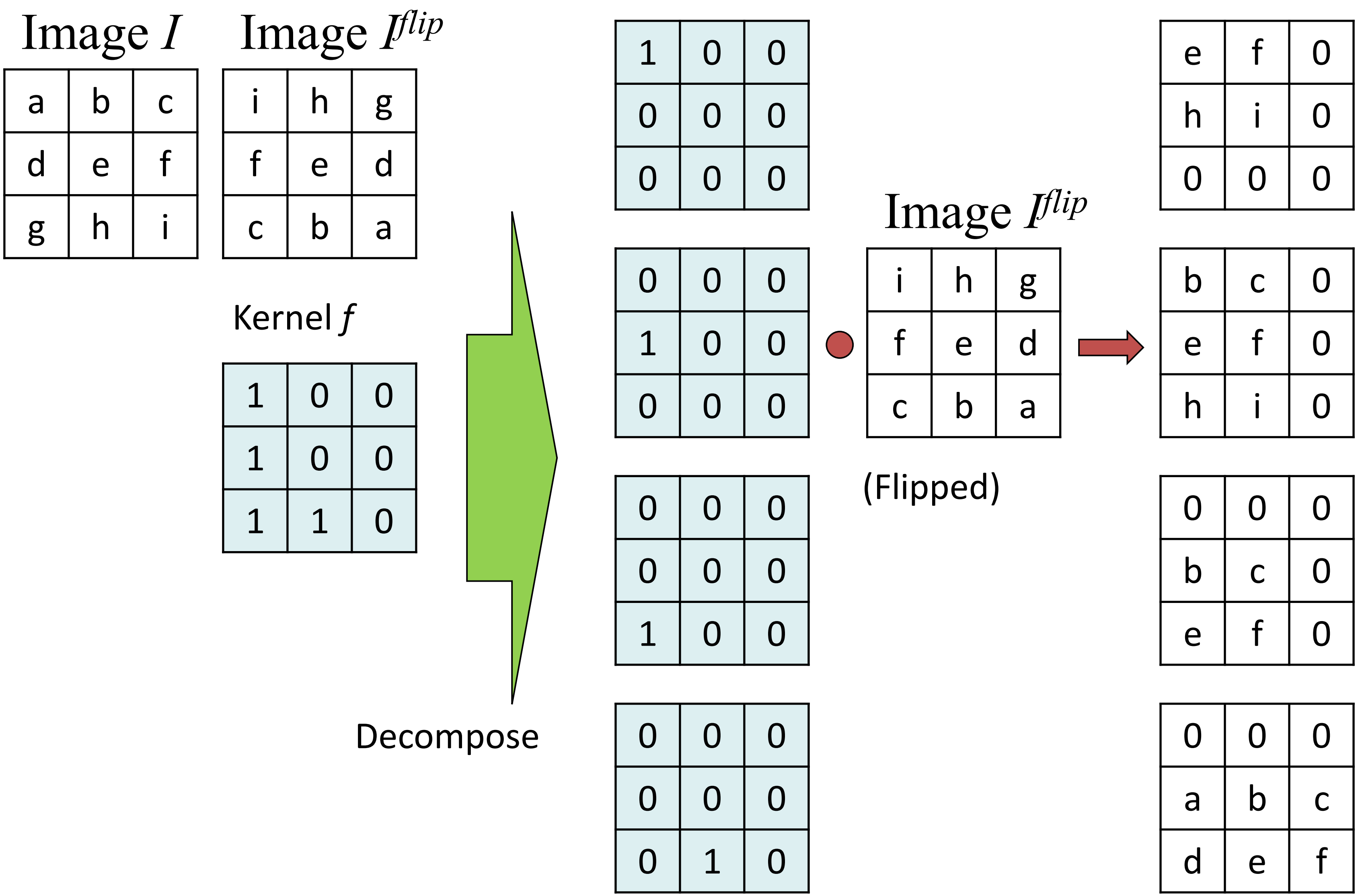
- Convolution has commutative property $I \otimes f$



- Convolution is commutative $I \otimes f = f \otimes I$



- Convolution is commutative $I \otimes f = f \otimes I$



Proof of Commutative property

- $g[m,n] = I \otimes f = f \otimes I$
- $g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$
- Let $k' = m-k, l' = n-l$,
  then $k = m-k', l = n-l'$
- $g[m,n] = \sum_{k',l'} I(k',l') * f(m-k', m-l') = f \otimes I$

Impulse functions shift images

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
0	0	0
0	0	0

Kernel f'

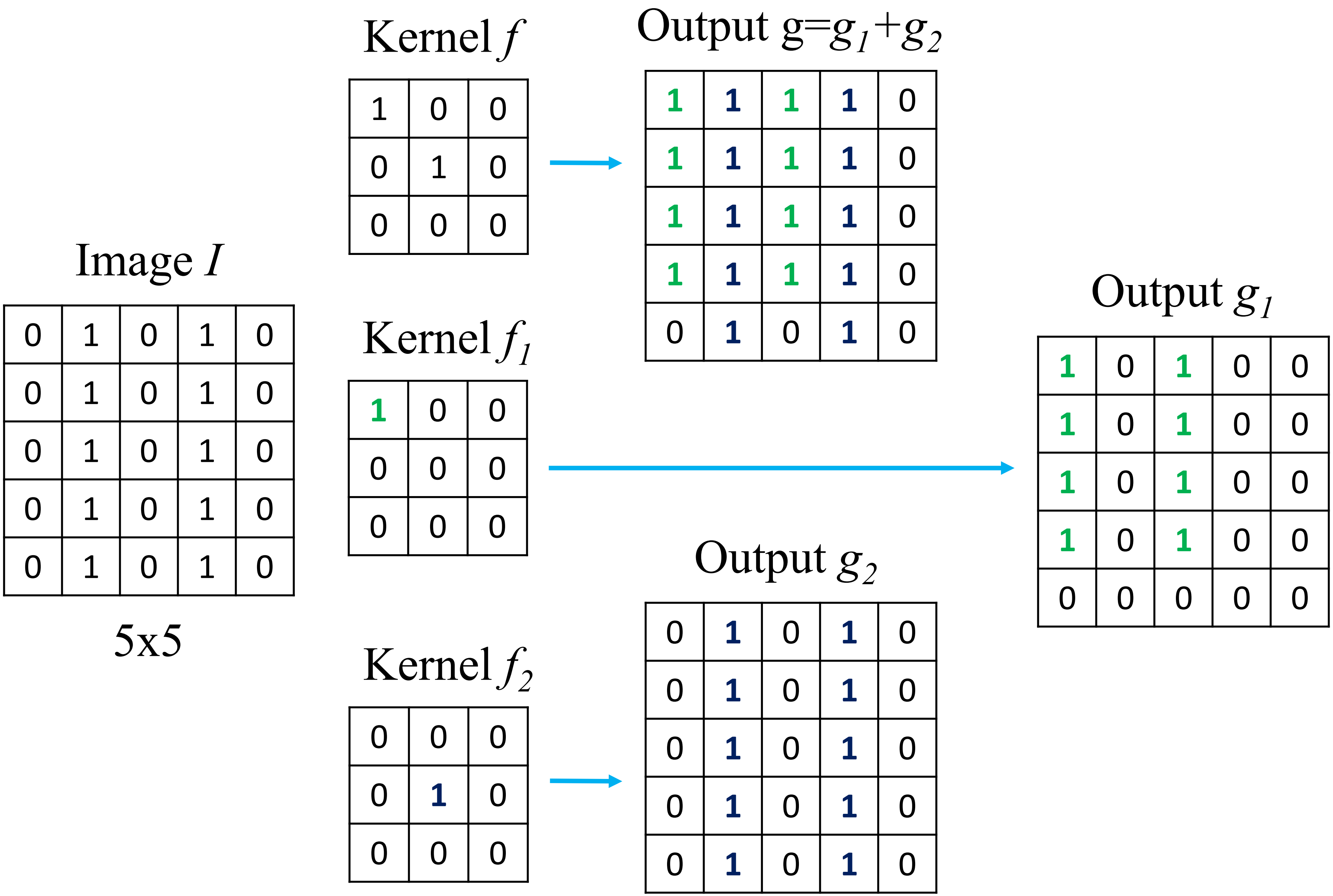
0	0	0
0	0	0
0	0	1

Result $I \otimes f$

e	f	0
h	i	0
0	0	0

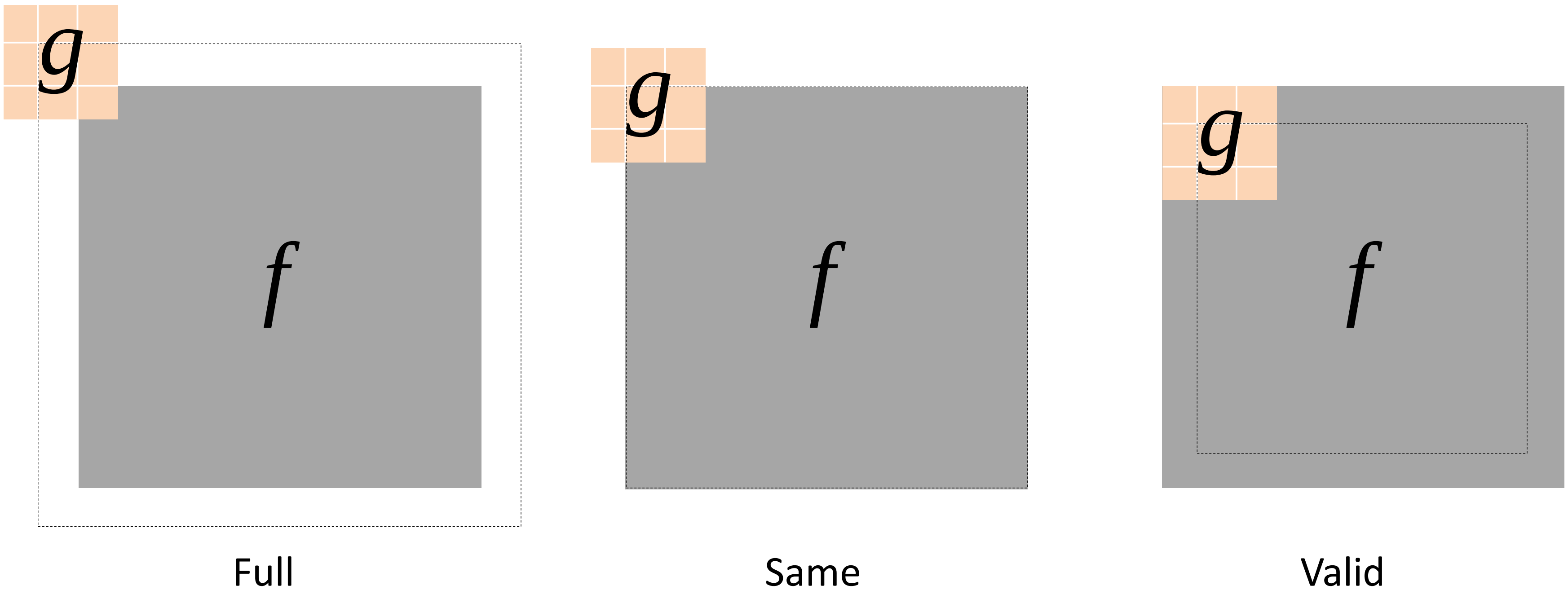
- In this case the resulting image shifted to the upper left

Linear independence



Output Size of Image Convolution

$f \otimes g$



filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

Same: output_size = f_size

Valid: output_size = f_size – (g_size – 1)

2D visualization of convolution (full)

Image *I*

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel *f*

1	2	3
4	5	6
7	8	9

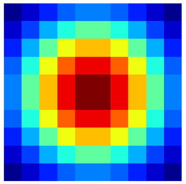
3x3

Output *g*

1	3	6	6	6	6	6	6	5	3
5	12	21	21	21	21	21	21	16	9
12	27	45	45	45	45	45	45	33	18
12	27	45	45	45	45	45	45	33	18
11	24	39	39	39	39	39	39	28	15
7	15	24	24	24	24	24	24	17	9
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

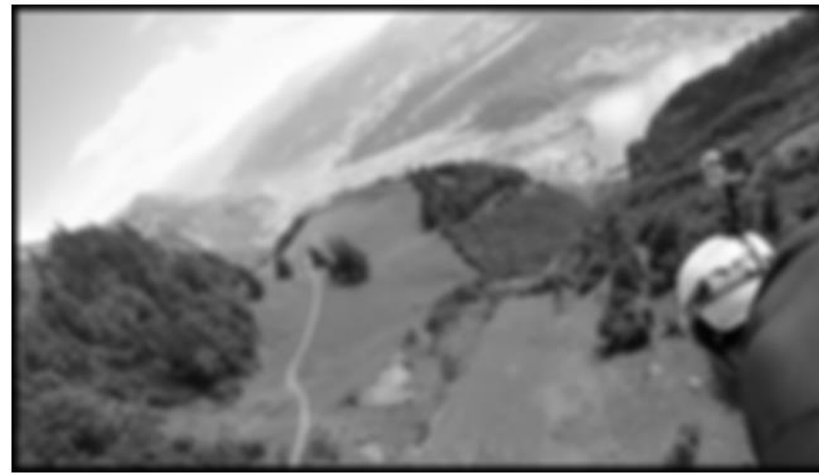
10x10

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Full

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

```
>> full = filter2(g, im, 'full');
>> size(full)

ans =

   369   649
```

2D visualization of convolution (same)

Image I

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel f

1	2	3
4	5	6
7	8	9

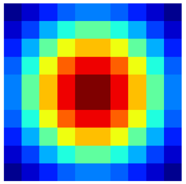
3x3

Output g

12	21	21	21	21	21	21	16
27	45	45	45	45	45	45	33
27	45	45	45	45	45	45	33
24	39	39	39	39	39	39	28
15	24	24	24	24	24	24	17
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

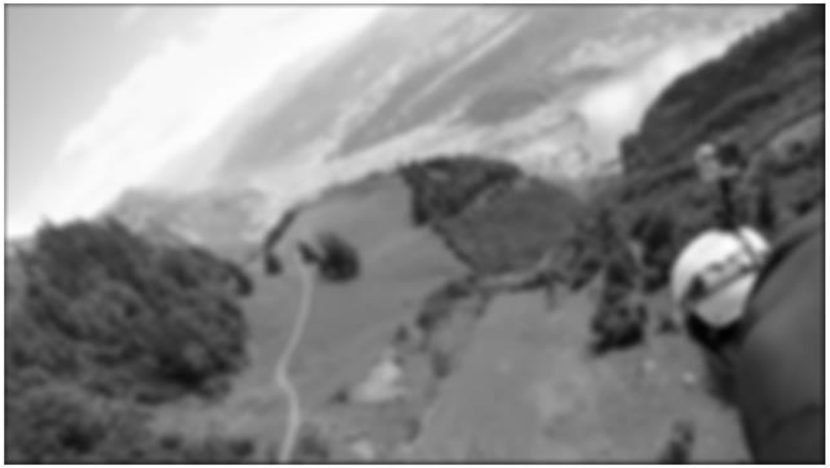
8x8

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Same

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

Same: output_size = f_size

```
>>same = filter2(g, im, 'same');
>> size(same)

ans =

   360   640
```

2D visualization of convolution (valid)

Image I

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel f

1	2	3
4	5	6
7	8	9

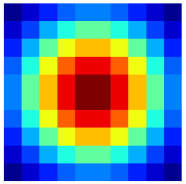
3x3

Output g

45	45	45	45	45	45
45	45	45	45	45	45
39	39	39	39	39	39
24	24	24	24	24	24
0	0	0	0	0	0
0	0	0	0	0	0

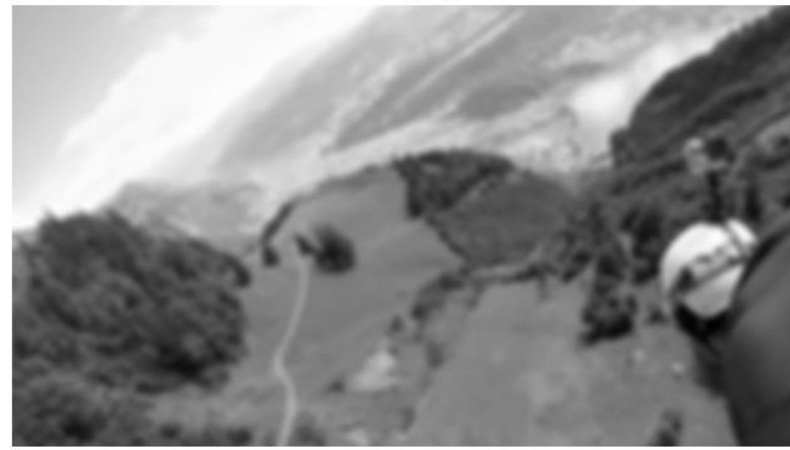
6x6

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Valid

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

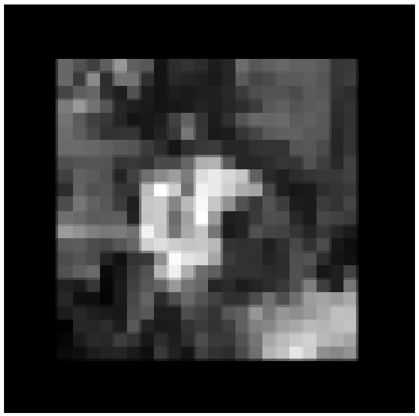
Same: output_size = f_size

Valid: output_size = f_size – (g_size – 1)

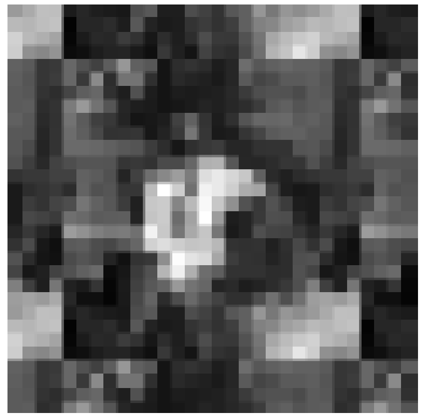
```
>> valid = filter2(g, im, 'valid');
>> size(valid)

ans =

  351   631
```

Image Boundary Effect

The filter window falls off at the edge of image.

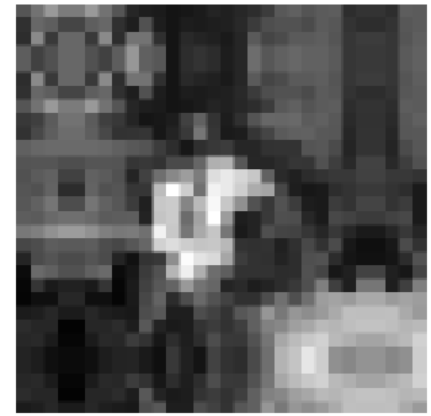


zero

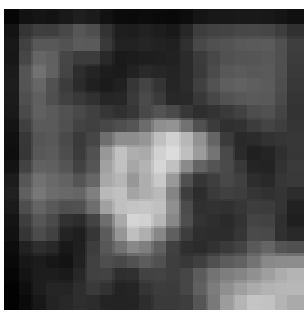


wrap

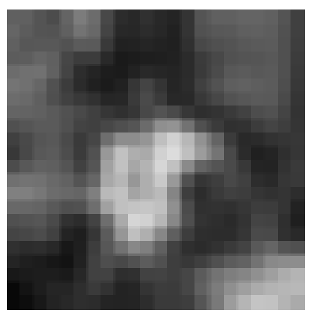
clamp



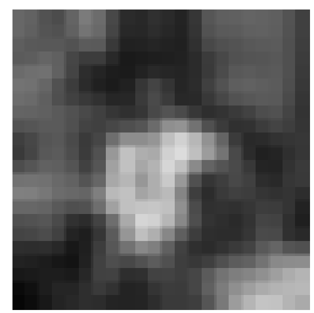
mirror



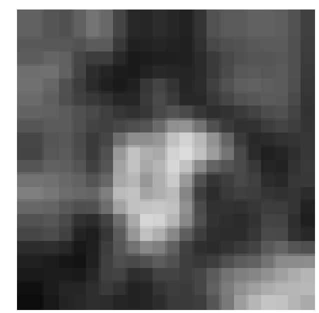
blurred zero



normalized zero



blurred clamp



blurred mirror

Image Extrapolation (Mirroring)

Code

```
J = imread('image.bmp');
figure; imshow(J);
```

J

Image Extrapolation (Mirroring)

Code

```
boarder = 40;
[nr,nc,nb] = size(J);
J_big = zeros(nr+2*boarder, nc + 2*boarder,nb);
J_big(boarder+1:boarder+nr,boarder+1:boarder+nc,:) = J;
```

J_big

Image Extrapolation (Mirroring)

Code

```
for i=1:border,
    for j=1:border,
        J_big(i,j,:) = J(border-i+1,border-j+1,:);
    end
end
```

Mirroring with respect to the board

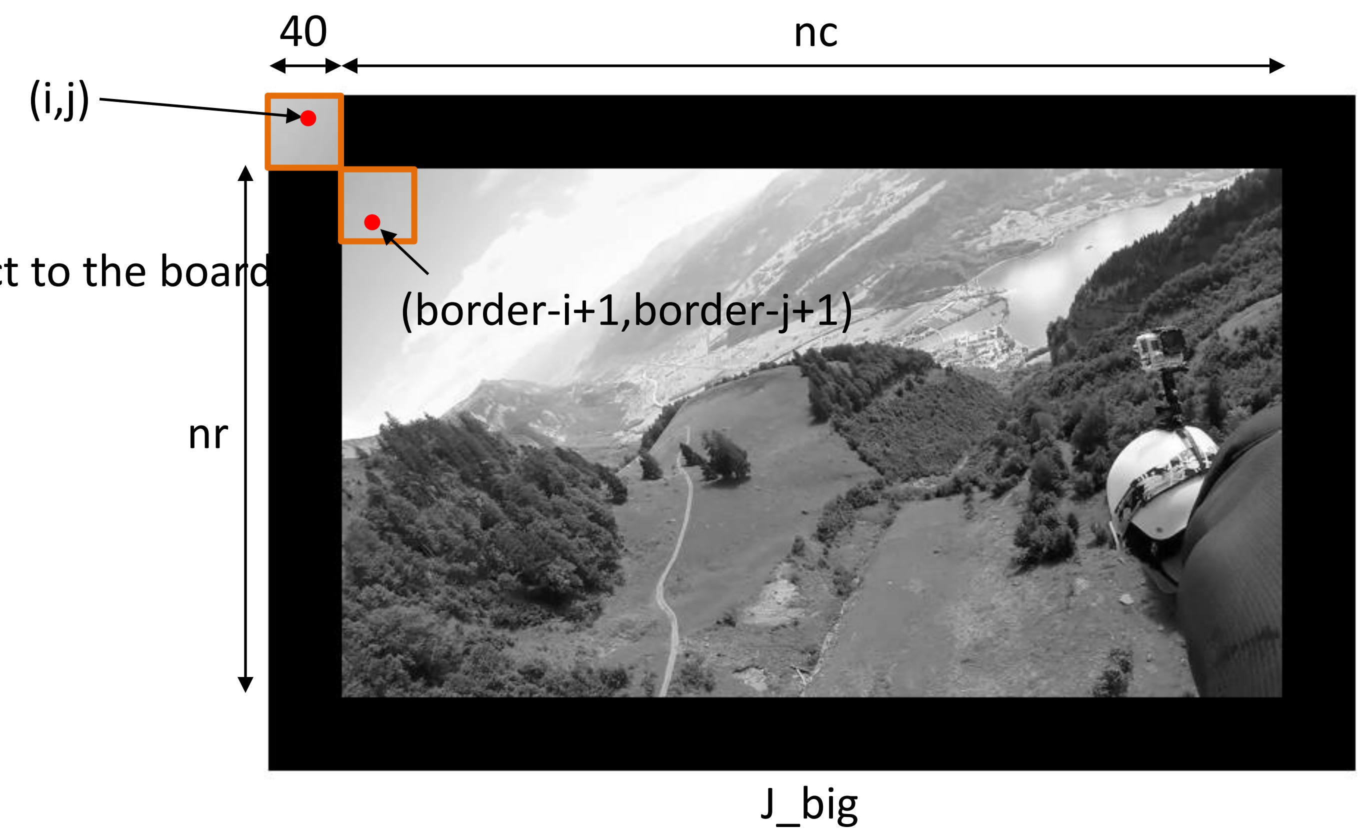


Image Extrapolation (Mirroring)

Code

```
for i=1:boarder,
   for j=border+1:border+nc,
        J_big(i,j,:) = J(border-i+1,j-border,:);
    end
end
```

Mirroring with respect to the board

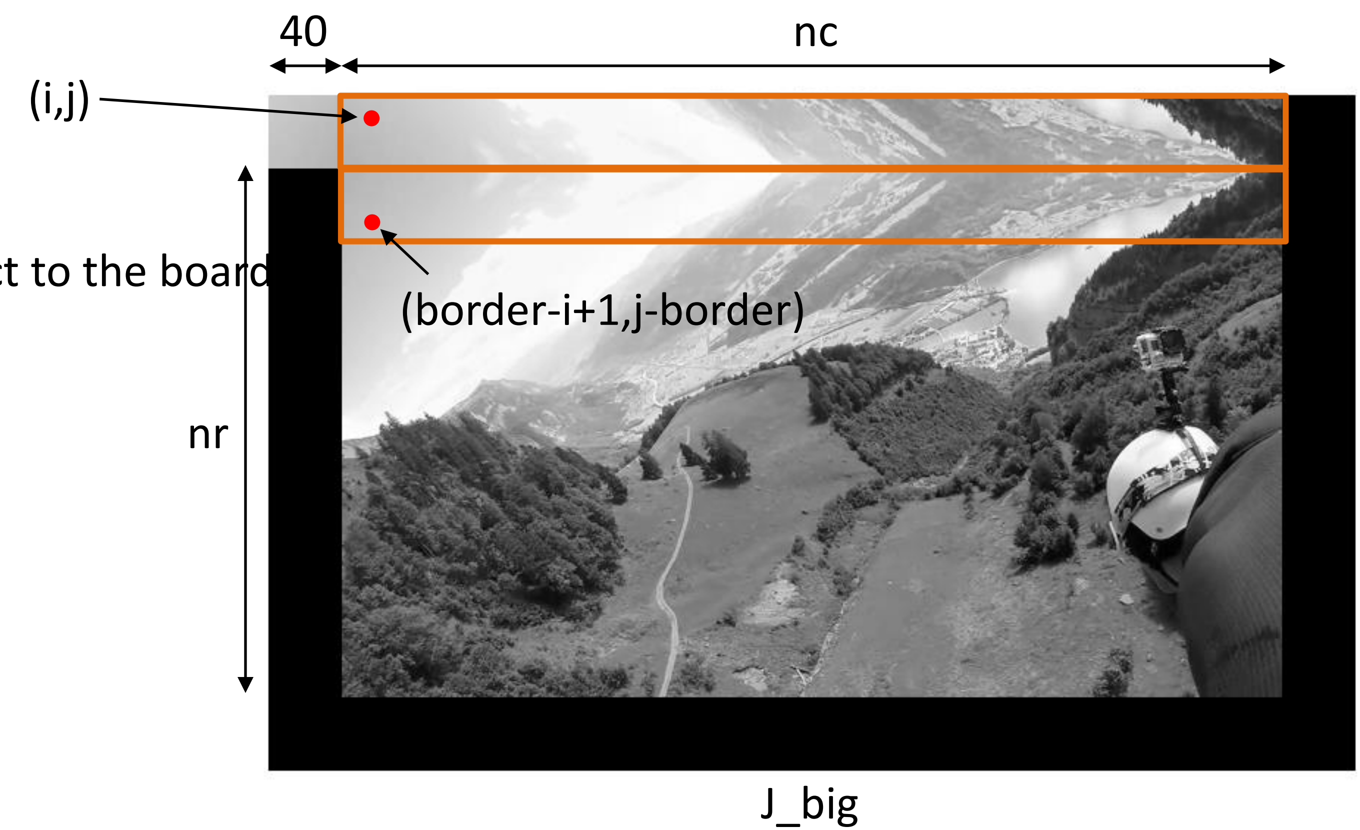


Image Extrapolation (Mirroring)

Code

```
for i=nr+border+1:border*2+nr,
    for j=border+1:border+nc,
        J_big(i,j,:) = J(2*nr-i+border+1,j-border,:);
    end
end
```

Mirroring with respect to the board

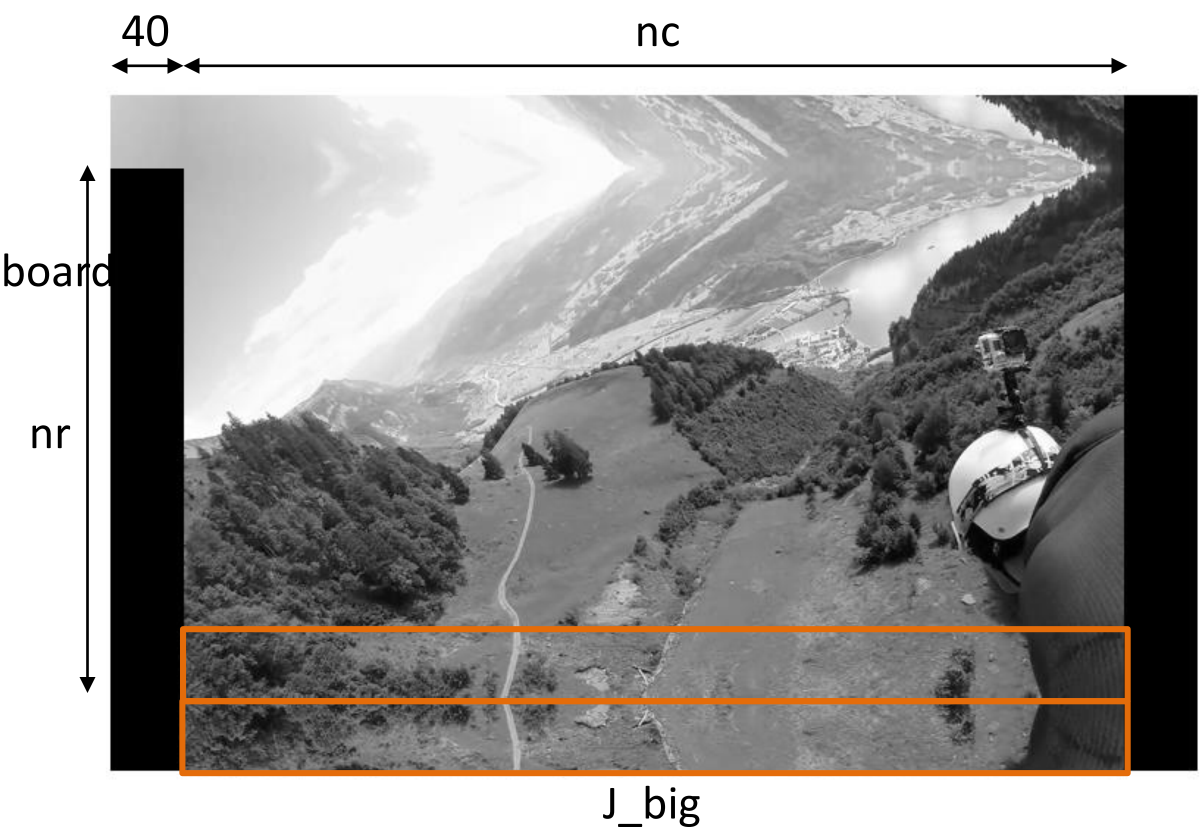
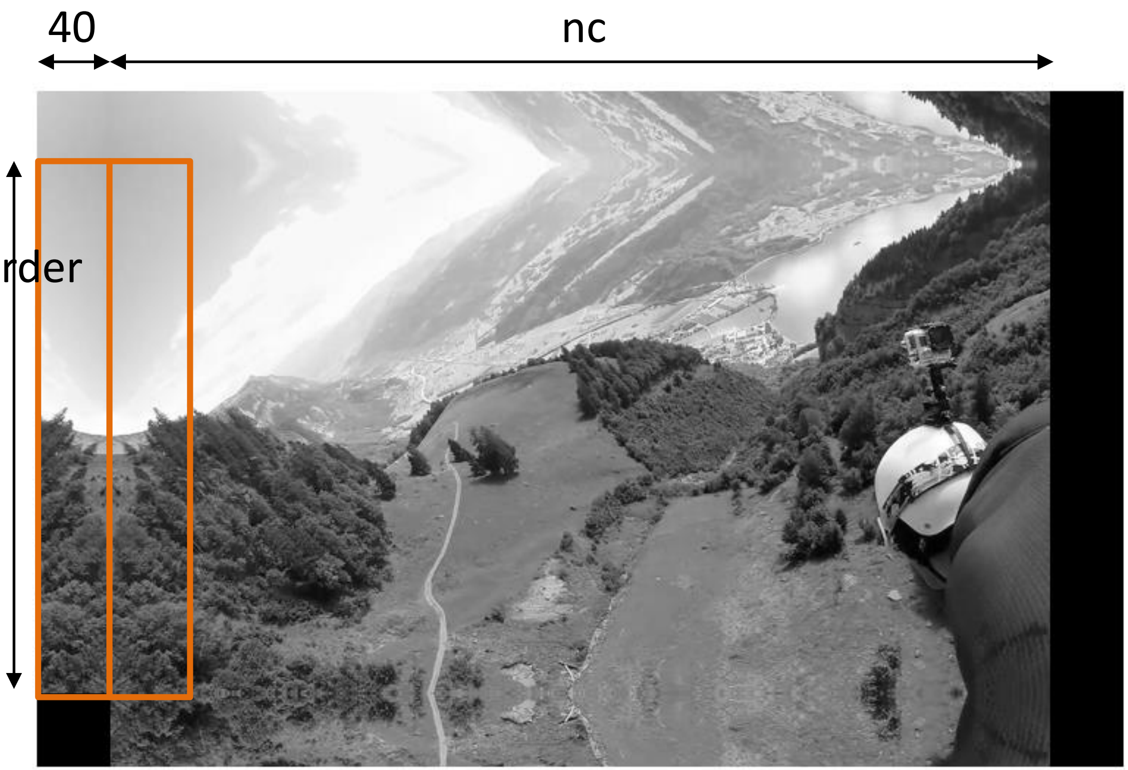


Image Extrapolation (Mirroring)

Code

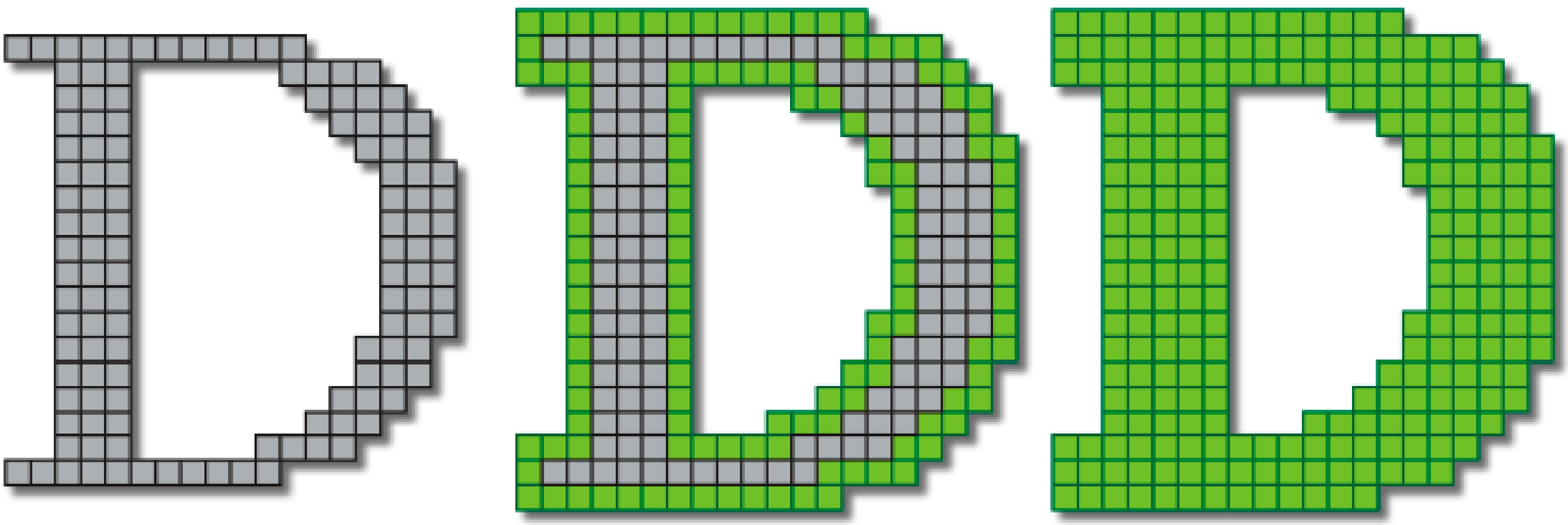
```
for i=border+1:border+nr;
    for j=1:border,
        J_big(i,j,:) = J(i-border,border-j+1,:);
    end
end
```

Mirroring with respect to the boarder



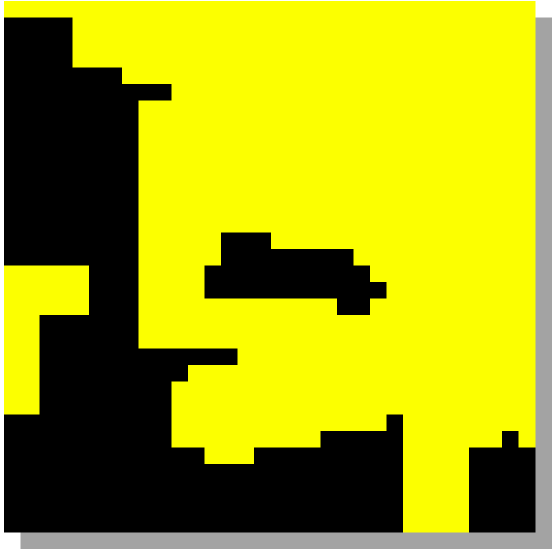
J_big

Dilation

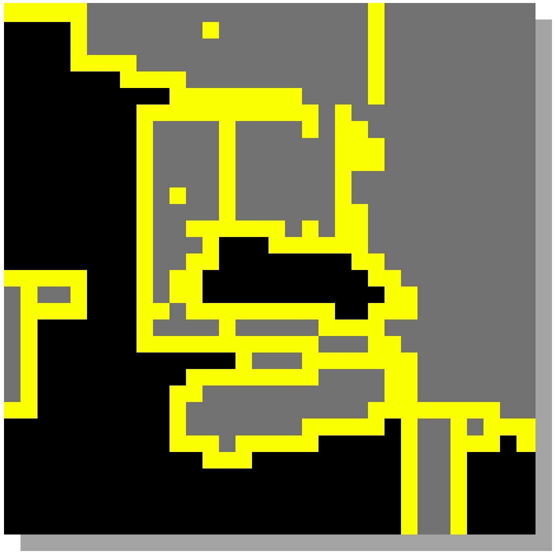


Dilation

The locus of pixels $\mathbf{p} \in S_{\mathrm{p}}$ such that $(\check{Z} + \mathbf{p}) \cap \mathrm{I} \neq \varnothing$.



dilated image

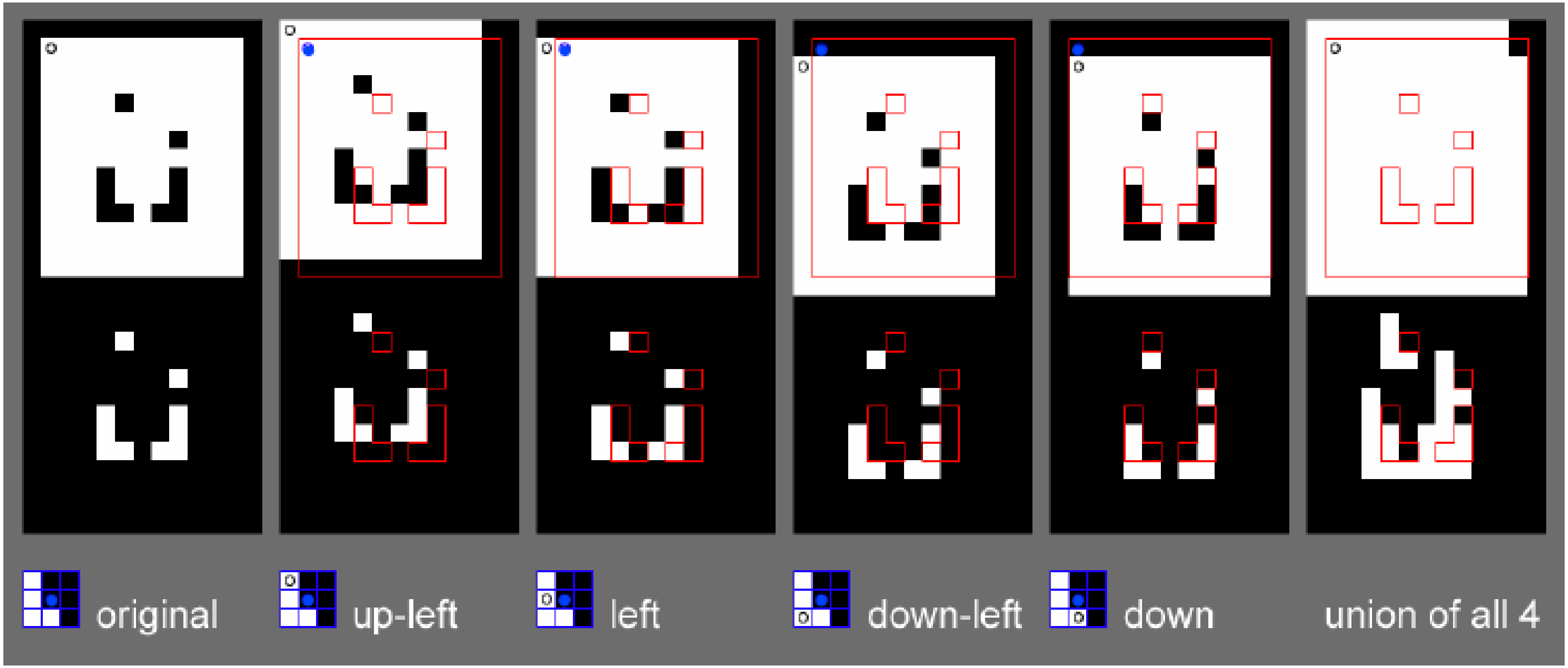


original / dilation

original image

SE = Z_8

Dilation through Image Shifting



Examples of image operation as convolution

Average Filter

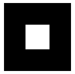
- Mask with positive entries, that sum 1.
- Replaces each pixel with an average of its neighborhood.
- If all weights are equal, it is called a BOX filter.

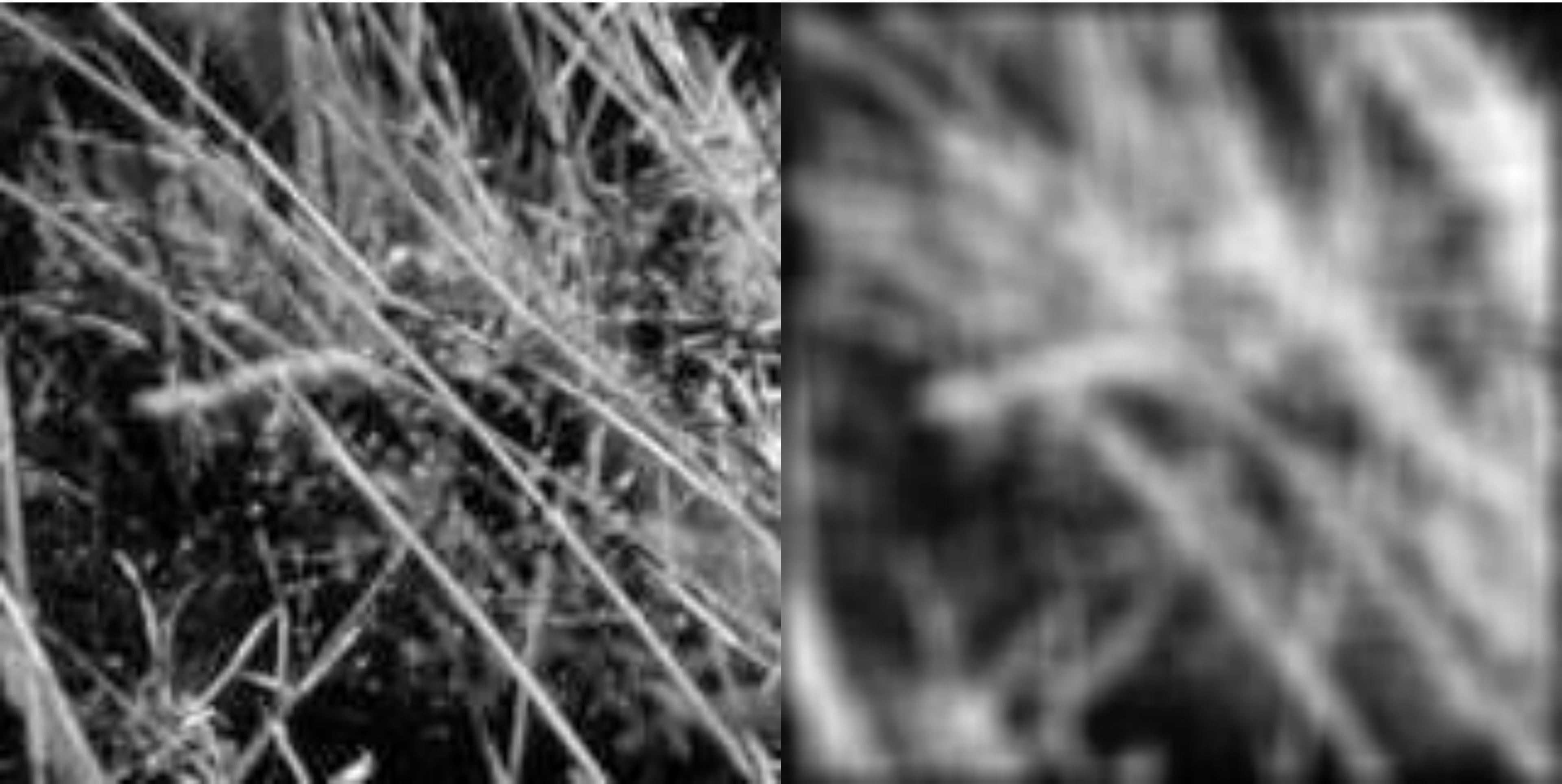
F

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

(Camps)

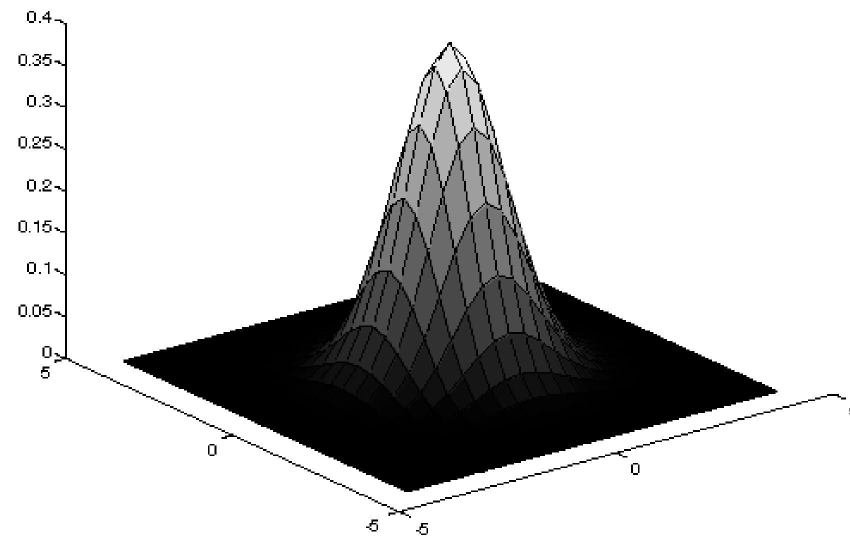
Example 1: Smoothing by Averaging





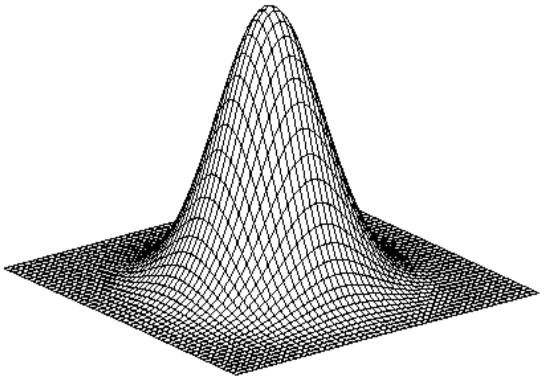
Gaussian Averaging

- Rotationally symmetric.
- Weights nearby pixels more than distant ones.
  - This makes sense as probabalistic inference.



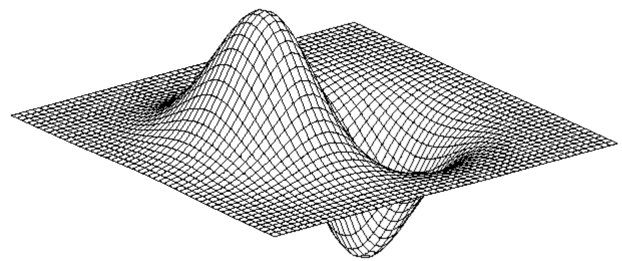
- A Gaussian gives a good model of a fuzzy blob

2D filters, more on this later…



Gaussian

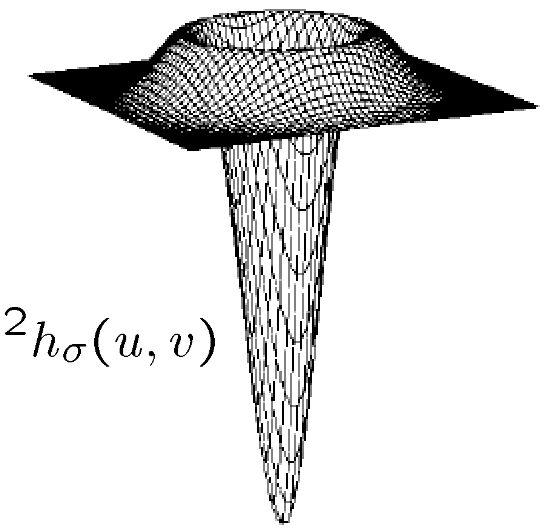
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian

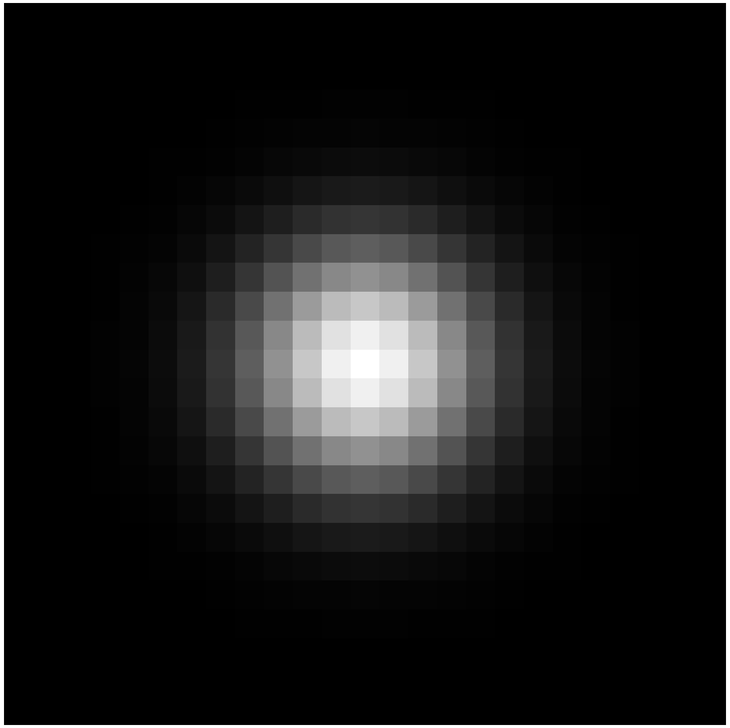


$$\nabla^2 h_\sigma(u,v)$$

- is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

An Isotropic Gaussian

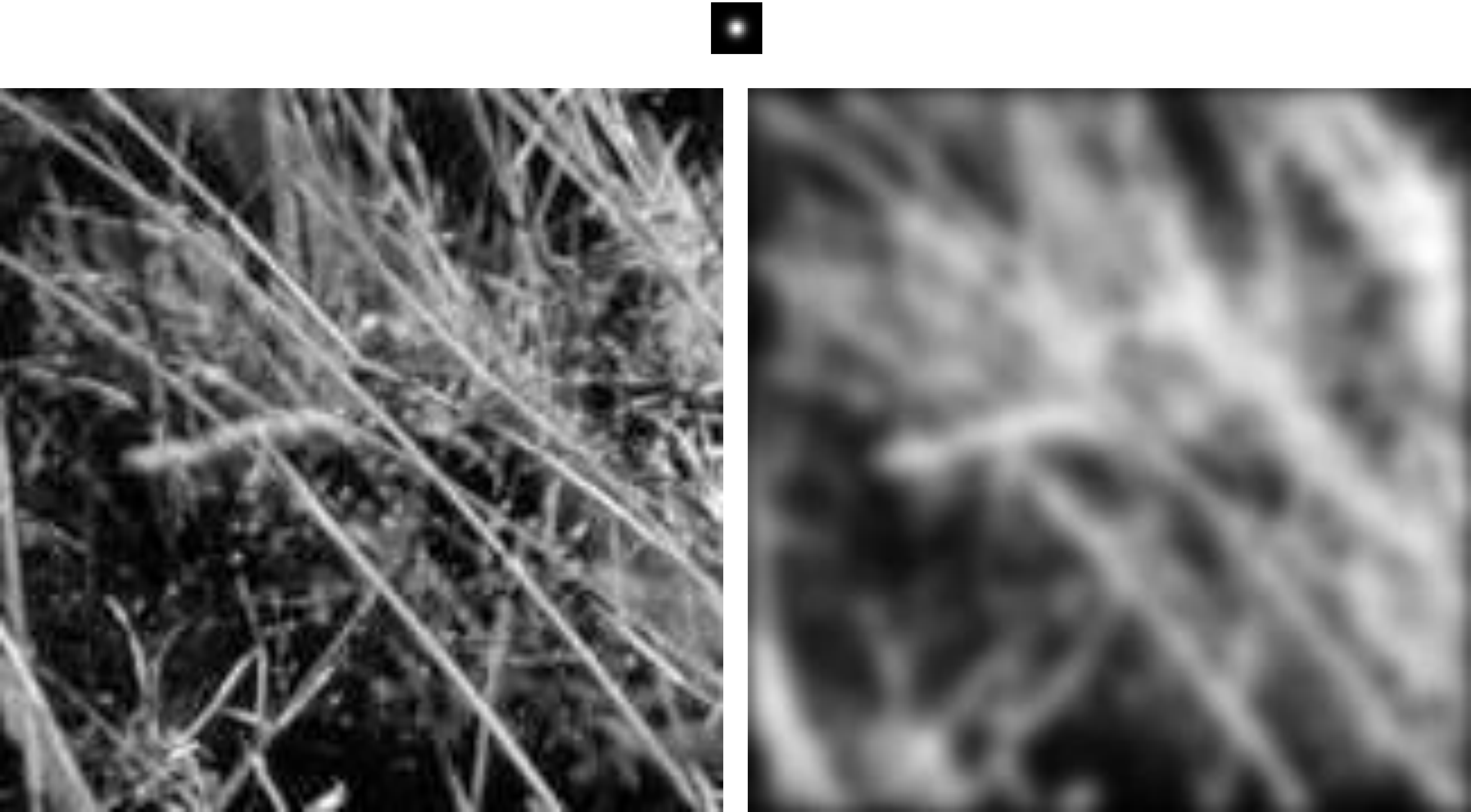


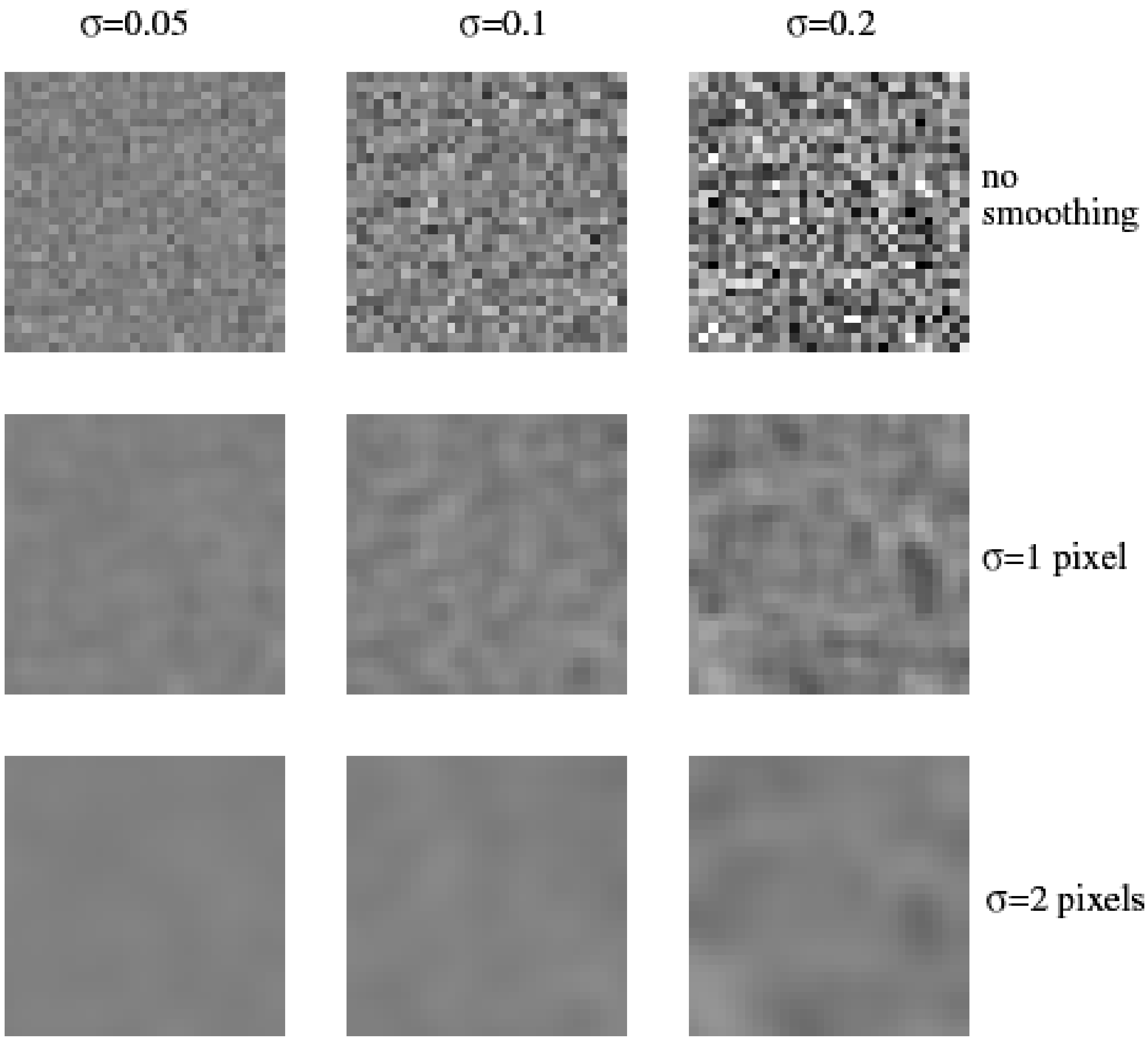
- The picture shows a smoothing kernel proportional to

$$e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- (which is a reasonable model of a circularly symmetric fuzzy blob)

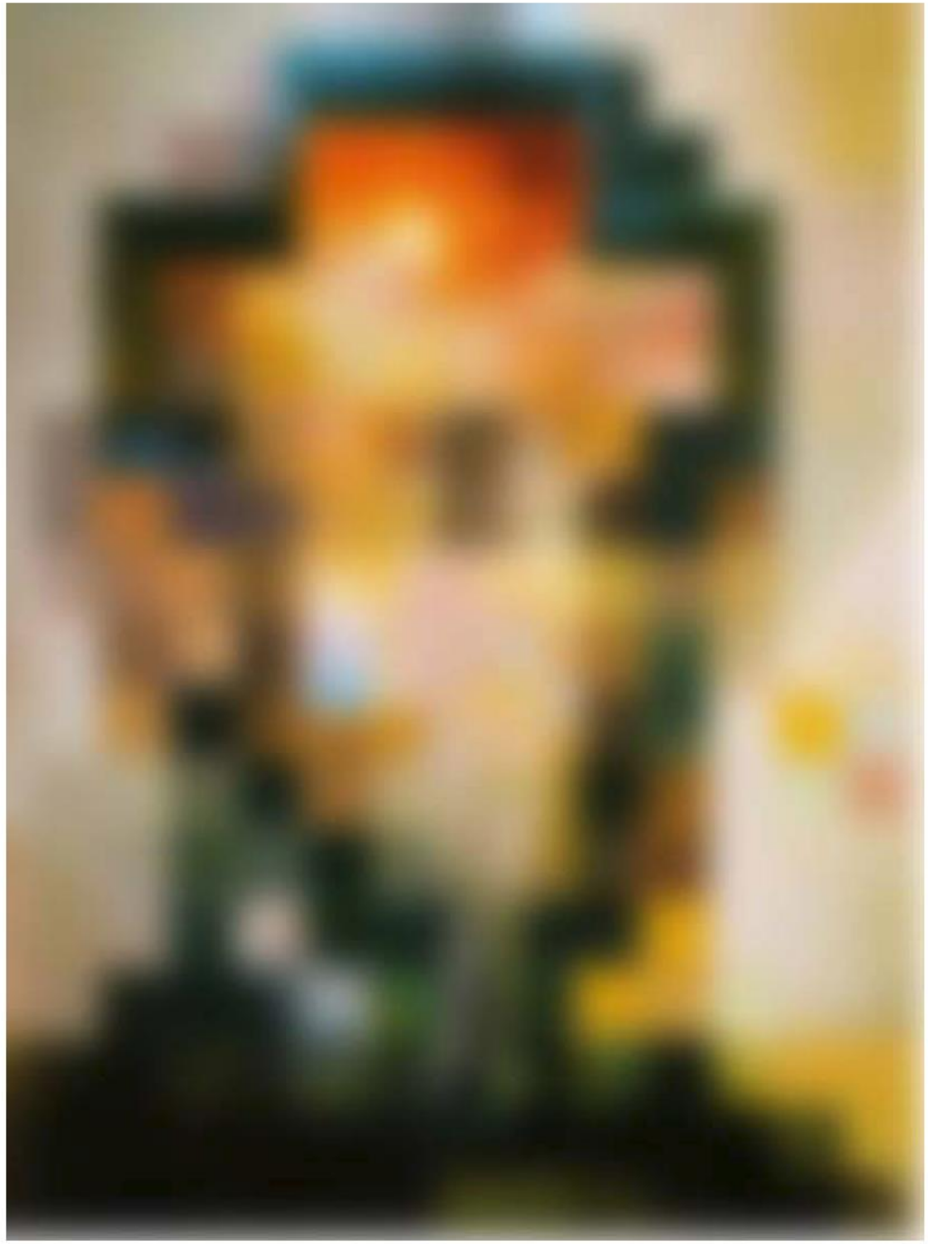
Smoothing with a Gaussian





The effects of smoothing
Each row shows smoothing with gaussians of different width; each column shows different realizations of an image of gaussian noise.

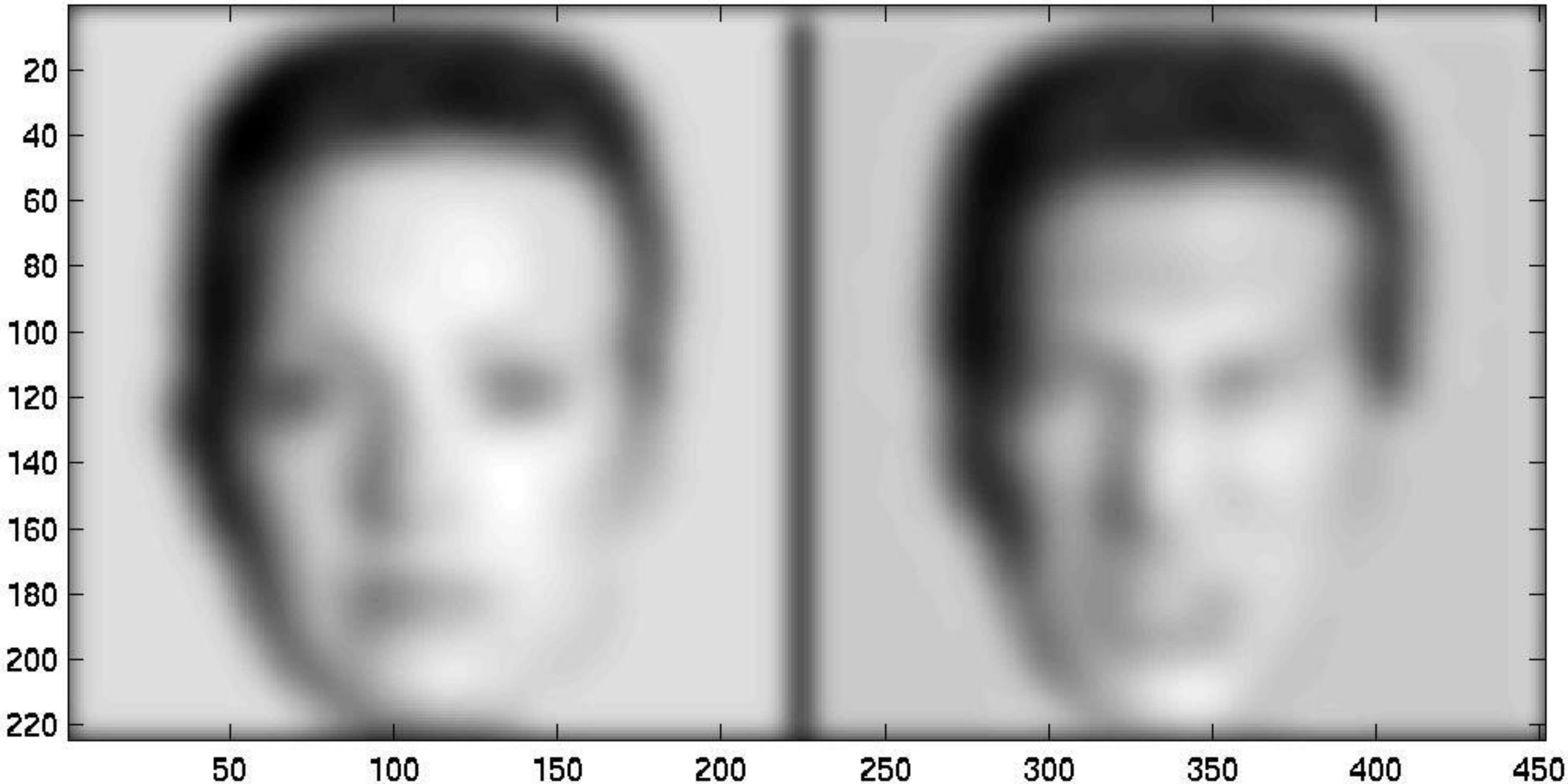
Salvador Dali, *"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln*", 1976



Salvador Dali, *“Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln*”, 1976

Image smoothing can remove noise, and also …


20
40
60
80
100
120
140
160
180
200
220
50
100
150
200
250
300
350
400
450



20
40
60
80
100
120
140
160
180
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